

Spectroscopic determination of the internal amplitude of frequency sweeping TAE

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Abstract

From an understanding of the processes that cause a marginally unstable eigenmode of the system to sweep in frequency, it is shown how the absolute peak amplitude of the mode can be determined from the spectroscopic measurements of the frequency sweeping rate, e.g. with Mirnov coils outside the plasma. In a first attempt to implement such a diagnostic calculation, the MISHKA code (Mikhailovskii A B *et al* 1997 *Plasma Phys. Rep.* **23** 844) is used to determine the global mode structure of toroidal Alfvén eigenmodes (TAEs) (Cheng C Z *et al* 1985 *Ann. Phys. (NY)* **161** 21) observed in the MAST spherical tokamak (Sykes A *et al* 2001 *Nucl. Fusion* **41** 1423). Simulations using the HAGIS code (Pinches S D 1996 *PhD Thesis* The University of Nottingham, Pinches S D *et al* 1998 *Comput. Phys. Commun.* **111** 131) are then made, replicating the experimentally observed sweeping phenomena. The fundamental theory is then used together with these simulation results to predict the internal field amplitude from the observed frequency sweeping. The calculated mode amplitude is shown to agree with that obtained from Mirnov coil measurements.

1. Introduction

Comparisons between numerical models and present day tokamak devices rely upon accurate diagnostic measurements of the key plasma parameters. In assessing the effect of energetic

⁴ See annexe 1 of Paméla J *et al* 2002 Overview of recent JET results, OV-1/1.4 *Proc. 19th International Conf. on Fusion Energy 2002 (Lyon, 2002)* (Vienna: IAEA).

particle-driven instabilities upon their confinement, it is extremely important to measure the amplitude of internal modes, something that is generally difficult to do accurately. In this paper, we demonstrate how from the measured spectrum of frequency sweeping that has been observed in the excitation of toroidal Alfvén eigenmodes (TAEs), it may be possible to calculate the absolute value of the amplitude of these modes. In addressing the case of global modes in the complex geometry of real tokamak plasmas, this necessitates the use of advanced numerical simulations to replace theoretical estimates.

The numerical simulations reported in this paper have been performed using the HAGIS code [4, 5]. This is a nonlinear drift-kinetic Hamiltonian δf code that models the interaction between a distribution of energetic particles and a set of Alfvén eigenmodes to obtain the linear growth rates, γ_L , and the nonlinear behaviour of the mode amplitudes and fast ion distribution function determined by kinetic wave–particle nonlinearities. The code uses realistic plasma geometry and allows for arbitrary particle distributions. The waves are described by their linear eigenfunctions that are calculated by the MISHKA code [1]. In HAGIS the mode varies in amplitude and phase in response to the energetic particles. Through the simulation of marginally unstable TAEs, the HAGIS code demonstrates self-consistent frequency sweeping, with the sweeping rate found to be $\approx 0.4\gamma_L(\gamma_d t)^{1/2}$, where γ_d is the mode damping rate, in agreement with theoretical predictions [6–8].

In this paper, the results obtained from these numerical simulations are presented together with the experimental observations for direct comparison. In particular, from a comparison of the frequency sweeping observed on MAST [3] with theoretical and numerical simulation predictions, an attempt is made to infer the mode amplitudes in the plasma core. The theory has produced an accurate description of the frequency sweeping process, and this analysis is the first attempt at developing a procedure to obtain an absolutely calibrated measurement of the internal mode amplitude.

2. Frequency sweeping

Numerical simulations of marginally unstable bump-on-tail kinetic instabilities [6] have shown how a hole and clump spontaneously appear in the particle distribution function and how this process supports a set of long-lived Bernstein–Greene–Kruskal (BGK) nonlinear waves that shift up and down in frequency. A similar nonlinear kinetic process of hole-clump production also occurs for modes driven by the radial gradient of the fast ion pressure, such as TAEs. This mechanism is a primary candidate to explain the fast frequency sweeping observed in experiment. Figure 1 shows such an experimental example of frequency sweeping nonlinear waves in the JET tokamak [9]. In this case, the frequency sweeps $\pm\delta\omega/\omega_0 \sim 5\%$ in $\delta t \sim 1$ ms, where $\omega_0 \approx v_A/2qR_0$ the TAE’s natural frequency and v_A is the Alfvén speed, q the safety factor and R_0 is the major radius at the magnetic axis.

Theoretical analysis and numerical studies [6, 7] have shown that plasma modes destabilized by kinetic drives can induce frequency sweeping. The sweeping results from the spontaneous excitation of phase-space structures when the system is near to marginal instability, i.e. when $|\gamma_L - \gamma_d| \ll \gamma_L$, and the phase-space transport of these structures that is needed to balance the power being extracted from the nonlinear waves due to the dissipation that is present in the system. Interestingly, particle simulation has shown that these structures do not spontaneously emerge in a strongly unstable system. The rate at which the frequency sweeps is determined by the wave–particle nonlinearity that relates the frequency shift $\delta\omega$ to the nonlinear bounce frequency ω_b of a particle trapped in the potential well of the wave.

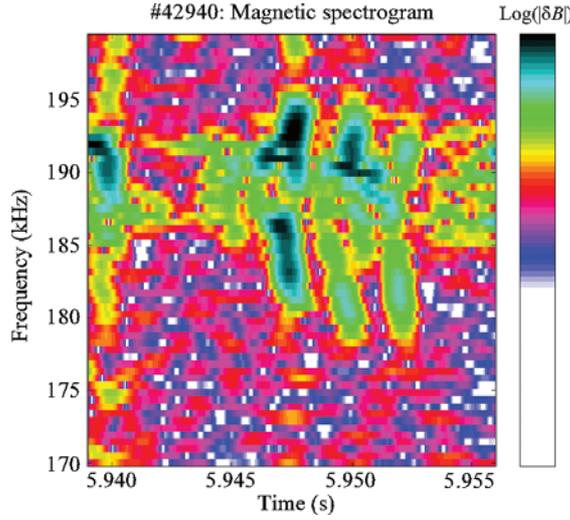


Figure 1. Example of frequency sweeping mode during a shear optimized D–T pulse in the JET tokamak. In this case, $\delta\omega/\omega_0 \sim 5\%$ in $\delta t \sim 1$ ms.

A simplified model of the process [6] gives the scaling as

$$\delta\omega = C_1 \left(\frac{\gamma_d}{\gamma_L} \right)^{1/2} \omega_b^{3/2} \delta t^{1/2} \quad (1)$$

$$\omega_b = C_2 \gamma_L \quad (2)$$

with C_1 and C_2 being constants of proportionality ($C_1 = \pi/(2\sqrt{2}) \approx 1$ and $C_2 = 16/3\pi^2 \approx 0.5$). More general simulations show that the estimates for C_1 and C_2 remain within 10% of these values when phase-space structures spontaneously form in the electrostatic problem [6,7] and when modelling TAE excitations in a tokamak [8]). As phase-space structures only spontaneously form when $\gamma_L \sim \gamma_d$, and as the rate of frequency sweeping is proportional to $(\gamma_L/\gamma_d)^{1/2}$, it is observed that the internal field amplitude and the rate of frequency sweeping are insensitive to the closeness to marginal stability. The insensitivity to $\gamma_L - \gamma_d$ means that the saturated nonlinear state has ‘forgotten’ that the system started near marginal instability and, if the identification for the mechanism for frequency sweeping is correct, then the recording of the time scale of the frequency sweeping determines the bounce frequency of trapped particles. In turn, the bounce frequency depends upon the mode amplitude through the relation,

$$\omega_b = C_3 A^{1/2} \quad (3)$$

where, here, the mode amplitude is defined in relation to the toroidal magnetic field on-axis, B_0 , as, $A = \delta B_r/B_0$ and C_3 is another constant of proportionality that generally has to be determined numerically (as discussed here). However, for the case of TAE in a large aspect ratio circular cross-section tokamak, C_3 may be estimated from the relationship [10],

$$\omega_b \cong \left| \omega - n \frac{v_{\parallel}}{R_0} \right| \left(\frac{S}{\omega r} \frac{2v_A}{m} \frac{\delta B_r}{B_0} \right)^{1/2} \quad (4)$$

where m , n are the poloidal and toroidal mode numbers, respectively, S is the magnetic shear at the position of the mode, r , v_{\parallel} and v_A are the parallel velocity of the resonant particles and the Alfvén speed, and B_0 and δB_r are the equilibrium magnetic field and the radial component of the perturbed magnetic field.

The time evolution of the frequency sweep described by equation (1), $\delta\omega \sim \delta t^{1/2}$, follows from the theory which shows that the mode amplitude, A , and thus the nonlinear bounce frequency, ω_b , remains constant in time as long as the frequency shift (which is assumed larger than the linear growth rate) is not too large as to break some of the underlying theoretical assumptions. Collisionality, which we neglect in this work, can be expected to cause the phase-space structures to collapse and lead to a decrease in mode amplitude. However, work done including such effects [6] shows that the frequency shift still follows the $\delta t^{1/2}$ scaling even when the mode amplitude decays.

Combining equations (1) and (3) provides an expression for how the absolutely calibrated mode amplitude in the core of the plasma can be derived from the observed sweeping rate of the TAE frequency,

$$\frac{\delta B_r}{B_0} = \frac{1}{C_3^2} \left(\frac{\gamma_L}{\gamma_d} \right)^{2/3} \left(\frac{\delta\omega^2}{C_1^2 \delta t} \right)^{2/3} \quad (5)$$

Whilst the analytical estimates for C_1 and C_3 may be used to give rough estimates of the mode amplitudes, numerical simulations are necessary to obtain accurate values.

3. Numerical modelling

Frequency sweeping occurs close to marginal stability, i.e. when the competing effects of fast ion drive and damping almost balance, such that the mode is marginally unstable. This situation has been numerically modelled in the HAGIS code [4, 5] by introducing an additional external damping mechanism, represented by the damping rate, γ_d . The equations that determine the wave evolution have been found to be

$$\begin{aligned} \dot{X}_k &= \frac{1}{2E_k} \sum_{j=1}^{n_p} \delta f_j \Delta \Gamma_j^{(p)} \sum_m (k_{\parallel m} v_{\parallel j} - \omega_k) S_{jkm} + X_k \gamma_d \\ \dot{Y}_k &= -\frac{1}{2E_k} \sum_{j=1}^{n_p} \delta f_j \Delta \Gamma_j^{(p)} \sum_m (k_{\parallel m} v_{\parallel j} - \omega_k) C_{jkm} + Y_k \gamma_d \end{aligned} \quad (6)$$

where X_k and Y_k represent the real and imaginary components of the k th wave's amplitude, S_{jkm} and C_{jkm} represent the real and imaginary parts of the scalar potential of the m th harmonic of the k th wave evaluated at the j th marker position. j is the summation index over numerical markers and m is the summation over poloidal harmonics.

Simulations are initially performed with $\gamma_d = 0$ to obtain the mode's linear growth rate, γ_L , without extrinsic dissipation. This then allows the selection of $\gamma_d \approx \gamma_L$, so that the system is near marginal instability. Then the simulations are repeated, but now near marginal instability, where conditions are favourable for the demonstration of spontaneous frequency sweeping.

As a first example, we consider the case of a circular cross-section plasma with an inverse aspect ratio, $a/R_0 = 0.3$ and a monotonically increasing safety factor from an on-axis value of $q_0 = 1.1$ to an edge value, $q_a = 3.5$ (as shown in figure 2). A linear eigenmode analysis of this equilibrium using the MISHKA code and the HAGIS code to simulate the case of a distribution of α -particles reveals that it is unstable to an $n = 3$ core-localized TAE mode shown in figure 3. The kinetic drive for this mode was provided by a centrally peaked slowing-down distribution of α -particles shown in figure 4 and described by

$$f(s, E) = C \left(\frac{1}{\exp[(\psi - \psi_0)/\Delta\psi] + 1} \right) \frac{1}{E^{3/2} + E_c^{3/2}} \text{Erfc} \left[\frac{E - E_0}{\Delta E} \right] \quad (7)$$

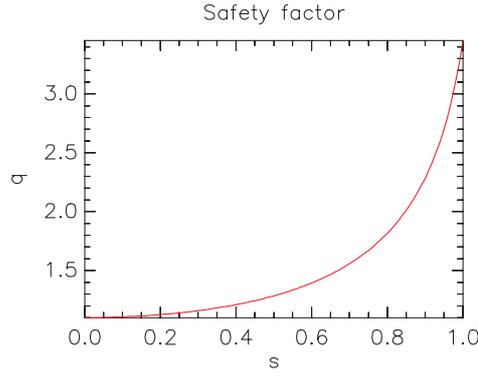


Figure 2. Safety factor (q) profile for a circular cross-section plasma with inverse aspect ratio, $a/R_0 = 0.3$. The radial co-ordinate s is defined through the normalized poloidal flux label as $s = \sqrt{\psi}$.

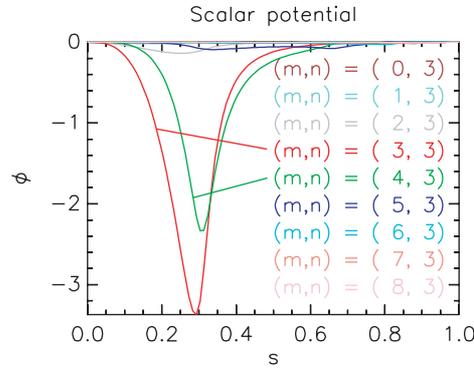


Figure 3. Radial mode structure of core-localized $n = 3$ TAE in a circular cross-section plasma.

where $s = \sqrt{\psi}$, ψ is the normalized poloidal flux and E is the energy. $\psi_0 = 0.1$, $\Delta\psi = 0.07$, $E_0 = 3.5$ MeV, $E_c = 494$ keV and $\Delta E = 410$ keV. The constant C was chosen by specifying the volume averaged fast particle beta, $\langle\beta_f\rangle = 3 \times 10^{-4}$. The background plasma density was chosen to be $2 \times 10^{19} \text{ m}^{-3}$ with a 50:50 D:T isotope mix meaning that an α -particle with energy 3.5 MeV has a velocity $1.234v_A$. In the modelling, all the particles had $\lambda = v_{\parallel}/v = 1$, i.e. were deeply co-passing.

In the absence of any additional external damping mechanisms, $\gamma_d = 0$, a simulation using 52 500 markers determines the linear growth rate for this mode to be $\gamma_L/\omega_0 = 0.027 \pm 0.002$. It is found that the mode saturates at an amplitude of $\delta B_r/B_0 \approx 10^{-3}$ as shown in figure 5. The mode is then made marginally unstable by adding an artificial external damping mechanism such that $\gamma_d/\omega_0 = 0.02$ and the simulation is repeated with 210 000 markers and run for 80 inverse growth times which equates to around 500 wave periods. In this case, the mode amplitude saturates at a much lower constant level, $\delta B_r/B_0 \approx 10^{-4}$ as shown in figure 6 and in agreement with the level predicted by the theory that is applied in the next section ($C_3 = 2.44 \times 10^6 \text{ radians s}^{-1}$ giving $\delta B_r/B_0 = 0.095(\gamma_L/\omega)^2$ for this case). Note that fluctuations of the mode from outside the frequency band of the sweeping make the raw $\delta B_r/B_0$ slightly larger than what the theory predicts. A Fourier spectrogram of the evolving mode reveals a predominantly down-shifted frequency sweeping branch as shown in figure 7.

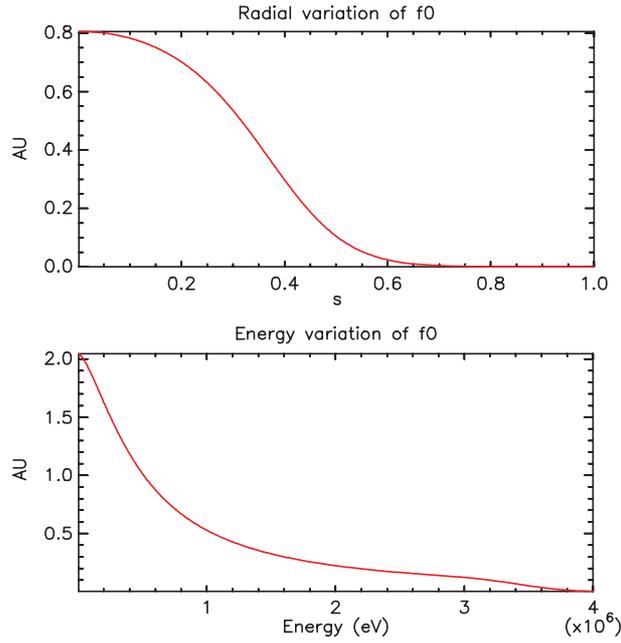


Figure 4. Distribution of α -particles used in simulations: centrally peaked in radius and slowing down in energy from a D–T fusion birth energy of 3.5 MeV.

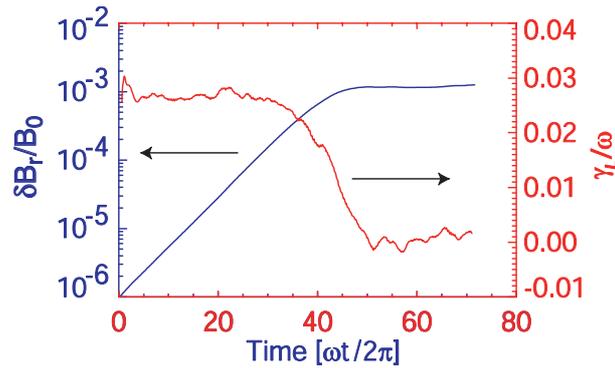


Figure 5. Evolution of mode amplitude (left-hand scale) and instantaneous mode growth rate (right-hand scale) calculated from changing mode amplitude. A linear growth phase is clearly seen up to around 40 wave periods whereupon a saturated final state is reached.

The reduced amplitude of the up-shifting frequency branch is attributed to the fact that the primary resonance, $v_{\parallel} = v_A$, is very close to the birth energy of the α -particles and there are, therefore, significantly less resonant particles at higher energies to drive the mode as it shifts up in frequency. Thus, in this case, a clump in the fast ion distribution was produced without a hole. The over-plotted white line in figure 7 is that of the theoretically predicted frequency sweeping, $\delta\omega = 0.4\gamma_L(\gamma_{dt})^{1/2}$, showing good agreement with the simulation for the frequency shift of the hole. As can be seen, it takes a certain amount of time, $\gamma_L t \approx 20$, for the adiabatic structure to emerge from its initial excitation, with the frequency shift's time evolution becoming clearer at later times. Nevertheless, the agreement with the theory is good.

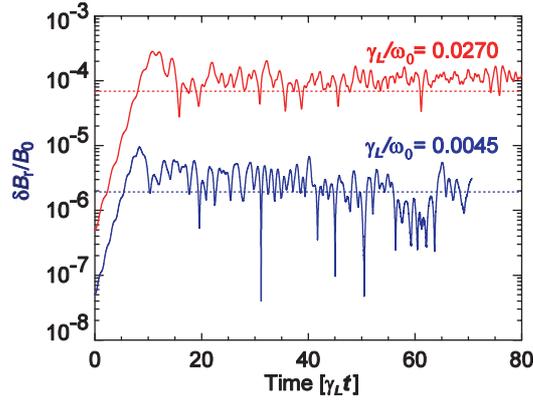


Figure 6. Evolution of mode amplitude for the cases $\gamma_L/\omega_0 = 0.027$, $\gamma_d/\omega_0 = 0.02$ and $\gamma_L/\omega_0 = 0.0045$, $\gamma_d/\omega_0 = 0.004$. The horizontal dashed lines are the theoretically predicted values of $\delta B_r/B_0$.

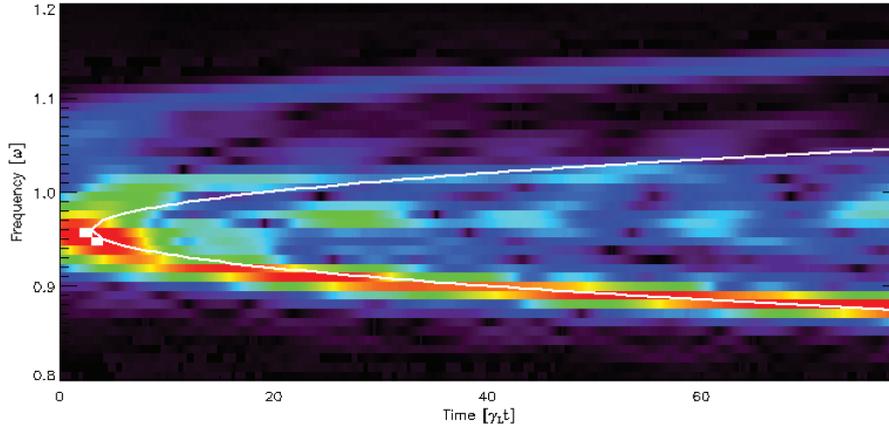


Figure 7. Sliding Fourier spectrum showing frequency evolution of marginally unstable TAE mode in response to kinetic α -particle drive ($\gamma_L/\omega_0 = 0.027$) and external damping ($\gamma_d/\omega_0 = 0.02$). The over-plotted white line shows the theoretically predicted frequency shift of $0.4\gamma_L(\gamma_d t)^{1/2}$.

Reducing the kinetic α -particle drive by reducing their pressure so that $\langle\beta_f\rangle = 7.5 \times 10^{-5}$ results in a linear growth rate of $\gamma_L/\omega_0 = (4.5 \pm 0.5) \times 10^{-3}$. Introducing an external damping rate, $\gamma_d/\omega_0 = 4 \times 10^{-3}$ makes this mode marginally unstable and again leads to a frequency sweeping regime but with smaller $\delta\omega$ as shown in figure 8. The over-plotted white line is again the predicted frequency sweeping and shows good agreement with the theory for the frequency sweeping of the hole.

4. Results

In this section the HAGIS code is used to determine the mode amplitude of a frequency sweeping global mode in the analytically demanding geometry of the tight aspect-ratio MAST tokamak. The experimentally observed frequency sweeping measured with magnetic coils near the edge of the plasma boundary is shown in figure 9. A mode number analysis using a

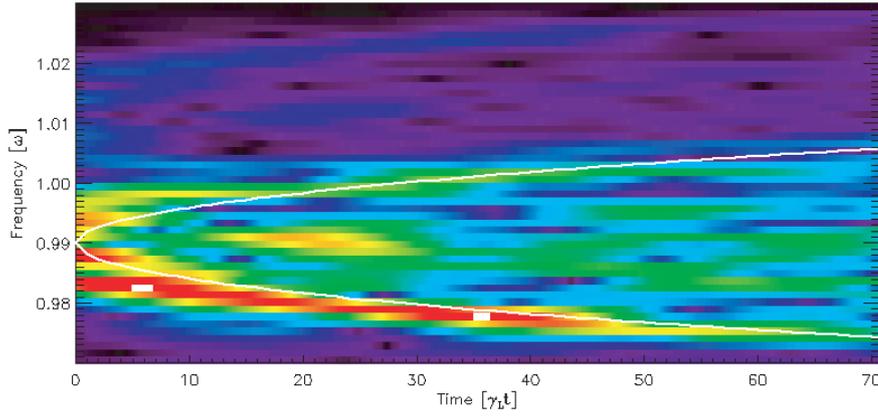


Figure 8. Sliding Fourier spectrum showing frequency evolution of marginally unstable TAE mode in response to kinetic α -particle drive ($\gamma_L/\omega_0 = 4.5 \times 10^{-3}$) and external damping ($\gamma_d/\omega_0 = 4 \times 10^{-3}$). The over-plotted white line shows the theoretically predicted frequency shift of $0.4\gamma_L(\gamma_d t)^{1/2}$.

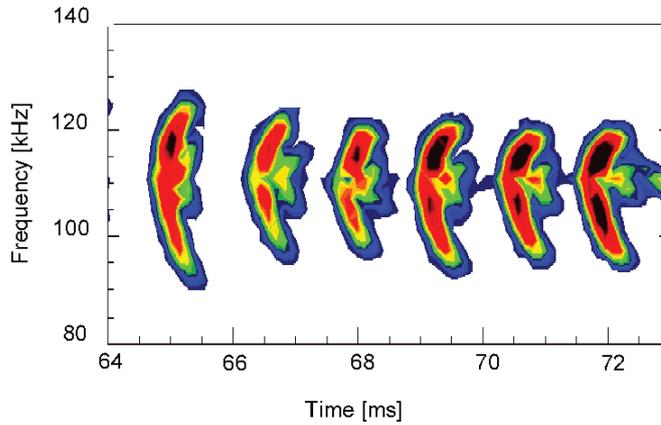


Figure 9. Magnetic spectrogram showing frequency sweeping $n = 1$ core-localized mode in MAST #5568. The first event shows a frequency shift of 18 kHz in a time of 0.8 ms.

toroidally distributed array of magnetic pick-up coils around the inside of the vessel identifies this mode as an $n = 1$ mode. This discharge was heated by neutral beam injection (NBI) with the injected deuterons having an energy of 40 keV. The magnetic field strength at the magnetic axis was 0.5 T, the major radius was 0.77 m and the minor radius 0.55 m. A careful linear stability analysis using the MISHKA code together with the EFIT [11] reconstructed equilibrium at this time and the HAGIS code identifies this mode as a global $n = 1$ TAE as shown in figure 10.

To identify the constant of proportionality between the nonlinear bounce frequency and the mode amplitude, simulations were performed with the HAGIS code. For a single particle moving in an axisymmetric toroidal equilibrium and trapped in a nearly constant amplitude TAE wave there is a symmetry between the toroidal and temporal variation of the wave field associated with the ability to describe it using the ansatz, $\xi(\mathbf{r}, t) = A\xi(r, \theta) \exp(in\phi - i\omega t)$. In this case, energy, E , and toroidal canonical momentum, P_ϕ , are no longer conserved quantities, but the quantity $H' = E - (\omega/n)P_\phi$ remains invariant.

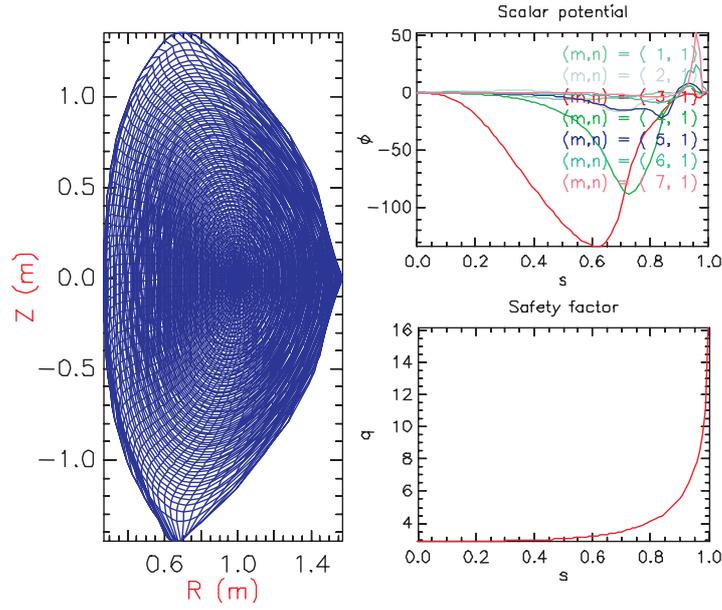


Figure 10. Equilibrium mesh and safety factor profile constructed for linear stability calculations of MAST #5568 at $t = 65$ ms. Also shown is the radial mode structure of the global $n = 1$ TAE.

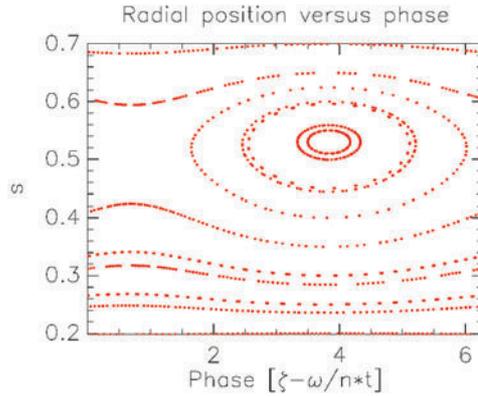


Figure 11. Poincaré plot for energetic deuterons ($H' = 20$ keV) in MAST #5568 at $t = 65$ ms interacting with a global $n = 1$ TAE. The amplitude of the mode is $A = \delta B_r / B_0 = 10^{-3}$.

By launching in HAGIS a set of particles at various plasma radii but with all the same value of H' it is possible to map out the island structure of fast ions trapped in the TAE. Such an example is shown for the MAST case under consideration in figure 11 where the value of $H' = 20$ keV was chosen such that the parallel velocity of fast ions at the centre (and the peak of the eigenfunction) was equal to the Alfvén velocity, $v_{\parallel} = v_A$. The amplitude of the mode was arbitrarily chosen to be $A = \delta B_r / B_0 = 10^{-3}$. By evaluating the nonlinear bounce frequency for ions that are deeply trapped in the TAE wave field and varying the TAE amplitude it is possible to obtain the scaling of ω_b with A as shown in figure 12. As expected theoretically, the nonlinear bounce frequency scales with the square root of the mode amplitude. It should be noted, however, that when repeating this scaling for earlier time points in the discharge

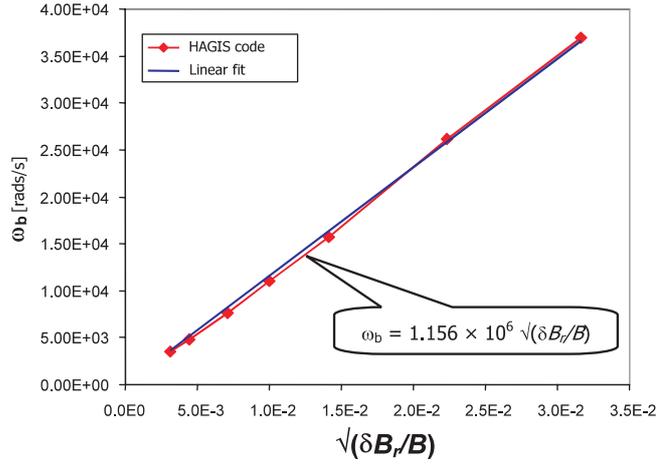


Figure 12. Scaling of the nonlinear bounce frequency of energetic deuterons ($H' = 20$ keV) in MAST #5568 at $t = 65$ ms with the square root of the mode amplitude.

when the safety factor profile was interpreted by EFIT to be reversed in the plasma core, this scaling broke down for amplitudes larger than $A > 2 \times 10^{-4}$.

Sufficient information is now available to estimate the amplitude of the TAE mode that is seen to sweep in frequency in the MAST discharge discussed above. Substituting the observed frequency sweep of 18 kHz in a time of 0.8 ms, together with the theoretical estimate of $C_1 \approx 1.1$ and the numerical calculation of $C_3 = 1.2 \times 10^6$ radians s^{-1} into equation (5) gives, $A = \delta B_r/B_0 \approx 4 \times 10^{-4}$. This corresponds to an absolute amplitude of $\delta B_r \approx 2 \times 10^{-4}$ T.

The measured amplitude of this mode, δB_r , at the position of the mid-plane Mirnov coil ($R = 1.85$ m) was 10^{-5} T. Using the MISHKA code with an ideally conducting wall at $R/a = 2$ to calculate the linear mode structure in the plasma and vacuum regions allows the peak amplitude to be determined as $\delta B_r \approx 5 \times 10^{-4}$ T. This value is in good agreement with that obtained above using the spectroscopic observation of the mode's frequency sweeping rate.

5. Conclusions

A spectroscopic technique has been formulated for inferring the internal amplitudes of frequency sweeping modes that start from the TAE frequency and remain in the TAE gap during the sweep. The example presented is that of a frequency sweeping TAE mode in the core of the MAST tokamak. In this case (MAST #5568 at $t = 65$ ms) the $n = 1$ global TAE amplitude was inferred to be $\delta B_r/B_0 \approx 4 \times 10^{-4}$.

This result assumes that the theory developed in [6, 7] is applicable and that the system is close to marginal stability. Alternative work [12], albeit for a different parameter regime where the system is initially in a configuration well above marginal stability, leads to different scalings for the frequency shift. Note, however, that in general it is difficult experimentally to create sustained conditions where the system is well above marginal instability. The mechanism invoked here is based on the often-made hypothesis that a dynamic system will hover near marginal stability. Note also that to have frequency sweeping in the experiment via the mechanism discussed here requires a sufficiently low level of extrinsic stochasticity in the particle orbits (e.g. due to collisions, RF heating or plasma turbulence, etc) to allow trapped particles' orbits to be well defined for many bounce times. Difficulty in obtaining

conditions where there is a low enough level of stochasticity has perhaps limited the number of experimental shots that have exhibited frequency sweeping.

More complete simulations of discharge conditions, including a realistic description of the beam distribution of energetic ions and the effects of collisionality, is the subject of future work. Such a simulation should also be capable of capturing the repetitive nature of the frequency sweeping TAE that has been observed.

Acknowledgments

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References

- [1] Mikhailovskii A B, Huysmans G T A, Kerner W and Sharapov S E 1997 *Plasma Phys. Rep.* **23** 844
- [2] Cheng C Z, Chen L and Chance M S 1985 *Ann. Phys. (NY)* **161** 21
- [3] Sykes A *et al* 2001 *Nucl. Fusion* **41** 1423
- [4] Pinches S D 1996 *PhD Thesis* The University of Nottingham
- [5] Pinches S D *et al* 1998 *Comput. Phys. Commun.* **111** 131
- [6] Berk H L, Breizman B N and Petviashvili N V 1997 *Phys. Lett. A* **234** 213
- [7] Berk H L, Breizman B N and Petviashvili N V 1998 *Phys. Lett. A* **238** 408
- [8] Vernon Wong H and Berk H L 1998 *Phys. Plasmas* **5** 2781
- [9] Wesson J A 2004 *Tokamaks* 3rd edn (Oxford, UK: Oxford Science) p 617
- [10] Berk H L, Breizman B N and Ye H 1993 *Phys. Fluids B* **5** 1506
- [11] Lao L L, St John H, Stambaugh R D, Kellman A G and Pfeiffer W 1985 *Nucl. Fusion* **25** 1611
- [12] Zonca F, Briguglio S, Chen L, Dettrick S, Fogaccia G, Testa D and Vlad G 2002 *Phys. Plasmas* **9** 4939