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SUSTAINED SELF-REVERSAL IN THE REVERSED FIELD PINCH

A. Y. Aydemir and D. C. Barnes
Institute for Fusion Studies
The University of Texas at Austin
Austin, Texas 78712

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The Institute for Fusion Studies,

University of Texas, Austin, Texas 78712

Abstract

Spontaneous reversal of the toroidal field in a Reversed Field Pinch as a result of low-β kink-tearing mode activity is investigated using a 3D incompressible MHD code. Helical and 3D steady reversed states are obtained. In 3D a critical value of the pinch parameter is observed above which only quasi-steady fluctuating states are seen.

The Reversed Field Pinch[1] (RFP), like the Tokamak, is an axisymmetric toroidal plasma confinement concept with both toroidal, B_ζ, and poloidal, B_θ, magnetic fields. In contrast to the Tokamak the toroidal current in an RFP is much larger. Thus, B_θ and B_ζ are of the same order and the safety factor $q = rB_\zeta/RB_\theta$ (r minor, R major toroidal radii) is less than one everywhere in the plasma. Magnetohydrodynamic (MHD) stability is provided by a close fitting conducting wall and high magnetic shear associated with a q profile that reverses near the wall where B_ζ reverses.

The RFP state has been observed to arise spontaneously as a result of MHD activity. This self reversal occurs when the pinch parameter $\Theta = B_{\theta w}/\langle B_\zeta \rangle$ is large enough, where $B_{\theta w}$ is the poloidal field at the wall and the brackets denote the volume average. Taylor[2] showed that dissipative relaxation of an isolated system in the presence of a conjectured global conserved quantity leads to a force-free metastationary state in which the field is given by the Bessel function model (BFM). This model predicts a universal relation between Θ and $F = B_{\zeta w}/\langle B_\zeta \rangle$ for such systems. A driven MHD system was shown to self reverse by Sykes and Wesson[3] who simulated only the initial phase of a rather dissipative system.

In this letter, results of numerical simulations of driven systems with more realistic dissipation and for very long times are reported. The results indicate that low-β kink-tearing modes lead to self reversal which may be sustained indefinitely.

A recently developed algorithm[4] is used to time advance the resistive incompressible MHD equations. The simulation domain is a cylinder with ends periodically identified (cylindrical torus). The initial state is a zero β, non-reversed, force-free equilibrium. The resistivity is taken to be constant in time and increasing in r.

Boundary conditions are chosen to simulate a driven system. The poloidal electric field $E_{\theta w}$ vanishes so that toroidal flux is conserved. The toroidal electric field $E_{\zeta w}$ is a function of time. It is zero from the initial time until the current resistively decays to a specified value. When $\Theta(t)$ reaches Θ_{min} (specified), $E_{\zeta w}$ is applied to keep the current constant subsequently.

Typical simulations are carried out with a magnetic Reynolds number $S = 5 \cdot 10^3$ at $r = 0$ and with unity aspect ratio. Fourier expansion is used to represent the poloidal and toroidal variations with 16-60 modes. A radial grid of 64 points is employed.

Initial toroidal current produces a safety factor profile which is unstable to various $m = 1$ modes (m:poloidal mode number). For the typical q profile of Fig. 1, $m/n = 1/2, 1/3, 1/4, \ldots$ modes are unstable with growth rates decreasing with n (toroidal mode number). The nonlinear evolution of a single mode can be followed in a single-helicity (2D) calculation in which only modes with a fixed helicity m_o/n_o are retained. Such a mode may saturate in a reversed state in a flux-conserving system. The helically deformed current channel increases the toroidal field on axis. A reversed $B_{\zeta w}$ then develops to conserve flux. This behavior is observed to be qualitatively the same for both resonant (such as the 1/3 mode which has a rational surface near $r = 0$) and non-resonant (such as the 1/2 mode with no rational surface) modes. The time evolution of $B_{\zeta w}$ for two different boundary conditions, $\Theta_{min} = 0$, and $\Theta_{min} = 1.6$, is shown in Fig. 2. The initial reduction in $B_{\zeta w}$ in both cases parallels the growth and saturation of the instability in a helical state. With no applied $E_{\zeta w}$ ($\Theta_{min} = 0$), there is no further decrease in $B_{\zeta w}$. In the second case, however, $B_{\zeta w}$ is driven negative after the current is clamped by a further increase in the amplitude of the helix. This behavior can be explained as follows in terms of the F-Θ relation. With $\Theta_{min} = 0$, after the saturation of the instability the plasma relaxes towards a point on the F-Θ curve (not necessarily that of the BFM[2]) along a trajectory with an approximately constant F and decreasing Θ (Fig. 3). Ultimately resistive decay would carry the system along the F-Θ curve to the point $(F,\Theta) = (1,0)$. With the constraint $\Theta \geqslant 1.6$, however, the plasma can approach the preferred F-Θ curve only along a trajectory with decreasing F, thus decreasing $B_{\zeta w}$. In this case, with a positive $E_{\zeta w}$ maintaining the toroidal current, the plasma eventually reaches a steady-state at $(F,\Theta) = (-0.10,1.60)$. This final equilibrium is characterized by large velocity flows maintaining the reversal against resistive diffusion. It differs from the BFM equilibrium in that the fluid velocity is not zero, and the currents vanish near the wall.

In a second, fully three-dimensional (3D) series of calculations, the equilibrium is perturbed with two modes of different helicities, $m/n=1/2$, and 1/3, and the evolution of these modes and others nonlinearly generated by them is followed. While the 2D simulations seem to lead always to a final steady state, the interaction of many helicities in 3D produces steady solutions only for $\Theta < 1.55$.

For these steady solutions, the modes with the highest linear growth rate, m/n= 1/2, 1/3, dominate throughout the calculation, and the final state is basically a superposition of these two modes. The total toroidal field on the wall shows a strong m=1 variation. The average field $B_{\zeta w}$ is reversed only for $\Theta > \Theta_r$, where $\Theta_r \cong 1.45$, the exact value depending on the resistivity profile, and the initial conditions. The time evolutions of $B_{\zeta w}$ and kinetic energy for $\Theta_{min} = 1.5$ are shown in Fig. 4 and clearly indicate a steady reversed state.

For $\Theta > 1.55$, the evolution of the plasma is markedly different in that the state variables, instead of becoming constant in time, exhibit fluctuations about steady-state mean values. The $\Theta_{min} = 1.6$ case shown in Fig. 5, has the same initial conditions as that pictured in Fig. 4. For this case an interesting period doubling occurs after t=800. As seen in Fig. 5 reversal is lost around t = 700, which is accompanied by the return of the current to a more symmetric state. However, in this non-reversed state, the m/n = 1/2 mode becomes unstable again (Fig. 6a), which drives the plasma into a state with half its original periodicity length. This is evident in the plots of mode amplitudes which show, for t > 900, periodic fluctuations in the energy of the modes with even toroidal mode number (Fig. 6a), while the energy in the modes with odd toroidal mode number decays exponentially (Fig. 6b). For t > 1000, the plasma is in a quasi-steady state in which the reversal is maintained by low level mode activity which limit cycles about a mean state. As Θ_{min} is increased, these fluctuations become larger in amplitude and aperiodic.

A characteristic of these 3D calculations that has a great bearing on confinement is the rapid annihilation of flux surfaces. In Fig. 7a, the shift of the flux surfaces is due to the saturated, non-resonant m/n = 1/2 mode. The resonant m/n = 1/3 mode that is still growing has produced an island chain, and a region of stochastic field lines around it. As the mode amplitude increases, this region quickly grows and essentially fills the whole plasma volume (Fig. 7b). At t=451 (Fig. 7c) when 1/3 is the dominant mode, except for a 1/3 island, there are no good surfaces left, although some reemerge near the wall after the plasma reaches a quasi-steady state (Fig. 7d).

In conclusion, low-β kink-tearing activity leads to reversal in both 2D helical, and full 3D calculations. If the plasma current is maintained, it is always possible to find 2D steady states in which dynamo action of large helical flows maintains the fields against resistive diffusion. In 3D calculations, for a given amount of toroidal flux, there is a critical value of toroidal current above which there are only quasi-steady solutions with fluctuation levels increasing with Θ. However, low level mode activity still maintains the mean fields against resistive diffusion indefinitely when external sources maintain the toroidal current. The fluctuations are analagous to other nonlinear systems in which the increase of a parameter leads from a stationary state to a limit cycle to aperiodic fluctuations. Finally, the implications of the observed stochastic magnetic field on confinement needs to be investigated.

Figure Captions

Fig. 1: Initial q-profile.

Fig. 2: Field-reversal parameter in helical calculations.

Fig. 3: F-Θ diagram for helical calculations.

Fig. 4: Kinetic energy and F for Θ_{min}=1.5 (3D).

Fig. 5: Kinetic energy and F for Θ_{min}=1.6 (3D).

Fig. 6: Selected mode energies: a) even toroidal mode numbers,

b) odd toroidal mode numbers (Θ_{min} = 1.6,3D).

Fig. 7: Field line traces (Θ_{min}=1.6, 3D).

References

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