INSTITUTE FOR FUSION STUDIES PROGRESS REPORT

for the period

September 1, 1982 to August 31, 1983

July 1983
I. ANNUAL PROPOSED BUDGET

Fiscal year 1984

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$1,135,000

Joint Institute for Fusion Theory $35,000
INSTITUTE FOR FUSION STUDIES PROGRESS REPORT

for the period

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II. ACTIVITIES TO BE PERFORMED

Fiscal year 1984

A. Research

Institute research in FY 1984 will follow, on the whole, lines established previously. Most of the projects mentioned here grow out of work summarized in the FY 1983 progress report, which is attached.

1. Equilibrium

The Institute's complementary approaches to stellarator equilibrium theory will be extended to finite pressure. An immediate goal is better understanding of the well-known singularities associated with rational surfaces in three dimensions. The ultimate goal is optimization of stellarator equilibria with respect to both stability and effective confinement volume.

2. Linear Stability

A wide variety of stability investigations are planned or in progress. A major theme, as in 1983, will be stability limitations on beta, with particular emphasis this year on stellarators and tokamaks. The high beta stability issue involves not only ballooning modes, but also tearing modes (including "tearing-ballooning" instability), resistive interchanges and a variety of kinetic effects; work in most of these areas has already begun. A closely related topic concerns suprathermal electron or ion components. Their effects on stability, their relation to fishbone oscillations, and their possible use in achieving high-beta tokamak stability will be further investigated. A
3. Transport

Recent understanding of the non-Maxwellian ion distribution function in tokamaks requires continued study to fully assess its transport implications, including ion parallel viscosity. Of course turbulent transport investigations will also continue; the goal is more accurate and more fully understood scaling laws for tokamaks as well as the RFP.

4. Nonlinear Dynamics

Analytical studies of plasma turbulence will remain a focal point of the Institute’s nonlinear research. A recently launched project in this area concerns tokamak edge turbulence; this will be studied using renormalized two-point theory, considering drift-wave as well as rippling-mode turbulence. The recent RFP dynamo studies will be extended, both to include additional physics, such as toroidal coupling, in the RFP problem and also to assess the importance of dynamo action in more general contexts. Numerical studies of turbulence will be intensified, especially with regard to further development of gyrokinetic implicit particle codes. Extension to higher dimensionality and inclusion of electromagnetic terms are the primary goals. Research on magnetic islands, solitons and nonlinear fluid models will continue. Toroidal coupling, in the nonlinear regime, of islands centered on different rational surfaces will be studied analytically. A more detailed evaluation of the solitary wave contribution to the tokamak fluctuation spectrum is planned. Studies of electromagnetic generalizations of drift-solitary waves will concentrate on experimental realism.
B. Meetings

Four workshops have been scheduled for FY 1984 by the Institute for Fusion Studies and the International Center for Fusion Theory (Nagoya), through the Joint Institute for Fusion Theory exchange program. These workshops are listed below; they are to be organized by participants from outside the IFS.

Additional meetings, not associated with the Joint Institute, are anticipated but not yet scheduled.

JIFT Workshops for 1983-1984

Japan to US:

(1) Wave Heating and Current Generation
Organizers: C. S. Liu and M. Porkolab
S. Tanaka and R. Sugihara
Location: General Atomic (Feb. 1984)

The Japanese delegation will also visit rf facilities at MIT and Princeton as part of the Workshop.

(2) 3D MHD Simulation Studies
Organizers: B. Carreras and T. Sato
Location: ORNL (March 1984)

US to Japan:

(1) Statistical Plasma Physics
Organizers: John Greene and Yoshi Ichikawa
Location: Nagoya (October 1983)

(2) Transport and Instabilities in Open Ended Systems
Organizers: R. Cohen and T. Kamimura
Location: Nagoya (August 1984)
I. INSTITUTE RESEARCH

A. Overview

Plasma physics research at the Institute for Fusion Studies has familiar goals: improved understanding of the equilibrium, stability, transport and nonlinear dynamics of confined plasmas under near-reactor conditions. The Institute's program is perhaps distinguished by its relatively strong emphasis on nonlinear processes, including both turbulent (drift-wave turbulence, stochasticity) and coherent (solitons, magnetic islands) phenomena. However, linear stability theory is nearly as popular a research area, and interest in equilibrium and transport is becoming increasingly active. Other topics are represented; most prominently, r-f heating theory has been explored, especially in conjunction with the Alfvén-wave studies of the Fusion Research Center. Here we summarize very briefly the major lines of Institute research during the past year. A more detailed discussion of most topics can be found in the survey of Subsection B. (Those projects which continue investigations launched in previous years are of course also discussed in earlier IFS progress reports.)

Virtually all Institute equilibrium work has concerned three-dimensional geometry: stellarators. A variational method for solving the three-dimensional inverse equilibrium problem (begun earlier in collaboration with scientists from other institutions) has been further developed; it yields significant gains in computational efficiency. An alternative technique exploits the Hamiltonian nature of field line "flow": the vacuum flux surfaces are optimized, under appropriate constraints, by small coil deformations which are chosen to
make the Hamiltonian nearly integrable. Magnetic island widths are thereby reduced and the effective confinement volume enhanced. A third approach is based on iterative solution of Ampere's law in general geometry; it is particularly useful for analyzing plasma currents. All three methods have yet to fully account for plasma pressure; this will be a major focus of future equilibrium work.

Much of the Institute's linear stability research concerns alternatives to the tokamak, such as EBT. Major progress has been achieved in understanding the effects of hot electron populations on stability, and the consequent limitations on beta. This work, done partly in collaboration with non-IPS scientists, uses previously developed generalized energy principles to obtain important modifications of previous stability limits. Rapid drift motion of a suprathermal species also occurs in beam-injected tokamaks, where it has been shown to destabilize the internal kink at the drift frequency as observed in the "fish-bone" oscillations.

Other recent achievements in stability theory include studies of free-boundary stellarator stability, field-reversed theta pinch stability, and finite pressure effects on collisionless tearing. The tearing mode work has found field curvature to be surprisingly innocuous in the collisionless limit.

A major present thrust of Institute linear stability work concerns ballooning modes. New methods have made the ballooning problem in stellarators much more tractable; important stabilizing influences of finite plasma pressure have thus been recognized. With regard to ballooning stability in tokamaks, it has been noted that a hot ion or
electron component of sufficient energy might permit access, for an initially low-beta plasma, to the second stability region.

Convective growth of whistler modes has been studied and appears dangerous for high \( \gamma \) EBT's and also for MFTFB.

Institute research on transport theory is nearly equally divided between neoclassical studies and turbulent transport. The neoclassical work emphasizes ion heat conduction in the banana regime. Collisional effects involving trapped particles have been shown to produce, in lowest order, a non-Maxwellian ion distribution function, \( f_{\text{i0}} \). The predicted high-temperature tail on \( f_{\text{i0}} \) is consistent with PDX observations; it substantially amplifies several neoclassical transport processes.

Turbulence theories characteristically include the reaction of transport upon the fluctuation spectrum which causes it. Previous Institute work in this area (in collaboration with scientists at Oak Ridge) led, in particular, to impressively accurate transport predictions for the ISX-B experiment. More recent related work concerns toroidicity-induced drift modes; analysis of their turbulent saturation has produced explicit scaling laws for this important transport mechanism.

Institute research in the area of nonlinear dynamics includes four major categories:

(i) Turbulence. Analytical work on drift wave turbulence is now mainly directed to understanding the fluctuation spectrum observed in tokamaks and its effects on transport and stability. Renormalization
of the two-point, toroidal kinetic theory, as reported last year, uncovered new mechanisms for the release of expansion free-energy, leading to spectral widths consistent with experiment. Recent extensions of this theory include incoherent ion fluctuations, two-species effects, and a collisional fluid version of the two-point formalism. The one-point theory has also been extended, in order to treat the case of toroidally induced drift modes. Such modes seem especially important because, unlike the conventionally studied slab-model drift waves, they are not shear stabilized.

Partly analytical study of nonlinear dissipative drift waves reveals solutions whose stochastic features indicate the presence of a chaotic attractor. The result is a well defined turbulent steady state with a broad frequency spectrum, similar to the observed spectrum.

Slab-like drift wave turbulence can damp plasma rotation, since rotation distorts the symmetry of the linear eigenmodes. An investigation of this issue, which should affect the tokamak radial electric field, has begun.

A new turbulence study concerns electromagnetic fluid fluctuations in the RFP. Viscous, resistive MHD is used to describe nonlinear evolution of the mean magnetic field, which is found to be governed by the difference between magnetic and fluid helicities. A major result is the clarification of the direction of RFP helicities, which has now been related to the helicity of the average field, $\mathbf{B}_0 \cdot \mathbf{j}_0$. Related work considers the effects of resistive interchange turbulence on RFP confinement. Scaling laws have thus been derived predicting the evolution of RFP behavior at higher temperatures.
(ii) **Numerical Simulation.** Of course simulation provides key support in all nonlinear areas, but certain numerical efforts have special intrinsic importance. The goal of implicit particle codes is a numerically stable, accurate treatment of low-frequency motions in collisionless plasmas. A variety of gyrokinetic techniques account for finite ion Larmor radius effects. A 2 1/2-dimensional, electrostatic version has been successfully tested in the study of low beta drift waves; 3-dimensional electrostatic, and 2 1/2-dimensional electromagnetic versions are under development.

Parallel research on the MHD-particle code has also achieved significant gains. A 2 1/2-dimensional version has been applied to magnetic reconnection problems, as discussed below. Recent extension of the code to three dimensions will allow a "non-reduced" description of nonlinear, ideal kink and ballooning perturbations in a tokamak. Nonlinear kink-tearing activity in a reversed field pinch has been studied using a distinct, resistive MHD code, which is also three-dimensional.

(iii) **Magnetic islands and solitons.** Nonlinear instability associated with the coalescence of two magnetic islands has been studied numerically, using the full (resistive) MHD equations. A new feature is the allowance for plasma compressibility; for certain initial current profiles, this leads to significant departures from Sweet-Parker behavior.

The evolution of islands associated with tokamak tearing instability is also under investigation; the goal is to examine the effects of finite beta, kinetic effects, turbulent diffusion and toroidicity on Rutherford’s description of tearing saturation. It is
found that these effects which appear quite important in linear theory, are much less important in the nonlinear regime.

Nonlinear fluid equations describing drift waves often possess solutions corresponding to localized structures which propagate coherently and interact nearly elastically. Such "drift-wave solitons" or solitary waves, may be related to certain tokamak observations and would enter the electrostatic fluctuation spectrum. Recent investigations concern the scattering of solitary waves; this is studied, both analytically and numerically, by means of the regularized-long-wave equation.

(iv) Fluid models and Hamiltonian theory. The success of reduced MHD in describing nonlinear electromagnetic disturbances in tokamaks has prompted several investigations. First, the Hamiltonian formulation of reduced MHD, constructed last year, has been extended to the high beta, multiple helicity system. It has also led to the discovery of new invariants, and exact nonlinear solutions, of certain versions of reduced MHD. Secondly, more elaborate versions of the reduced model have been developed; there include, self-consistently, finite gyroradius effects (such as nonlinear diamagnetic convection) and key features of the electron response at long mean-free path. Nonlinearly, the model has been found to yield a reasonably accurate version of transport due to magnetic field stochasticity, as well as providing a natural electromagnetic generalization of electrostatic solitary-wave theory.
B. Topical Survey

1. Equilibrium

Equilibrium Stellarator Calculations (H. L. Berk, K. V. Roberts, M. N. Rosenbluth)

The stellarator ballooning mode calculation indicated that given a vacuum field and good vacuum flux surfaces, plasma currents are accurately calculated. We are therefore attempting to calculate equilibrium fields with finite pressure for given vacuum fields and zero average current on a flux surface. The result should yield relevant steady state equilibrium. The calculation generalizes some of the techniques developed from the ballooning mode code and the computation is currently in progress.

Stellarator Coil Design (J. R. Cary, J. D. Hanson)

The goal of this work is to understand the factors that control the existence of magnetic surfaces in three-dimensional stellarators. A specific problem is to determine the extent to which toroidicity destroys surfaces and imposes a lower limit on stellarator aspect ratio. Secondly, for design purposes one would like to know whether this surface destruction can be ameliorated with proper coil designs. This research has shown that, surprisingly, aspect ratio per se is not limited, even when standard coil designs are used. Instead, toroidal effects impose a limit on \( I_{eq} = \tau \epsilon \), an estimate of the equilibrium \( \beta \)-limit, in which \( \tau \) is the average rotational transform, and \( \epsilon \) is the inverse aspect ratio. However, it has also been found that one can bypass this limit by modifying the coil winding law. Improvements in \( I_{eq} \) by factors of two to three have already been obtained.

In last year's report, a method for calculating the vacuum toroidal harmonics needed to partially restore magnetic surfaces was discussed. The present method is a significant improvement. First of all, one can use this method to restore surfaces to all orders. In practice this means that one can make the islands arbitrarily small. Secondly, this method is easily adopted to realistic coil configurations. Hence, this method is immediately available as a design tool.

References:

2. Linear Stability

Free Boundary Stellarator Stability (D. C. Barnes, J. R. Cary)

The free-boundary stability of a high-\( \beta \) helical system with straight geometrical axis (straight stellarator) has been investigated. Stability at high-\( \beta \) is found to be nearly equivalent to stability at infinitesimal; i.e. there is no ballooning behavior in such a sharp-boundary system.
Stability at small $\beta$ is shown to be rigorously equivalent to the presence of a vacuum magnetic well for systems with no net axial current. This conclusion, at odds with much of the lore of high-$\beta$, sharp-boundary theory is modified if a small ($O(\beta)$) net axial current is present. In the latter case, stability results from a sufficiently large current in such a direction that the increment in rotational transform across the plasma boundary is increased relative to the vacuum shear.

These techniques were used to analyze the Heliac configuration, where stable configuration were found with no net axial current. The allowable $\beta$ was found to be the maximum for which an equilibrium exists and is in the range of 10–20%. The equilibrium $\beta$ limit generally increases with decreasing magnetic well depth, indicating a tradeoff between equilibrium and stability. The equilibrium $\beta$ limit is also increased for increased helical wavelength.

Stability of systems with a magnetic hill and non-zero axial current has been shown to persist at least to $\beta$ of order 5%.

During the past year the heliac results mentioned above were provided to Prof. Fred Ribe of the University of Washington as a basis for the design of a proposed straight heliac experiment.

Hidden Constitutive Relations in WKB Theory (H. L. Berk, R. R. Domínguez (to be published Phys. Fluids)(IFS #770)).

For modes that depend on local spatial gradients, and for which $\omega/\omega_c \sim 1$ where $\omega_c$ is the cyclotron frequency, the correct local dispersion relation requires a subtle eikonal analysis. A simple formula has been obtained to find the correct result. For example, the dispersion relation for electrostatic waves in a sheared magnetic field of constant $|B|$ but with density varying in $x$ perpendicular to $B$, is given by

$$0 = D(k, \omega) \equiv 1 + \sum_j \frac{4\pi e_j^2}{m_j} \int \frac{\hbar}{k_L^2} \left( 1 + \frac{n k_y}{\partial x} \right)$$

$$\left[ (k_\parallel v_\parallel + m_c \omega_c) \frac{\partial F(E,x)}{\partial E} + \frac{k}{\omega_c} \frac{\partial F(E,x)}{\partial x} \right]$$

$$\cdot \int d^3v \frac{\omega - m_c \omega_c - k_\parallel (x) v_\parallel}{(\omega - m_c \omega_c - k_\parallel (x) v_\parallel)}.$$

where the distribution, $F(E,x)$, is a function of energy (per unit mass) $E$ and guiding center, $x_\parallel = x - v_y/\omega_c$.

The term $\frac{n k_y}{k_L} \frac{\partial}{\partial x}$ is needed to give the correct description. Otherwise, when $\omega/\omega_c \sim 1$, the dispersion relation would not agree with fluid theory in the cold plasma limit, and spurious effects would arise for modes whose dispersion relations depends directly on spatial gradients.
Stellarator Ballooning Mode Calculations (H. L. Berk, M. N. Rosenbluth, J. L. Shohet)

We have shown how one can simply use magnetic vacuum field solvers to implement ballooning mode calculations in a stellarator, by following two nearby field lines. This is a direct way to investigate MHD stability of precise experimental designs. We have further been able to construct the Mercier coefficient as the asymptotic limit of the ballooning mode equation and we show for equilibria with steep pressure profiles, that the equilibrium correction of the vacuum field cancels the intrinsically destabilizing Mercier term. Thus, if a configuration is Mercier stable at zero beta, in this model it will be Mercier stable for all beta. Shafranov has shown for systems where the helical field is small compared to the toroidal field, that if the rotational transform increases radially, its stability is determined by the Mercier criterion. We have found a counterexample to this, in a system where Shafranov’s ordering of the helical fields fails. However, it is clear that Mercier stability is often the key criterion for stellarator stability and achieving average well stability at zero beta, should yield a large critical beta limit. With moderately long pressure gradients, equilibrium also depends on global shifts for which more sophisticated finite pressure equilibrium codes need to be developed.


EBT stability calculations for the investigation of the stabilizing effect of hot particles have made progress in several important areas.

(1) Long wavelength decoupling condition (Phys. Fluids 3, 606(1983)).

Analysis of long radial wavelength modes show that they appear to yield the most stringent stability conditions for the stabilizing effect of the hot component in an EBT configuration. At zero core beta the decoupling condition of the hot species is more difficult to achieve than with WKB-like modes, and the Lee-Van Dam critical beta condition can be reduced by a substantial factor.

(2) Long wavelength finite Larmor effects.

The constant curvature model was extended to describe the hot electron finite Larmor radius effect for the long radial wavelength modes. We found that the modes appear to be locally rigid on the outside of the layer but small on the inner half of the layer. Further any FLR effect one might expect from extrapolating WKB theory or from dimensional arguments is cancelled for the self-consistent eigenfunction. In addition we find a new dissipative destabilizing mechanism associated with the drift resonances at the peak pressure of the layer. Thus, as in conventional theory, the hot component FLR effect hardly affects the stabilization of the long wavelength modes, and there is even a new dissipative destabilizing source.

(3) Compressional effects of long wavelength modes are currently being examined. It may be possible to show that additional compressibility effects, not previously discussed in the literature, will have a stabilizing effect on long wavelength modes, while FLR effects can stabilize the shorter wavelength modes. The calculation is currently in progress.
(4) The numerical solution of the integral equation that is obtained when the curvature and magnetic field gradients are comparable is being calculated. In this regime the standard analytic techniques fail.

*Lawrence Livermore Laboratory
*Oak Ridge National Laboratory

Particle Orbits Near a Linear Magnetic Null (J. R. Cary with J-S. Kim, Lawrence Berkeley Laboratory)

Field-reversed theta pinches have proven to be stable for many Alfvén times in experiments at Los Alamos National Laboratory, Kurchatov, and Nagoya, Japan. At first glance this is remarkable considering that such systems are predicted to be violently unstable by MHD calculations. However, this contradiction is not completely unexpected, since field-reversed theta pinches have a magnetic null, a closed curve encircling the symmetry axis on which the magnetic field vanishes. Near this null, MHD theory breaks down. It is therefore believed that an understanding of the stability will come via a kinetic calculation properly treating the motion of particles near the magnetic null.

The present research, now completed and accepted for publication, is a step in this direction. This work consists of an analytical and numerical study of particle orbits near a magnetic null. The study shows that the orbits can be classified by two parameters: \( \varepsilon \), the ellipticity of the flux surfaces, and \( Q \equiv \sqrt{2mE/p_z} \), a parameter involving the particle's energy and canonical momentum. Three regimes have been found:

1. For \( Q >> 1 \) and \( Q \ll -1 \), the particle motion is observed to be regular. When \( \varepsilon \) is close to unity the additional invariant is the canonical angular momentum \( p_\theta \). When \( \varepsilon \ll 1 \) holds, the additional invariant is given approximately by the adiabatic invariant of the \( y \) motion. Significant resonance structure occurs only for \( \varepsilon \approx 0.5 \).

2. For intermediate values, \( \varepsilon^{3/2} \ll Q \ll 1 \), nearly all orbits are stochastic. For \( Q \) in the vicinity of unity, there exists a small fraction of integrable particles which are trapped near the tips of the ellipses. For \( Q \) near \( \varepsilon^{3/2} \), the particle motion is like guiding-center motion with random jumps in the magnetic moments.

3. For very small values, \( |Q| \ll \varepsilon^{3/2} \), guiding-center motion with magnetic moment conservation is observed.

These results have been applied to the field-reversed theta pinch experiments of the Los Alamos National Laboratory. This application shows that most of the particles in this system are stochastic. A significant fraction of the particles near the center of the machine are integrable and conserve the adiabatic invariant of the more rapid \( y \)-motion. However, almost none of the particles obey guiding-center dynamics. These results may explain why small-gyroradius theories incorrectly predict instability.

References:

Collisionless Tearing and Drift Modes in a Plasma Cylinder
(J. R. Cary, B. S. Newberger (Los Alamos National Laboratory))

In resistive MHD stability theory the effects of curvature are paramount. Localized modes are stable unless the curvature is bad. Bad curvature can also greatly enhance the growth rate of tearing modes, in which case they are more widely known as resistive interchanges. Good curvature can stabilize tearing modes.

The goal of this research is to determine the effects of curvature on collisionless, kinetic plasma stability theory. At this early stage of development it seems appropriate to analyze a plasma cylinder, which is the simplest system with generic curvature. As the effects of \( B_\parallel \) and \( V_B \) drifts are of the same order, they are also included.

Hamiltonian methods are used to analyze this system. The equilibrium distribution depends on three invariants, the energy \( h \) and the canonical momenta \( p_\theta \) and \( p_z \). A canonical Hamiltonian guiding-center theory of the particle drifts is obtained by expanding the Hamiltonian about the radius of its minimum. Knowledge of the unperturbed particle motion allows one to obtain the perturbed distribution, complete with the resonances due to curvature and \( V_B \) drifts. Taking moments of this distribution yields the perturbed charges and currents. These formulae together with Maxwell's equations yield a closed set of differential equations. Remarkably, an algebraic elimination reduces this set to a fourth-order system.

The analytic results obtained by using the methods of Antonsen, Lee and Chen, are as follows: (1) Bad curvature does not destabilize electron drift modes. (2) Bad curvature does not destabilize the kinetic tearing mode, (when \( \Lambda^* \) is negative) unless the Suydam condition of ideal theory is violated. This second result is surprising, since it implies that the slow interchange mode of resistive MHD is not present in collisionless kinetic theory.

A numerical analysis of this work is still in progress. Early results indicate that \( \Lambda_c \), a measure of the external free energy needed to drive tearing modes unstable, can be enhanced by a factor of two over the results obtained previously in slab calculations. On the other hand \( \Lambda_c \) can be reduced to nearly zero by bad curvature.

Stability of the Low-Frequency Hot Electron Interchange Mode
for an EBT Model
(J. N. Dawson, Y. Ohsawa (UCLA), J. W. Van Dam)

Simulation results of the low-frequency interchange instabilities of a hot electron plasma obtained with a 2 1/2-dimensional relativistic electromagnetic particle code that models EBT geometry were compared with the analytical predictions of linear stability theory. The growth rates of the long-wavelength modes tend to agree with those predicted for zero Larmor radius, whereas for the short-wavelength modes, the Larmor radius effects are strongly stabilizing.

References


Ballooning Instabilities in Hot Electron Plasmas
(T. M. Antonsen (Univ. of Maryland), H. L. Berk, Y. C. Lee (Univ. of Maryland), M. N. Rosenbluth, J. W. Van Dam)

A general treatment of short wavelength, low frequency modes in a hot electron plasma shows that inclusion of the circulating class of hot electrons, field line bending effects, and dissipative contributions from wave-particle resonances all reduce the region in parameter space where stable operation would be expected. Nevertheless, modes with sufficiently short wavelengths can still be entirely stabilized by finite hot electron gyroradius effects.

Energetic Particle Stabilization of Ballooning Modes in Tokamaks (D. C. Barnes, M. G. Engquist, M. N. Rosenbluth, S. T. Tsai, (Inst. of Physics, Peking) and J. W. Van Dam)

The presence of an anisotropic, highly energetic ion (or electron) population in a tokamak device is found to allow direct access at constant shear into the regime of second ballooning stability, after which the energetic population is no longer required. This new concept could have significant application to the attainment of high beta in tokamaks (or stellarators).

We investigated the situation in which an anisotropic population of hot ions is mirror-trapped on the outer side of the torus. The temperature of the hot ions is such that their magnetic drift frequency is taken to be much larger than the MHD growth rate. A large-aspect-ratio model equilibrium for which the pressure gradient is localized in a thin layer was adopted for simplicity. The linear stability of high-mode-number modes was obtained by numerical solution of a ballooning integro-differential equation that was derived from a lower bound to the low-frequency kinetic energy principle. Although this procedure underestimates stability, the results nevertheless indicate that hot ions trapped, for example, between $\beta = \pi r/4$ are able to stabilize ballooning for shear values up to $r_{g^*}/q \approx 0.9$ and for beta values beyond the second stability limit ($\beta_T \approx r/R$).

When the drift frequency is not high enough to completely decouple the energetic species, there is a tendency for the MHD mode to acquire a real frequency close to the drift frequency of the energetic species, which could correlate with recent observations of "fishbone" fluctuations.

Currently, work is underway to treat the stability problem for a realistic equilibrium, obtained by solving the anisotropic Grad-Shafranov equation self-consistently for finite aspect ratio. This approach includes the global magnetic well and also shaping effects of outer conductors.
Variational Gyrokinetic Theory (H. V. Wong)

A variational form of the eigenmode equations, with inclusion of finite Larmor radius effects, has been derived for low frequency, long wavelength, electromagnetic perturbations of inhomogeneous magnetized plasmas. The linearized kinetic equation is solved by a perturbation expansion in the Larmor radius. Cartesian tensors are used to carry out this perturbation in an efficient manner. The independent variables employed are canonical, and the properties of Poisson brackets are exploited to construct variational quadratic forms. A manuscript describing this investigation is presently being prepared.

Whistlers in EBT’s and Tandems
(M. N. Rosenbluth, R. Sudan)

Studies of convective growth and propagation of whistlers have shown that \( \omega / \Omega_{ce} > \frac{1}{2} \) whistlers are propagated away from the plasma, while at lower frequencies they are confined. This means that for EBT layers with \( \gamma \sim 2-3 \) whistlers remain trapped and could lead to ring spreading. Whistler growth in MFTF also appears dangerous.

3. Transport

Non-Maxwellian Ion Distribution in Tokamaks Caused by Neoclassical Heat Conduction
(A. A. Ware)

Measurements of the ion distribution function (\( f_i \)) on PDX using active charge exchange have shown \( \ln f_i \) versus energy (\( \epsilon \)) plots which exhibit two slopes. The lower energy part has an effective temperature \( (-\partial \ln f_i / \partial \epsilon)^{-1} \) which decreases with minor radius as expected, but the higher energy part has an effective temperature which remains approximately constant with radius at a value close to the central ion temperature. These non-Maxwellian distributions can be traced to the large poloidal Larmor radii of the tail particles; the parameter \( \rho_0/L \) is not small compared with unity and the basic argument of neoclassical theory, that \( f_i \) must be Maxwellian in lowest order, is vitiated.

The problem of ion heat conduction has been restudied without assuming \( f_{i0} \) is Maxwellian. The Fokker-Planck drift kinetic equation has been solved to give \( f_{i0} \) for the tail particles assuming the low energy part of \( f_i \) is given. It is found that if ion self-collisions are dominant for energy scattering, the banana regime diffusion requires the tail particles to diffuse outwards in radius and downwards in energy maintaining a constant effective temperature as observed experimentally. Energy scattering collisions with electrons or circulating beam ions modify this conclusion. The discontinuity in \( \partial \ln f_i / \partial \epsilon \) is interpreted as the equivalent in the phase space \( r,v \) of a contact discontinuity in gas dynamics.

The presence of the non-Maxwellian tail to \( f_i \) substantially increases the ion heat conduction, the ion energy content per unit volume and the rate of energy transfer to electrons by collisions. Other transport processes will also be modified.

Reference

1. IFS Report #90, "Non-Maxwellian Ion Distributions caused by Neoclassical Heat Conduction", A. A. Ware, April, 1983, submitted for publication.
Turbulent Transport due to Toroidally Induced Drift Modes

(P. H. Diamond, P. L. Similon)

The saturated state spectral intensity for toroidally induced drift modes has been calculated from renormalized one-point turbulence theory [see below, part 4(i)]. The resulting electron radial diffusion coefficient is calculated for a number of collisionality regimes. Using the assumption of balance of electron diffusion losses with Ohmic heating, the electron temperature is eliminated and final forms for the electron energy confinement times are obtained. The predicted confinement time density scaling increases from \(n^{3/8}\) in the collisionless regime to \(n^{9/8}\) in the dissipative trapped electron regime.

The effects of finite \(\beta_P\) corrections and transport due to magnetic braiding are discussed. At low \(\beta(\beta_P < 1)\) magnetic braiding does not contribute substantially to electron energy transport, which is dominated by electrostatic convection. However, electron conduction losses may be more significant at higher \(\beta(\beta_P < 1)\).

4. Nonlinear Processes

(i) Turbulence: analytical studies

Analytical Investigations of Effects of Resistive Interchange Turbulence on Energy Confinement in Reversed-Field Pinch

(Z. G. An, P. H. Diamond)

This work is an analytical investigation of the confinement deterioration mechanism in a reversed-field pinch caused by electron conduction losses due to magnetic flutter produced by resistive interchange instabilities.

Using approximate analytical solutions of the (compressible) MHD equations for even and odd potential parity slow and fast resistive interchange modes, mixing length theory is applied to estimate the potential and magnetic perturbation levels. These results are used to calculate the stochastic magnetic field diffusion coefficients for weak and strong magnetic field diffusion coefficients for weak and strong magnetic turbulence. An expression for the anomalous electron thermal conductivity is then derived for various regimes, and thus a scaling law for energy confinement time is obtained.

Dynamics of Magnetic Fields in the R.F.P.

(P. H. Diamond, Z. G. An)

This project is concerned with the development of a dynamical theory for the evolution of and maintenance of the magnetic field configuration in the RFP. An equation for \(\langle B \rangle\) has been derived from the resistive MHD equations with viscosity. This equation incorporates the electric field from turbulent fluid motion. Treating the Ohm Law and equation of motion on an equal footing, the induced electric field is related to fluids and magnetic helicities and energies. In particular, the drive of \(\langle B \rangle\) is related to the difference of magnetic and fluid helicity. \(\langle B \rangle\) decays by collisional resistivity and by fluid and Alfvénic diffusion processes.

The RFP dynamo problem differs from the earth dynamo problem in that helicity is not generated by rotation, which is absent. Hence, it has not been apparent that MHD turbulence in an RFP carries helicity. Here this issue has been resolved by relating the fluctuating kinetic
and magnetic helicities to the helicity of the average magnetic field \( B_0 \cdot J_0 \), thus removing uncertainty as to the direction content of the helicities. Also, it follows that:

i) dynamo action is proportional to the deviation from equipartition \( (E_M - E_K) \) and the helicity of the average field.

ii) fluctuations with large \( k_B \) respond to the "twist" of the average field faster, hence generating larger helicity.

A simple model of the fluid and Alfvénic eddy turn-over time is used throughout to determine the effect of renormalization on the propagators.

The steady state equations for \( \langle B \rangle \) and \( B_Z \) are then constructed. It is shown that:

i) dynamo action requires \( E_M > E_K \)

ii) a steady force-free \( B_Z \) magnetic configuration cannot be maintained against resistive decay. However, the significance of a force-free current and field in a turbulent fluid is obscure at best, (i.e. the equilibrium is dynamic, not static). Hence, estimates of fluctuation levels needed for quasi-maintenance are made and used to estimate heat conduction losses due to the dynamo action.

Work is continuing on:

i) effects of mode-coupling

ii) effects of pressure and average flows

Theory of Two-Point Correlation for Drift Wave Instabilities

in Tokamaks

(P. H. Diamond, P. W. Terry)

The theory of two-point phase space density correlation is formulated for drift wave instabilities in order to understand the physics underlying the wavenumber and frequency spectrum, transport and nonlinear stability of low frequency turbulence in tokamaks. Several studies in realistic geometries are undertaken. By elucidating a novel mechanism for the extraction of expansion free energy, these studies detail the impact of incoherent fluctuations associated with microscale correlation on the collective resonances of the system. Dynamics of both electrons and ions are considered under a variety of conditions. The theory is extended to collisional regimes and shown to be germane to a fluid description.

a. Trapped Electrons

Using the bounce-averaged gyrokinetic equation in the ballooning representation, incoherent fluctuations in a trapped electron species is investigated. Incoherent fluctuations excite the collective resonances of the system. In the steady state, the collective modes are damped to balance the excitation. This damping is the width of the collective resonance \( \Delta \omega_k \) centered at the mode frequency \( \omega_k \). From the solution of the two-point phase space density equation, an analytic formula for the width \( \Delta \omega_k \) of the frequency spectrum is obtained and related to "clump-induced" mechanism of extraction of expansion-free energy. Ion Compton scattering is invoked in order to specify the details of the saturation of this overdriven steady state, and yields a formula for the fluctuations spectrum. The impact of incoherent fluctuations on transport is considered. A formula for the energy flux is derived, showing an enhancement of the flux arising from the nonlinear destabilization. An appropriate energy confinement scaling law is also obtained. The effects of a temperature
gradient in electron temperature are also investigated and shown to result in greater excitation and hence a broader frequency spectrum.

b. Incoherent Fluctuations in Ions

Incoherent fluctuations in the ion species are studied noting that broad mode structure in the direction of the magnetic field and uncertainty in the frequency due to its spread make ion incoherent fluctuations at the ion magnetic drift frequency possible for $\omega_{ce}$ turbulence. For dissipative electrons and a collision-driven unstable mode, incoherent fluctuations in the ions are shown to access the expansion-free energy and overdrive the mode.

c. Two Species

Incoherent fluctuations in both species are investigated. In this study two-point phase space density correlation is calculated for ions resonant with the ion magnetic drift as described in Section b above and trapped electrons as described in Section a. For a steady state, the net broadening of the collective resonance, given the linearly destabilizing electron dissipation, is obtained from the solution of coupled spectrum balance equations. It is demonstrated that ion incoherent fluctuations access expansion-free energy and compliment the electron "clump-induced" nonlinear destabilization. The width of the frequency spectrum is found to be significantly broader than the width obtained from electron incoherent fluctuations (Section a) alone.

d. Universal Mode

Nonlinear stability from incoherent emission is considered for the quasi-marginally stable universal mode. "Clump-induced" expansion-free energy accessibility from incoherent mode coupling in the electrons drives a nonlinear instability. In this steady state calculation, the net damping at finite amplitude required to balance the enhanced growth is computed.

e. Dissipative Drift Waves, Fluid Description

The theory of two-point correlation is extended to fluid systems and the effect of collisions on clumps is investigated employing a fluid model with parallel collisional viscosity. Details of the source term of the evolution equation of the two-point correlation (which embodies the effects relating to the extraction of expansion-free energy) remain unaltered in the fluid description. Dissipation and inertial ranges occur in this problem depending on the relative strengths of collisional and $\text{EXB}$ turbulent diffusion. In the dissipative range hollowing out of the two-point correlation is found to occur as a result of collisional decorrelation. Ion dynamics with nonlinear parallel ion heating and parallel and perpendicular viscosities is considered in order to obtain a description of the saturated state as a paradigm for edge turbulence in tokamaks.

Statistical Description of Drift Wave Turbulence

(Wendell Horton, Jr.)

The dissipative drift wave equations have stochastic solutions with broad frequency spectra for each $k$ mode. The simplest explanation for this behavior is the presence of a chaotic attractor in the phase space of the system. The chaotic attractor exists in each (typical) triplet in the $k$ spectrum as shown in the three wave study of Terry and Horton. The statistical properties of the signals on the attractor are independent of initial data due to the exponential
separation of neighboring orbits, a property also present in each triplet. The flow is mixing on the attractor.

The basis of attraction defined as the domain of initial data \( \{ \phi_k(t_0) \} \) which are pulled into the attractor after a transient may be very large as indicated from a few randomly selected initial data sets. No attempt has been made at determining the dimensionality or the boundary of the attractor.

On the attractor the time average appears equal to the phase average over the random-phase ensemble. Thus, the attractor gives rise to a well defined turbulent steady state with a unique spectral distribution and a well defined anomalous transport flux. The turbulent state has no obvious inconsistencies with the \( k_n \) spectra and anomalous transport of the so-called microturbulence measured by electromagnetic wave scattering experiments in tokamaks such as in the Mazzucato experiment.2

References:


Nonlinear Interaction of Toroidicity Induced Drift Modes
(P. H. Diamond, P. L. Similon)

Drift waves have frequently been associated with the observed low frequency density fluctuation and energy confinement degradation in tokamaks. Previous investigation has focused on the development of theoretical models for the nonlinear evolution and saturation of drift mode instabilities and on the comparison of the theoretical predictions with experimental observations. In all cases, a shearless or sheared slab model of a tokamak has been used in the theory. In toroidal geometry, there exist two branches of the basic electron drift mode. The slab-like branch is the toroidal analogue of the familiar Pearlstein-Berk mode structure in the sheared slab mode, and is characterized by a rapid eigenfunction variation along the magnetic field line. The slab-like branch is damped by energy convection toward the ion Landau resonance point. The toroidicity-induced branch, which results from the inclusion of ion magnetic drifts, has a bound wave function which is slowly varying over the scale of the connection length. The toroidicity-included branch, which results from the inclusion of ion magnetic drifts, has a bound wave function which is slowly varying over the scale of the connection length. The toroidicity-induced modes do not experience shear damping. Hence this branch is destabilized by any inverse electron dissipation. For this reason, the toroidicity induced branch appears to be more relevant to considerations of drift wave stability and transport. The purpose of this work is to elucidate the mechanism of nonlinear evolution and saturation of the toroidicity induced branch.

Many possible mechanisms for saturation of drift wave instabilities have been advanced. Here, nonlinear ion-wave interaction is considered. Starting from the nonlinear ion gyrokinetic equation, a nonlinear dielectric operator is constructed by a perturbative expansion to third order in field amplitude. It is important to note that the toroidicity-induced mode structure has significant effects on the nonlinear interaction of the modes. Specifically, the nonlinear dielectric operator is non-local in the extended poloidal coordinate and the beat fluctuation-ion resonance operator must be renormalized.
The latter point follows from the fact that typically $d_G > G_{\nu} v_t \sim \omega_{t \nu}$, where $d_G$ is the ion turbulent decorrelation frequency and $\omega_{t \nu}$ is the ion transit frequency.

Using the saturation condition, a spectral intensity equation is derived. The spectral intensity is determined by the balance of the electron-induced growth with transfer of wave energy to long wavelengths by the ion Compton scattering process. Ultimately, wave energy is deposited in the longest wavelengths, which are not bounded in poloidal angle and hence are shear damped. The spectral intensity equation is solved to obtain the wave number spectrum and total intensity at saturation.

**Drift-wave Turbulence and Plasma Rotation**

(M. Kotschenreuther)

Plasma rotation significantly distorts the symmetry of slab-like driftwave eigenmodes, which can lead to rapid turbulent rotation damping. This effect might give rise to a particular profile of the radial electric field, which could then serve as an experimental test of the presence of this type of mode on the plasma turbulence.

**ii) Numerical Simulation**

**3D MHD Studies of RFP’s (A. Y. Aydemir, D. C. Barnes)**

Numerical studies of the Reversed-Field Pinch (RFP) devices are being performed using a recently developed 3D code that solves the resistive, incompressible, primitive (non-reduced) MHD equations in a cylindrical geometry. Results indicate development and maintenance of a reversed toroidal field on the wall by the dynamo action associated with low-$\beta$ kink-tearing mode activity.

**Implicit Gyrokinetic Particle Codes (D. Barnes, T. Kamimura (IPP-Nagoya), J. N. Leboeuf, F. W. Perkins (PPPL), T. Tajima)**

Gyrokinetic codes are being developed on three fronts. The first (Barnes, Kamimura, Leboeuf, Tajima) is an extension of our implicit algorithm for magnetized plasmas\textsuperscript{1} to include finite ion gyroradius effects through a $J_0(k_r \rho_i)$ weighting factor attached to ion density and ion force. This decentered Lorentz force algorithm with implicit electric forces and FLR effects is similar to W. W. Lee’s modified Poisson’s equation algorithm\textsuperscript{2} but retains all wave-particle mode coupling. The implicit code has the advantage that the time step can greatly exceed the explicit limit. The second approach to implicit gyrokinetics\textsuperscript{3} (Barnes, Kamimura, Leboeuf, Tajima) is to use the modified Poisson’s equation algorithm but solve implicitly for parallel electron and ion motion and parallel forces. Preliminary results in the shearless drift wave case show qualitative agreement with existing codes and linear theory. The third approach\textsuperscript{4} (Perkins, Tajima) is in the developmental stage. It is a gyrokinetic algorithm for the perturbed distribution using the characteristic orbits. As an illustration, for the unmagnetized plasma, the solution of the perturbed Vlasov equation
\[ \partial_t \delta f + \gamma \cdot \nabla \delta f + qE/m \gamma \cdot \nabla \delta f - qE/m \gamma \cdot \nabla \delta f \langle f \rangle, \]

with \( \partial_t \langle f \rangle = 0 \) (Maxwellian) is represented by

\[ \delta f = \sum_i W_i(x_i(t), v_i(t), t) \delta(x-x_i(t)) \delta(v-v_i(t)) \],

where \( W_i = -qE/m \gamma \cdot \nabla \delta f \), \( x_i = v_i = qE/m \). This solution can be provided by a particle code with an individual particle carrying its position \( x_i \), velocity \( v_i \) and weight \( w_i \), where \( x_i \) and \( v_i \) satisfy the characteristics. The same procedure can be applied to the resonant part of the distribution function satisfying the gyrokinetic equation.\(^2\)

References

Three-Dimensional MHD Particle Code for High Beta Tokamak
(F. Brunel)

The MHD-particle code has been extended in 3 dimensions, with conducting boundaries, to simulate tokamak in toroidal geometry. We use a cylindrical coordinate system with \( \theta \) as the toroidal direction and we use a rectangular cross-section. The code pushes the full set of MHD equations (i.e. \( \partial \rho / \partial t + \nabla \cdot \rho \mathbf{v} = 0 \), \( \partial \mathbf{v} / \partial t = -\nabla P + (1/\mu) (\nabla \times \mathbf{B}) \times \mathbf{B} \), \( \partial \mathbf{B} / \partial t = \mathbf{v} \times (\nabla \times \mathbf{B}) + \eta \nabla^2 \mathbf{B} \)) which make this code well suited to describe tokamak in the high-beta small aspect ratio regime. This code can also be used to describe the reversed field pinch configuration. In the 2 1/2D case (two spatial dimensions and three velocity components), due to the non-diffusive Lax-Wendroff scheme coupled with the particle description of the fluid which satisfies exactly the continuity equation, the code has already shown it can handle strong density gradient and can describe the ideal ballooning mode in a natural way without using artificial viscosity terms. The code can also describe fast reconnection regime when sharp current sheet appears. The 3D version of this code is presently tested against the m=1, n=1 kink mode and the growth rates obtained agree well with theory.

Other Particle Codes Development (D. C. Barnes, F. Brunel, J. Geary (student), J. N. Leboeuf, M. Lebrun (student), T. Tajima, E. Zaidman)

Further particle code development embraces 3 distinct areas and codes. The first effort (Geary, Leboeuf, Tajima) pertains to long-time scale magnetostatic guiding-center modes in 2 1/2D with applications to Alfvén wave heating. Since ion time scales are of interest, we treat electron perpendicular motion in the guiding-center approximation and
use standard predictor-corrector algorithms for particle pushing. Preliminary results confirm the linear dispersion relation and fluctuation spectra in the guiding-center limit. A second effort (Barnes, Tajima, Zaidman) involves large spatial scales bounded electromagnetic or magnetostatic codes in 2 1/2D with applications to collisionless tearing. The method is to use different grid spacings, coarse away from the rational surface, very fine around it, to better resolve the tearing layer. Feasibility studies are now in progress. The third effort (Brunel, Lebrun, Tajima) consists in developing a 3D toroidal electrostatic particle code along the lines of Brunel's toroidal particle MHD code in ($r$,θ,$Z$) space (full 3D grid). The non-self-consistent version reproduces the correct drift orbits of the particles in toroidal geometry. A combination of Fast Fourier Transform and finite difference schemes is being built to solve Poisson's equation.

Particle Simulations of Drift Wave Turbulence (P. H. Diamond, J. N. Leboeuf, R. D. Sydora (student), T. Tajima, P. Terry)

Three-dimensional electrostatic particle simulations, using a multi-surface sheared slab model, are being carried out to investigate the nonlinear physics of drift waves. The particle code used is a standard 3D normal mode expansion (Z direction) method with guiding-center electrons and full dynamic ions. All ion Landau resonances are made to lie within the system and many electron resonant surfaces are packed in order to incorporate nonlinear electron effects. The properties of drift waves depend upon rational surface separation $\Delta x_{mn}$ and initial fluctuation levels. The simulations reveal that when the resonant island width of the drift mode $W_{mn}$ is less than the separation $\Delta y_{mn}$, stable drift eigenmodes occur. Preliminary results show that when the resonant island width $W_{mn} > \Delta x_{mn}$, weakly unstable modes are observed with low saturation level. The ratio of the electron decorrelation frequency $\omega_c$ to the wave frequency is of order 1 in this case. The drift wave spectrum shows mode localization about $x_{mn}$ and amplitude decay beyond $x_1$.

Studies with $\omega_c/\omega > 1$ are ongoing.

Reference:

Particle Simulations of Drift Waves in Sheared Slab Geometry with a Single Rational Surface (P. H. Diamond, J. N. Leboeuf, R. D. Sydora (student), T. Tajima, P. Terry)

Verification of the linear theory of drift waves in sheared slab geometry by particle simulations has been elusive at best. We have carried out simulations of drift waves using a standard 2 1/2D particle code with guiding-center electrons and full dynamic ions. For strong shear cases ($L_s/L_n = 28$, $M_i/m_e = 100$, $T_i = T_e$), we find that we can recover the linear theory results as to the absolute stability of the drift waves and their eigenmode structure as long as the ion resonance point lies inside the simulation domain. For the physical even parity of the drift modes with respect to the rational surface, the simulation eigenfrequencies and eigenmodes match the shooting code results. For modes with odd parity with respect to the rational
surface, we can recover the linear theory results as long as the rational surface is not at the walls. For even modes and weaker shear cases \( \mathcal{L}_e / \mathcal{L}_n > 70 \), with the ion resonance point kept inside the simulation domain, the possibility of convective amplification is actively being studied. Preliminary results indicate amplification occurs.

References:

Magnetized Plasma Clump Simulations (R. H. Berman (MIT), T.-Z. Chiueh (student), P. H. Diamond, J. N. Leboeuf, T. Tajima)

Extremely collisionless particle simulations of ion acoustic instabilities carried out at MIT have shown that in the presence of clumps the threshold to instability can be significantly reduced from the conventional linear one.\(^1\) We are studying the effect of clumps on a typical magnetized plasma instability such as the ion cyclotron wave one.\(^2\) This instability is excited by a drifting electron Maxwellian parallel to the magnetic field and generates cyclotron modes propagating almost perpendicular to the magnetic field. We are investigating the effect of clumps on the instability threshold and the enhancement of resistivity predicted by recent calculations of Chiueh and Diamond. A 2 1/2D electrostatic guiding-center electron, full dynamic ions particle code is used for this purpose.

References:

(iii) Magnetic Islands and Solitons

Fast Magnetic Reconnection Driven by the Coalescence Instability
(A. Bhattacharjee, F. Brunel, T. Tajima)

We have studied the effect of compressibility and the magnitude of the toroidal field on the coalescence instability, using a 2 1/2D resistive MHD particle code. We have considered a well-known family of exact, two-dimensional hydromagnetic equilibria in rectangular geometry, with a periodic chain of islands, for which \( \mathbf{B} = \nabla \psi (x,y) \times \mathbf{Z} + B_0 (x,y) \mathbf{Z} \) with \( \psi = (B_0/k) 2 \ln (\cosh ky + \varepsilon \cos kx) \) and \( v_2^2 = - \nabla^2 \psi = B_0 k \exp (2i \psi / B_0) \). The parameter \( \varepsilon \) tells about the current distribution inside the islands; for \( \varepsilon = 1 \), the current density is singular at the center of the island while for \( \varepsilon = 0 \), the current is homogeneous in \( x \) and no island exists. We have considered the cases where \( \varepsilon = .3 \) and \( \varepsilon = .7 \). When \( \varepsilon = .3 \), we observe that for high magnetic Reynolds number that the reconnection rate is linear in time and scales as \( \eta^{1/2} \), i.e. \( \eta \propto \eta^{1/2} t^m \) with \( m = 1 \), as described by Sweet and Parker. The exponent \( m \) does not change as we vary the ratio of toroidal (along \( z \)) or poloidal (in the \( x-y \) plane) pressure with respect to the background plasma pressure; the plasma compressibility plays no role in this case. When \( \varepsilon = .7 \), for which the force of attraction between two neighboring current channels is strong, we observe a reconnection rate \( \psi \propto t^m \) with \( m \) varying from 1.09 for \( B_1 = .25 \) to 1.89 for \( B_1 = 3.5 \).
(the poloidal field B₁ is expressed in unit of the Alfvén wave velocity with respect to the sound velocity of the background plasma): a faster reconnection rate is observed for large value of B₁, as the plasma becomes more compressible. We also observe a reduction in the value of m as we increase the toroidal field B₂, thus making the plasma more incompressible. Such a fast reconnection process may play a role at the formation stage (as well as at the destruction stage after tilting) of the spheromak or reversed theta pinch geometry, where the toroidal and poloidal field are of the same order of magnitude and where the plasma can be considered as compressible.

References:


Nonlinear Tearing Mode Analysis (M. Kotschenreuther,
    P. J. Morrison, R. D. Hazeltine)

A treatment of nonlinear tearing modes in the Rutherford regime is being developed which includes effects such as toroidal curvature with compressibility, density and temperature gradients. These latter effects are known to be highly significant in the linear regime, but in the more relevant nonlinear stage their importance has not previously been examined analytically.

Scattering of Regularized-Long-Wave Solitary Waves
    (J. R. Cary, J. D. Meiss, P. J. Morrison)

In recent times it has become clear that coherent features within turbulent flow are of importance. An example of this may be edge turbulence in tokamaks. It has been suggested that certain tokamak experimental observations of localized density perturbations²,³ can be explained in terms of "soliton-like" structures⁴ that are a kind of nonlinear drift wave (more precisely solitary drift wave). The broadness of the experimentally observed drift wave spectrum⁴ can be explained by utilizing the ideal gas approximation⁵ for an ensemble of solitary waves.

Unlike real solitons, solitary waves emit radiation upon collision. Radiation in this context means that the "pulse-like" density perturbation loses some of its area to small amplitude "oscillatory-like" perturbations. Although the radiation is small, it is clearly not ideal gas behavior. Furthermore, besides radiation more radical outcomes can occur. Our scattering studies are concerned with the investigation of these outcomes.⁶

The most tractable nonlinear drift wave model is the regularized-long-wave (RLW) equation, a nonlinear partial differential equation, which upon linearization yields the drift wave dispersion relation. The nonlinearity balances dispersion to produce two types of solitary drift waves. In one case the pulse and velocity are positive; in the other case both are negative. Hence several scattering combinations are possible. We investigate these scattering situations using numerical methods and approximate analytical techniques.

The analytical techniques are based upon the field Lagrangian for which the RLW equation is external. Noether's theorem enables us to assign mass, momentum, and energy to solitary waves. These quantities
are conserved during collisions and hence, provide rigorous constraints on the collision outcome.

For detailed analysis we use the trial function or Rayleigh-Ritz approximation. We chose a trial function that corresponds to the superposition of the two impinging solitary waves. Variation yields a set of ordinary differential equations that describe the collision.

These equations are analyzed in the Born and "high energy" approximations, which are compared with direct numerical integration of the PDE utilizing the split-step-fast-Fourier transform method. Excellent agreement is obtained between the results of the "high energy" approximation and the integration. The phase shift obtained from the Born approximation indicates a resonance. At this resonance the numerics show that two new solitary waves are produced. We have some evidence that in a certain parameter range simple solitary production may occur. For a wide range of parameter space collisions are nearly elastic - only a small amount of radiation is produced.

References:

(iv) Fluid Models and Hamiltonian Theory

Nonlinear Fluid Models (R. D. Hazeltine, M. Kotschenreuther, P. J. Morrison)

Reduced MHD (RMHD) especially with resistivity, has been found to predict nonlinear fluid motions in tokamaks with impressive accuracy. However, at long mean-free path, the RMHD system can be seen to yield unphysical results; for example, it predicts an incorrect spatial variation of the parallel electric field. Furthermore, it omits a number of FLR and compressibility effects which are known to crucially affect linear stability. Improved versions of RMHD have been constructed by means of a systematic ordering procedure which attempts to model high temperature tokamak conditions more realistically. The new model simulates long mean-free path electron dynamics with sufficient accuracy to describe several kinetic linear and nonlinear processes, including diffusion due to magnetic field stochasticity. The ion dynamics includes non-linear diamagnetic correction, Pfirsch-Schlüter return flows and plasma compressibility. Fortunately, the model is not much more complicated than (high beta) RMHD: only one additional independent field variable is required, and only quadratic nonlinearities appear.

Hamiltonian Description of Reduced MHD (R. D. Hazeltine, P. J. Morrison)

In recent times, a large number of equations have been shown to be Hamiltonian in a generalized sense by the construction of Lie-Poisson structures. This affords a compact way of expressing symmetries inherent in a system of equations. We have obtained this Lie-Poisson
structure for the equations of reduced MHD in both the high and low-$\beta$
approximations. An immediate result is the existence of a new class of
integral invariants that appear to be analogous to potential vorticity
- a quantity that has great physical utility in oceanography. An
additional byproduct of our formulation is the discovery of exact
nonlinear solutions (c.f. below). The formulation also led the way to
a nonlinear variational principle for which the RMHD equations are
external.

References:
I. P. J. Morrison, in Mathematical Methods in Hydrodynamics &
Integrability in Dynamical Systems, M. Tabor & Y. Treve (eds.), New

Exact Nonlinear Solutions to Reduced Fluid Equations
(F. J. Morrison, R. D. Hazeltine)

Exact analytical solutions to the nonlinear reduced MHD system,
and to a generalization of that system, have been constructed. In the
reduced MHD case, the solutions correspond to nonlinear shear-Alfven
waves. Their rapid poloidal and toroidal propagation (without rapid
plasma motion) implies high-frequency components in the electromagnetic
frequency spectra, as frequently observed in Tokamak experiments. The
nonlinear stability of such propagating structures, and their relation
to electromagnetic solitary waves, are under investigation.

Lie-Poisson Structure for the Liouville Equation and the
BBGKY Heirarchy Representation
(F. Morrison, E. Marsden, A. Weinstein (University of
California, Berkeley))

The Lie-Poisson structure for the Liouville equation has been
obtained; hence, we have a Hamiltonian field theory where the sole
dynamical field variable in the n-particle distribution function
defined on $\Gamma$-space. The conventional way of making this system
tractable is to decompose it into the heirarchy of simple and
multiple-particle distribution functions; and then make arguments that
justify truncation. Truncations may be Hamiltonian (e.g. the Vlasov
equation) or non-Hamiltonian (e.g. any collision term that introduces
irreversibility). Representation of functions of the n-particle
distribution as a direct sum of functionals of marginal distribution
functions cause the Lie-Poisson structure to unfold into pieces that
kinematically couple the $i$ and $i+1$ point functions. The
mathematical structure thus obtained is identified as a filtered Lie
Algebra. It is a natural starting point for investigating truncations
that preserve the Hamiltonian property. These truncations can be
independent of the particular Hamiltonian that instills the dynamics.
The structure appears to naturally separate kinematical behavior from
dynamical. Expressions in this formulation are quite complicated so a
Poisson bracket diagram technique has been developed. Here a Poisson
bracket that involves multiparticle distribution functions is
represented diagrammatically. The entire structure is a sum over many
such brackets. Future developments will couple this kinematical
diagram system with dynamics.
II. PERSONNEL

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III. ACTIVITIES

A. Workshops

1. U.S.-Japan Workshop on Hot Electron Physics (organized by J. W. Van Dam and T. Watanabe)

A miniworkshop on "Hot Electron Physics" was held at the Institute for Fusion Studies of the University of Texas at Austin during the week of January 10-14, 1983. This workshop was conducted under the auspices of the U.S.-Japan Joint Institute for Fusion Theory.

Although one in a continuing series of U.S.-Japan workshops, this was the first to employ a "working group" format. A relatively small number of specialists were invited to participate, and the meeting itself was focused on a few specific problems. These problem areas were determined to be (a) the equilibrium and stability of hot electron plasmas and (b) the heating of hot electrons - both with application primarily to bumpy torus and tandem mirror devices.

The purpose of this miniworkshop was likewise twofold. Its intention was, first, to report on recent developments in these two specific areas of hot electron physics and, also, to initiate or continue research collaborations among the Japanese and American participants. To these ends, informal presentations involving a blend of theory, computation, and experiment, as well as small-group working sessions, were scheduled. The success of this miniworkshop format may be judged from the vigorous dialogue between speakers and audience during the scheduled presentations and from the variety of individual scientific interaction during the afternoon free sessions.
2. U.S.-Japan Workshop on Statistical Physics and Chaos in Fusion Plasmas (organized by C. W. Horton, Jr., L. E. Reichl and Yoshi Ichikawa)

On December 13-17, 1982, the Institute was host to a U.S.-Japan Workshop on Statistical Physics and Chaos in Fusion Plasmas. The workshop was sponsored by the Joint Institute for Fusion Theory, with the cooperation of the University's Center for Statistical Mechanics. Over twenty-five scientists attended the workshop; the Japanese delegation was led by professor Yoshi Ichikawa. The meeting was enriched by the participation of several scientists from outside the plasma-fusion community.

An exceptionally broad range of topics was represented, including solitons, fluid turbulence, drift-wave turbulence, plasma transport due to magnetic stochasticity and clumps. The interdisciplinary context proved valuable in several respects. In particular, plasma physicists were offered a wider perspective from which to view the confinement aspects of turbulence and chaos. The workshop proceedings are soon to be published (Statistical Physics and Chaos in Fusion Plasmas, C. W. Horton, Jr., and L. E. Reichl, editors, John Wiley and Sons).
B. IFS Reports (9/82 – 6/83)

#68/68R – IMPLICIT PARTICLE SIMULATION OF MAGNETIC PLASMAS
(sub. to Jour. Comp. Phys.) Barnes, D., Kamimura, T., Leboeuf, J. N., Tajima, T.,

#69 – FINITE LARMOR RADIUS STABILITY THEORY OF EBT PLASMAS

#70 – LOCAL WKB DISPERSION RELATION FOR THE VLASOV-MAXWELL
EQUATIONS (sub. to Phys. Fluids)
- Berk, H. L., Domingues, R. R.

#71 – INSTITUTE FOR FUSION STUDIES PROGRESS REPORT, PERIOD
SEPTEMBER 1981 TO AUGUST 1982 – IFS Staff

#72 – HOT PLASMA DECOUPLING CONDITION FOR LONG WAVELENGTH

#73 – DRIFT WAVES IN A STELLARATOR [Phys. Fluids 26
880(1983), under the title DRIFT WAVES IN A STRAIGHT
 STELLARATOR – Bhattacharjee, A., Sedlak, J. E., Similon, P.
L., Rosenbluth, M. N., Ross, D. W.

#74 – KINETIC THEORY OF RF WAVES IN A PLASMA IN AN
INHOMOGENEOUS MAGNETIC FIELD (sub. to Phys. Fluids) –
Itoh, S-I., Itoh, K.

#75 – KINETIC THEORY OF GLOBAL n=1 INSTABILITIES IN
TOROIDAL PLASMA, (sub. to Phys. Fluids) – Itoh, S-I.,
Itoh, K., Tuda, T., Tokuda, S.

#76 – 3-D NONLINEAR INCOMPRESSIBLE MHD CALCULATIONS (sub.
to Jour. Comp. Phys.) – Aydemir, A., Barnes, D.

#77 – PROCEEDINGS OF U.S.—JAPAN WORKSHOP ON HOT ELECTRON
PHYSICS – Van Dam, J. W. (ed.)

#78 – CHARGED PARTICLE MOTION NEAR A LINEAR MAGNETIC WELL
(sub. to Phys. Fluids) – Kim, J-S., Cary, J. R.


#80 – SCATTERING OF REGULARIZED—LONG—WAVE SOLITARY WAVES
(sub. to Physica D.) – Morrison, P. J., Meiss, J. D.,
Cary, J. R.

#81 – THE FAST ALPHA PARTICLE DISTRIBUTION FUNCTION IN AN
OPEN FIELD-LINE PLASMA WITH ELECTROSTATIC CONFINING
POTENTIAL – Hanson, J. D., Hamnen, H.

#82 – SOLITONS IN TURBULENT FLOW [appears in 1983
U.S.—Japan Workshop, IFSR #79] – Meiss, J. D.
#83 - NON-INTRINSIC AMBIPOLAR DIFFUSION IN TURBULENCE THEORY (sub. to Phys. Fluids) - Berk, H. L., Molvig, K.

#84 - KINETIC THEORY OF ALFVEN WAVES (sub. to Phys. Fluids) - Mahajan, S. M.

#85 - VARIATIONAL METHODS FOR THE THREE-DIMENSIONAL INVERSE EQUILIBRIUM PROBLEM IN TOROIDS--PART 2 (sub. to Phys. Fluids) - Bhattacharjee, A.

#86 - THREE-DIMENSIONAL STRUCTURE OF ICRF WAVES IN TOKAMAK PLASMAS - Itoh, S-I., Itoh, K.

#87 - BALLOONING MODE CALCULATIONS IN STELLARATORS (accept. by Phys. Fluids) - Berk, H. L., Rosenbluth, M. N., Shohet, L.

#88 - REDUCED MAGNETOHYDRODYNAMICS AND THE HASEGAWA-MIMA EQUATION (sub. to Phys. Fluids) - Hazeltine, R. D.

#89 - SEARCHING FOR INTEGRABLE SYSTEMS - Cary, J.

#90 - NON-MAXWELLIAN ION DISTRIBUTIONS CAUSED BY NEOCLASSICAL HEAT CONDUCTION (sub. to Phys. Rev. Lett.) - Ware, A. A.


#92 - STATISTICAL DESCRIPTION OF DRIFT WAVE TURBULENCE - Horton, W.

#93 - MAGNETIC RECONNECTION DRIVEN BY THE COALESCENCE INSTABILITY (sub. to Phys. Fluids) - Bhattacharjee, A., Brunel, F., Tajima, T.


#95 - CONSTRUCTION OF THREE-DIMENSIONAL VACUUM MAGNETIC FIELDS WITH DENSE NESTED FLUX SURFACES (sub. to Phys. Fluids) - Cary, J.

#96 - THE HEAT CONDUCTION PROCESS IN TOKAMAK HOT ION PLASMAS - Ware, A.

#97 - HAMILTONIAN DESCRIPTION OF REDUCED MHD - Hazeltine, R., Morrison, P.