# Thermal density fluctuations and correlations in homogeneous plasmas

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The spatial correlation function for thermal plasma density fluctuations is computed from the plasma entropy. The method is demonstrated by three examples: a Maxwellian plasma, a strongly magnetized plasma and a plasma dominated by Coulomb collisions. In each case the entropy is computed from the one-particle distribution function and then, following the Einstein method, used to construct the probability distribution for density fluctuation.

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## I. INTRODUCTION

A straightforward way [1–3] to compute fluctuation spectra is based on Boltzmann's entropy formula, which can be inverted to express the probability distribution for a fluctuating system in terms of its entropy. This use of the entropy to study small departures from the equilibrium state was first suggested by Einstein [4]. It has the advantages, compared to other means of computing correlations and fluctuations, of conceptual simplicity and transparency. Its most important drawbacks are

1. The method requires knowledge of the entropy. The present work uses Shannon's formula [5] (or, equivalently, Boltzmann's H-function [3]) to compute entropy from the one-particle distribution function—a function that is known in certain idealized situations. Knowledge of the distribution cannot be presumed for more realistic cases, such as turbulent plasmas [6]. However, even in such cases an approximate form for the distribution might be accessible.

2. The method requires a separation of time scales, in order for its entropy to be physically meaningful. The time scale associated with the disordering or mixing process must be very short compared to the scale associated with observation of the system. Only in this case can the system be assumed to have explored the available phase space, so that statistical mechanics pertains. (In Feynman's words [7], it must be assumed that "all the fast things have happened and all the slow things not.") In the standard applications of statistical mechanics, these time scales differ by orders of magnitude; as the two scales become closer in size, statistical predictions become less reliable.

3. The method requires the system to be isolated, at least approximately: loss and source rates must be slow on the time scale of interest.

Using the entropy to compute probabilities and correlation functions is called[1] "the Einstein method." It is closely related [2] to application of the "fluctuation-dissipation theorem, "[8] but has a distinct objective. While the fluctuation-dissipation theorem allows one to compute *temporal* correlations (the two-time correlation function) from the rate of entropy production, the Einstein method allows calculation of *spatial* correlations from the entropy itself. Similarly, while the Einstein method requires time-scale separation, the fluctuation-

dissipation theorem requires knowledge of the linear response function. In structure the two approaches are analogous; in application they are complementary rather than competitive.

The following section shows how the entropy is computed and then used to study fluctuations, for an arbitrary distribution. The succeeding two sections apply this formalism to density fluctuations in the two directions transverse to the magnetic field, in a magnetized plasma (Sec. III); and to electron density fluctuations in one dimension mediated by Coulomb collisions (Sec. IV). The third application, to a simple Maxwellian plasma, is a limiting case; it is discussed near the end of Sec. II.

Our results for the magnetized and collisional plasmas appear to be new. Although the forms of the derived correlation functions are not surprising, their straightforward derivation suggests that the same method might be useful in more realistic cases.

## II. ENTROPY

### A. Shannon form

We denote the entropy by $S$ and its density by $s(\boldsymbol{x})$:

$$S = \int d^3x s(\boldsymbol{x}).$$

The Shannon formula [5] is

$$s = -\int d^3v f \log\left[\left(\frac{h}{m}\right)^3 f\right] \tag{1}$$

where $f(\boldsymbol{x}, \boldsymbol{v})$ is the one particle distribution function, $m$ is the particle mass and $h$ is Planck's constant. (The variables $\boldsymbol{x}$ and $\boldsymbol{v}$ refer to position and velocity.) All the distribution functions that we consider have the form

$$f(\boldsymbol{x}, \boldsymbol{v}) = f_{M0}(\boldsymbol{v})[1 + \alpha + \widehat{f}(\boldsymbol{x}, \boldsymbol{v})] \tag{2}$$

where

$$\alpha \equiv \frac{\widetilde{n}(\boldsymbol{x})}{n_0}$$

is the normalized fluctuation in the density $n$, $\widehat{f} \ll 1$ describes the plasma response to that fluctuation and $f_{M0}$ is a spatially constant Maxwellian:

$$f_{M0} \equiv \frac{n_0 m^{3/2}}{(2\pi T)^{3/2}} e^{-mv^2/T} \tag{3}$$

with $n_0$ the lowest-order, constant density and $T$ the constant temperature, measured in energy units. We consider density fluctuations only; including temperature fluctuation introduces technical complications that we reserve for a separate study.

After substituting Eq. (2) into Eq. (1), we take the following steps:

1. denote the lowest order (constant Maxwellian) term by

$$s_0 \equiv -\int d^3v f_{M0} \log f_{M0} = n_0 \left(\Gamma + \frac{3}{2}\right)$$

   where $\Gamma$ is a familiar [9] constant,

$$\Gamma = -\log\left[\left(\frac{h}{m}\right)^3 \frac{n_0 m^{3/2}}{(2\pi T_0)^{3/2}}\right].$$

   This is the entropy of an ideal gas.

2. ignore terms linear in the perturbation $\alpha$, since they do not contribute to the spatial integral. In this regard, notice that the response function $\widehat{f}$ is also linear in $\alpha$.

3. neglect terms of cubic or higher order, assuming the statistics to be dominated by the quadratic term.

The result is conveniently expressed as

$$s = s_0 - \Delta s \tag{4}$$

where [6]

$$\Delta s \equiv -\frac{1}{2}\int d^3v f_{M0}\left(\alpha + \hat{f}\right)^2. \tag{5}$$

### B. Einstein distribution

Consider a system parameterized by a set of variables $\boldsymbol{\alpha} = \{\alpha_i\}$, chosen so that the thermal equilibrium state has $\boldsymbol{\alpha} = 0$. Following Einstein [4], we assume that, for sufficiently small $\alpha_i$, the system entropy $S(\boldsymbol{\alpha})$ can be defined for nonequilibrium states and that it satisfies the Boltzmann formula

$$S(\boldsymbol{\alpha}) = \log P(\boldsymbol{\alpha})$$

where $P$ measures the probability for observing the fluctuation $\boldsymbol{\alpha}$. Then we can compute all statistical properties of the $\alpha_i$ from the Einstein distribution

$$P(\boldsymbol{\alpha}) = e^{S(\boldsymbol{\alpha})}. \tag{6}$$

Since $S$ is maximal at the equilibrium state, it contains no terms linear in the $\alpha_i$; since the $\alpha_i$ are small, the probability distribution is well approximated by the quadratic terms in the entropy, which we denote here by

$$\Delta S = -\frac{1}{2}\alpha_i \sigma_{ij} \alpha_j, \tag{7}$$

thus defining the entropy matrix $\sigma_{ij}$. Note that the restriction to quadratic terms implies Gaussian statistics.

We now can compute any correlation of the form

$$\langle \alpha_i \alpha_j \cdots \rangle = \int d\boldsymbol{\alpha} P(\boldsymbol{\alpha}) \alpha_i \alpha_j \cdots$$

using the well-known manipulations of Gaussian statistics [2]. In particular one finds that the correlation matrix

$$c_{ij} \equiv \langle \alpha_i \alpha_j \rangle \tag{8}$$

is simply the matrix inverse to $\sigma_{ij}$:

$$c_{ik}\sigma_{kj} = \delta_{ij}. \tag{9}$$

### C. Spatial correlations

This work considers the continuum case, in which the various $\alpha_i$ are density fluctuations at various spatial locations $\boldsymbol{x}$:

$$i \to \boldsymbol{x}.$$

Thus the correlation matrix becomes a correlation function

$$c(\boldsymbol{x}, \boldsymbol{x}') = \langle \alpha(\boldsymbol{x}) \alpha(\boldsymbol{x}') \rangle \tag{10}$$

and the entropy matrix becomes the *entropy kernel*, $\sigma(\boldsymbol{x}, \boldsymbol{x}')$, defined by the continuum version of Eq. (7):

$$\Delta S = \int d^3x \Delta s(\boldsymbol{x}) = -\frac{1}{2} \int d^3x d^3x' \alpha(\boldsymbol{x}) \sigma(\boldsymbol{x}, \boldsymbol{x}') \alpha(\boldsymbol{x}'). \tag{11}$$

Finally Eq. (9) becomes the integral equation

$$\int d^3x'' c(\boldsymbol{x}, \boldsymbol{x}'') \sigma(\boldsymbol{x}'', \boldsymbol{x}') = \delta(\boldsymbol{x} - \boldsymbol{x}'). \tag{12}$$

Solving this equation for the correlation function is in general a challenging task, but we now specialize to a situation in which the solution is easy: spatially homogeneous statistics. Then $\sigma(\boldsymbol{x}, \boldsymbol{x}') = \sigma(\boldsymbol{x} - \boldsymbol{x}')$, and similarly for $c(\boldsymbol{x}, \boldsymbol{x}')$, so that Eq. (12) is a convolution integral, trivially solved by means of Fourier transforms. If we indicate the Fourier transform with a tilde,

$$\widetilde{c}(\boldsymbol{k}) = \int d^3x e^{i\boldsymbol{k}\cdot\boldsymbol{x}} c(\boldsymbol{x})$$

then (12) implies

$$\widetilde{c} = \frac{1}{\widetilde{\sigma}} \tag{13}$$

and the correlation function is

$$\langle \alpha(\boldsymbol{x})\alpha(0)\rangle = c(\boldsymbol{x}) = \frac{1}{(2\pi)^3}\int d^3k \frac{e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}}{\widetilde{\sigma}}. \tag{14}$$

Note that $\widetilde{c}(\boldsymbol{k})$, the spectral density, is itself of considerable interest.

The assumption of spatial homogeneity approximates the true statistics if the fluctuations vary on a much shorter spatial scale than the background plasma. This is frequently but not always the case in experimental plasmas.

Our procedure is simply outlined. Starting with kinetic theory for the one-particle distribution $f$, one computes the entropy change due to fluctuations from Eq. (5). Comparing the result to Eq. (11), one extracts the entropy kernel, and then uses Eq. (13) to get the spectral density and correlation function. The physics underlying this straightforward method is also clear: if the system has time to find its most probable state (without significant external interference), then the fluctuations one observes will be the most likely fluctuations, determined by the form of the perturbed entropy. The most demanding step in this recipe is the first: finding $f$.

We point out that a closely parallel method, applied to the entropy production rate rather than $S$, can be used to compute time correlations of fluctuations [2].

### D. The local approximation

In the examples that we consider, the distribution function is determined by a mixing process that acts on a length-scale short compared to the gradient scale of the fluctuations. In a magnetized plasma, for example, we consider fluctuations that vary slowly on the scale

of the gyroradius. It is then natural to derive an entropy kernel with the following form:

$$\sigma(\boldsymbol{x}) = \sigma_0\delta(\boldsymbol{x}) + \sigma_2\lambda^2\nabla^2\delta(\boldsymbol{x}) + \sigma_4\lambda^4\nabla^4\delta(\boldsymbol{x}) - \cdots \tag{15}$$

where the $\sigma_i$ and $\lambda$ are constants. It is evident that $\lambda$ has the dimensions of length; in the magnetized plasma case, we will find that $\lambda = \rho_t$, the thermal gyroradius; in the collisional case, $\lambda$ is the mean-free path.

This localized approximation to the kernel allows straightforward calculation of the correlation function. Keeping terms up to fourth order in Eq. (14), we obtain

$$c(\boldsymbol{x}) = \frac{1}{(2\pi\lambda)^3}\int d^3\zeta \frac{e^{-i\boldsymbol{\zeta}\cdot\boldsymbol{y}}}{\sigma_4(\zeta^4 - B\zeta^2 + C)} \tag{16}$$

where $\zeta = k\lambda$, $\boldsymbol{y} = \boldsymbol{x}/\lambda$, $B = \sigma_2/\sigma_4$ and $C = \sigma_0/\sigma_4$.

Note that the integral is uniquely specified, and the spectral density is well behaved, only if the roots of the denominator are displaced from the real-$\zeta$ axis; thus we require $B^2 < 4C$ or

$$\sigma_2 < 2\sqrt{\sigma_0\sigma_4}. \tag{17}$$

This requirement is met for the distributions considered in the following sections.

### E. Maxwellian plasma

In the simplest application of the Einstein method the response function $\widehat{f}$ is neglected. Then the distribution is a Maxwellian with a small, spatially dependent density perturbation. The corresponding entropy kernel is

$$\sigma(\boldsymbol{x}) = \sigma_0\delta(\boldsymbol{x}). \tag{18}$$

By comparing Eqs. (5) and (11) one finds that $\sigma_0 = n_0$. The spectral density is constant (white noise spectrum), so the correlation length vanishes:

$$c(\boldsymbol{x}) = n_0^{-1}\delta(\boldsymbol{x}).$$

Hence

$$\langle\widetilde{n}(\boldsymbol{x})\widetilde{n}(\boldsymbol{x}')\rangle = n_0\delta(\boldsymbol{x} - \boldsymbol{x}') \tag{19}$$

so that the fluctuation amplitude is proportional to $\sqrt{n_0}$. This proportionality is well known [3], although the present derivation is not standard and relatively compact.

The entropy kernel, Eq. (18), corresponds to that of an ideal gas, in which particle interactions are neglected. Hence the result, Eq. (19), is not surprising. However, it is possible for a gas to be Maxwellian and yet contain finite spatial correlations; the fact that the Einstein method mistreats this case may be related to our use of the Boltzmann entropy, rather than the Gibbs entropy [10].

The following two sections consider somewhat more interesting applications of the method.

## III. MAGNETIZED PLASMA

### A. Distribution function

We consider a plasma magnetized by a uniform magnetic field $\boldsymbol{B} = \boldsymbol{B}$; the corresponding gyrofrequency is denoted by $\Omega = eB/m$ and the thermal gyroradius is

$$\rho_t = v_t/\Omega$$

where $v_t = \sqrt{2T/m}$ is the thermal speed. The plasma is magnetized when $\rho_t$ is small compared to the scale length $L$ for variation perpendicular to $\boldsymbol{b}$. For definiteness we consider the plasma electrons, but the ion problem is not significantly different. We also neglect collisions, assuming the collision frequency to be small on time scales of interest.

In this and the following section we use velocity coordinates $(\eta, \xi, \gamma)$ where $\eta$ is the normalized speed,

$$\eta = v/v_t,$$

$\xi$ is the direction cosine parallel to the magnetic field,

$$\xi = \boldsymbol{b} \cdot \boldsymbol{v}/v,$$

and $\gamma$ is the gyrophase angle,

$$\gamma = \arctan(v_3/v_2)$$

where $v_2$ and $v_3$ are the components of $\boldsymbol{v}$ perpendicular to the field. Thus the electron velocity is written as

$$\boldsymbol{v} = v_t\eta[\boldsymbol{b}\xi + \sqrt{1-\xi^2}(\boldsymbol{e}_2 \cos\gamma + \boldsymbol{e}_3 \sin\gamma)]$$

where the $\boldsymbol{e}_i$ are the obvious unit vectors. It also is convenient to introduce the vector gyroradius

$$\boldsymbol{\rho} = \frac{\boldsymbol{b} \times \boldsymbol{v}}{\Omega}.$$

We do not review the well-established kinetic theory of a magnetized plasma (see, for example [11]). The key point is that, when the magnetic field is uniform and the electric field sufficiently weak, approximate constants of the motion are the kinetic energy, the magnetic moment, and the guiding-center position,

$$\boldsymbol{x}_{gc} = \boldsymbol{x} - \boldsymbol{\rho}. \tag{20}$$

For the present case of a uniform magnetic field, it follows that the solution to the kinetic equation can depend on $\eta$ and $\xi$ in an arbitrary way, but must depend on position and on gyrophase through the combination $\boldsymbol{x} - \boldsymbol{\rho}$.

The displacement Eq. (20) in itself has no effect on entropy; to obtain an entropy change from gyromotion, we must also average over the gyrophase ("coarse-graining"). Such an average makes sense when observation of fluid fluctuations occupies a time interval long compared to the gyroperiod. Hence the pertinent distribution for a magnetized plasma has the general form

$$f(\boldsymbol{x}, \boldsymbol{v}) = \langle F(\boldsymbol{x} - \boldsymbol{\rho}, \eta, \xi) \rangle_\gamma \tag{21}$$

in which gyrophase dependence enters only through $\boldsymbol{\rho}$, the angle brackets indicate a gyrophase average,

$$\langle \cdots \rangle_\gamma = \oint (d\gamma/2\pi) \cdots$$

and the function $F$ is to be chosen consistently with Eq. (2). Thus we obtain the distribution

$$f(\boldsymbol{x}, \boldsymbol{v}) = f_{M0} \left[ 1 + \langle \alpha(\boldsymbol{x} - \boldsymbol{\rho}, \eta) \rangle_\gamma \right]. \tag{22}$$

In the notation of Eq. (2) we have

$$\alpha + \widehat{f} = \langle \alpha(\boldsymbol{x} - \boldsymbol{\rho}, \eta) \rangle_\gamma = \langle e^{-\boldsymbol{\rho} \cdot \nabla} \alpha(\boldsymbol{x}, \eta) \rangle_\gamma. \tag{23}$$

Keeping terms through fourth order in $\rho$, we find that

$$\widehat{f} = \frac{\rho^2}{4} \nabla^2 \alpha + \frac{\rho^4}{64} \nabla^4 \alpha. \tag{24}$$

Here all gradients are taken in the plane perpendicular to $\boldsymbol{b}$: $\nabla = \nabla_\perp$. However, for simplicity we assume $\nabla_\parallel \alpha = 0$ and suppress the $\perp$-subscript; in the resulting two-dimensional problem $S$ represents the entropy per unit length along $\boldsymbol{b}$.

### B. Correlation function

We substitute Eq. (24) into Eq. (5), collect second- and fourth-order terms, and perform the integrals to obtain the entropy density perturbation

$$\Delta s = -\frac{n_0}{2}\left[\alpha^2 + \frac{\rho_t^2}{2}\alpha\nabla^2\alpha + \frac{\rho_t^4}{16}\left(\alpha\nabla^4\alpha + 2(\nabla^2\alpha)^2\right)\right]. \tag{25}$$

The entropy change $\Delta S$ is of course the integral of this function over the perpendicular plane; partial integration then produces replacement

$$(\nabla^2\alpha)^2 \to \alpha\nabla^4\alpha$$

and we find

$$\Delta S = -\frac{n_0}{2}\int d^2x\alpha\left(\alpha + \frac{\rho_t^2}{2}\nabla^2\alpha + \frac{3}{16}\rho_t^4\nabla^4\alpha\right). \tag{26}$$

Comparing this expression to (11) we see that (15) is reproduced, with

$$\sigma_0 = n_0, \;\; \sigma_2 = \frac{n_0}{2}, \;\; \sigma_4 = \frac{3n_0}{16}.$$

Notice that Eq. (17) is satisfied.

Thus the spectral density is

$$\widetilde{c}(\boldsymbol{k}) = \frac{16}{3n_0}\frac{1}{\zeta^4 - (8/3)\zeta^2 + (16/3)} \tag{27}$$

while the correlation function is given by the two-dimensional version of Eq. (16):

$$\begin{aligned} c(\boldsymbol{x}) &= \frac{1}{(2\pi)^2}\frac{16}{3n_0}\int d^2k e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}\frac{1}{\zeta^4 - (8/3)\zeta^2 + (16/3)} \\ &= \frac{4}{3(\pi\rho_t)^2 n_0}\int_0^\infty \frac{\zeta d\zeta}{\zeta^4 - (8/3)\zeta^2 + (16/3)}\oint d\theta e^{-i\zeta y\cos\theta} \\ &= \frac{4}{3(\pi\rho_t)^2 n_0}\int_0^\infty \frac{J_0(\zeta y)\zeta d\zeta}{\zeta^4 - (8/3)\zeta^2 + (16/3)} \end{aligned} \tag{28}$$

where

$$y \equiv x/\rho_t$$

and $J_0$ is the Bessel function. To evaluate the integral, we decompose the integrand into partial fractions,

$$\frac{1}{\zeta^4 - (8/3)\zeta^2 + (16/3)} = \frac{3}{8\sqrt{2}i}\left(\frac{1}{\zeta^2 + \zeta_0^2} - \frac{1}{\zeta^2 + \zeta_0^{*2}}\right)$$

where

$$\zeta_0 = \left[(4/3)(\sqrt{2} - i)\right]^{1/2} \tag{29}$$

and $\zeta_0^*$ is its complex conjugate. Thus

$$c(\boldsymbol{x}) = \frac{1}{\sqrt{2}(\pi\rho_t)^2}\Im\left[\int_0^\infty \frac{d\zeta\zeta J_0(\zeta y)}{\zeta^2 + \zeta_0^2}\right]$$

where$\Im$ refers to the imaginary part. The integral appearing here is known [12], and we have

$$c(\boldsymbol{x}) = \frac{1}{\sqrt{2}(\pi\rho_t)^2}\Im[K_0(\zeta_0 y)] \tag{30}$$

where $K_0$ is the MacDonald function. This function is displayed in Fig. 1.

## IV. COLLISIONAL PLASMA

### A. Distribution function

We now consider density gradients parallel to a uniform magnetic field in a collisional dominated plasma. In this case the response is determined by the Spitzer [13] equation

$$v_\parallel \boldsymbol{b}\cdot\nabla f - C(f) = -f_{M0} v_\parallel \boldsymbol{b}\cdot\nabla\alpha + Q$$

where $Q$ represents a particle source, included to balance the losses from parallel diffusion. (The system remains effectively isolated despite the source term because, as shall be seen, the required $Q$ is too weak to affect the lowest order solution.) For simplicity, we consider the electron response in a plasma whose ions have large ionic charge, so that the collision operator can be approximated by a Lorentz gas operator:

$$C(f) = \nu\eta^{-3}\frac{\partial}{\partial\xi}(1-\xi^2)\frac{\partial f}{\partial\xi},$$

Here $\nu\eta^{-3}$, with $\nu$ a constant measure of the collision frequency, is the speed-dependent collision frequency. Generalization to allow energy scattering and electron-electron collisions is straightforward. The kinetic equation has become

$$\frac{\partial}{\partial\xi}(1-\xi^2)\frac{\partial\widehat{f}}{\partial\xi} - \lambda\eta^4\xi L\nabla_\parallel\widehat{f} = \lambda\eta^4(\xi\alpha' + \widehat{Q}). \tag{31}$$

Here

$$\lambda \equiv \frac{v_t}{\nu}$$

is the mean-free path ,

$$\alpha' \equiv \nabla_{\|}\alpha = d\alpha/dx$$

is the parallel gradient, $\widehat{Q}$ is the appropriately normalized source, and distance along the field is represented by the coordinate $x$.

In the collision-dominated case,

$$\lambda(d/dx) \ll 1.$$

We therefore expand $\widehat{f}$ in powers of $\lambda$ and quickly find the well-known [14] solution

$$\widehat{f} = -\frac{\lambda}{2}\alpha'\eta^4\xi + \mathcal{O}(\lambda^2). \tag{32}$$

Use of the full collision operator merely requires replacing the $\eta^4$ appearing here by the so-called Spitzer[13] function. We point out that Eq. (31) is easily solved to any order in $\lambda$, by expansion in Legendre polynomials. However, the correlation function computed from the higher order theory is unphysically singular, presumably because the source term enters the analysis in second order.

### B. Correlation function

Substitution of the distribution into Eq. (5) yields

$$\Delta s = -\frac{1}{2}\int d^3v f_{M0}[\alpha^2 + (1/12)(\eta^4\lambda\alpha')^2 + \mathcal{O}(\Delta^4)].$$

After performing the velocity integral we integrate over the parallel spatial coordinate to find

$$\Delta S = -\frac{n_0}{2}\int dx(\alpha^2 + \sigma_2\alpha\alpha'') \tag{33}$$

where $\sigma_2 = -315/64$. The spectral density is therefore

$$\widetilde{c}(k) = n_0^{-1}\frac{1}{1+|\sigma_2|(k\lambda)^2} \tag{34}$$

and the correlation function is

$$c(x) = \frac{\kappa e^{-\kappa|x|/\lambda}}{2n_0\lambda} \tag{35}$$

where

$$\kappa \equiv \sqrt{64/315} \approx 0.451.$$

The slope discontinuity at $x = 0$ reflects inability to resolve fine structure in the small gyroradius ordering; the same discontinuity appears in typical temporal correlation functions, and for the same reason [2].

## V. SUMMARY

In many plasma systems, one can identify a time-scale that is slow compared to some mixing process (such as collisions, gyromotion, or neoclassical banana motion) and yet rapid compared to loss rates and source terms. In that case the entropy is usefully predictive: Einstein's formula, Eq. (6), provides the probability distribution for fluctuations. Even when the time scale separation is moderate, one expects the method to have approximate predictive value.

We have used the Einstein method to compute the spatial correlations of density fluctuations, and therefore the amplitude of such fluctuations, for three plasma systems. A summary of our procedure is provided in Sec. II, after Eq. (14). The (well-known) correlation function for a Maxwellian plasma is given by Eq. (19); for a magnetized plasma by Eq. (30); and for a collisional plasma by Eq. (35).

Of course plasma physics often studies situations without sufficient isolation or time-scale separation. Note, however, that these properties are assumed, at least implicitly, in any approximation to the kinetic equation that omits sources and the time-derivative of the distribution. In particular, magnetically confined plasmas, if sufficiently quiescent, should be amenable to analysis based on entropy.

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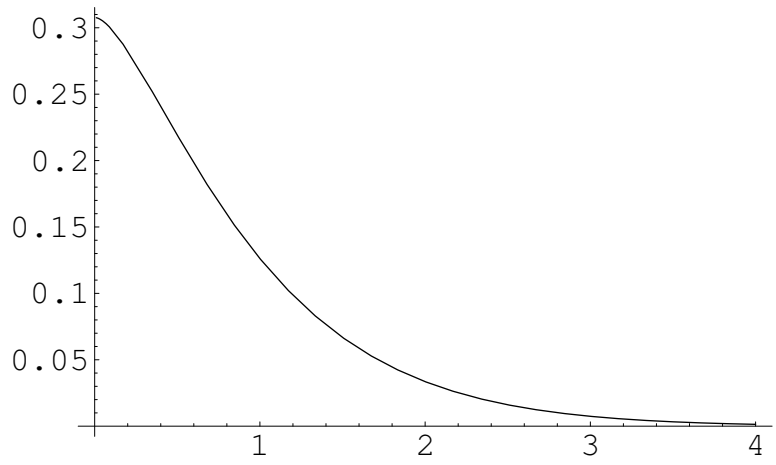
## FIGURE CAPTIONS

FIG. 1. Correlation function for density fluctuation in a magnetized plasma. The abscissa measures separation in units of the thermal gyroradius, $\rho_t$

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