NON-MODAL RENORMALIZED KINETIC THEORY OF THE DRIFT TURBULENCE OF THE NON-STATIONARY SHEAR FLOWS

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A great variety of experimental results are now available, which confirm a suppression of turbulence by $E \times B$ velocity shear as a key feature of the regimes of improved confinement of plasma and formation of transport barriers in tokamaks.

Some experimental results, detected at L-H transition

1. Rapid suppression or reduction of the drift turbulence and anomalous transport.
2. Suppression predominantly of the short wavelength part of the drift turbulence spectrum.
3. Reduction of the cross phase between the perturbed potential and perturbed density.
4. Growth of the correlation time in shear flow turbulence.

For low frequency modes of drift type the non–modal approach was developed in the series of our papers

- Mikhailenko V.S., Mikhailenko V.V., and Stepanov K.N. Initial value problem solution for Hasegawa-Wakatani equations for plasma with radial electric field shear. IAEA Technical Committee Meeting on First Principle - based Transport Theory, June 21-23, 1999, Kloster Seeon, Germany, Abstract P15
- Mikhailenko V.S., Mikhailenko V.V., and Stepanov K.N. Temporal evolution of linear drift waves in a collisional plasma with homogeneous shear flow, Physics of Plasmas, 2000, vol.7, N.1, p.94-100.
- Mikhailenko V.S., Mikhailenko V.V., M.F.Heyn, and S.M.Mahajan, Temporal evolution of drift Alfven waves and instabilities in an inhomogeneous plasma with homogeneous shear flow. Phys. Review E66, p.066409.1-066409.12, 2002.
- Mikhailenko V.S., Weiland J. Toroidal ion temperature gradient driven instability in plasma with shear flow, Physics of Plasmas, vol. 9, p.529-535, 2002
- Mikhailenko V.S., Mikhailenko V.V., and Weiland J., Rayleigh-Taylor instability in plasmas with shear flow, Physics of Plasmas, vol. 9, 7, p.2891-2895, 2002
- Mikhailenko V.S., Scime E.E., Mikhailenko V.V., Stability of stratified flow with inhomogeneous shear, Physical Review E, vol.71., 026306, 2005
- Mikhailenko V.S., Mikhailenko V.V., Azarenkov N.A., Stepanov K.N. Evolution of Anomalous Transport in Shear Flow of Toroidal Devices, 16th IAEA Fusion Conference, Chengdu, China, 2006
- Mikhailenko V.S., Mikhailenko V.V., Stepanov K.N., Azarenkov N.A. Anomalous transport of particles in plasma flow with strong inhomogeneous velocity shear, Physics of Plasmas, vol. 15, 072102, 2008
- Mikhailenko V.S., Mikhailenko V.V., Stepanov K.N., Renormalized theory of drift turbulence in plasma shear flows, Plasma Physics and Controlled Fusion, vol.52 , 055007, 2010

$$\phi(\mathbf{R},t) = \phi_0 \exp\left(i\omega t - ik_x\xi - ik_y\eta\right)$$

in shear flow with velocity

$$V_0(\xi)\,\mathbf{e}_\eta = V_0'\xi\mathbf{e}_y.$$

In laboratory set of reference

$$x = \xi, \quad y = \eta + V_0'\xi t \quad or \quad \xi = x, \quad \eta = y - V_0' x t$$

$$\begin{aligned}\phi(\mathbf{r},t) &= \phi_0 \exp\left(i\omega t - ik_x x - ik_y\left(y - V_0' x t\right)\right)\\ &= \phi_0 \exp\left(i\left(\omega + k_y V_0' x\right)t - ik_x x - ik_y y\right)\\ &= \phi_0 \exp\left(i\omega t - i\left(k_x - k_y V_0' t\right)x - ik_y y\right).\end{aligned}$$

For $V_0' \simeq \gamma$ and $t > \gamma^{-1}$, $V_0' t > 1$!!!

In the normal-mode approach, the solution is sought in the form $\phi(\mathbf{r},t) = \phi(x)\exp\left(ik_y y - i\omega t\right)$.

Hasegawa–Wakatani system of equations for plasma with strong shear flow

Mikhailenko V.S., MikhailenkoV.V., and Stepanov K.N., Initial value problem solution for Hasegawa-Wakatani equations for plasma with radial electric field shear. IAEA Technical Committee Meeting on First Principle – based Transport Theory, June 21–23, 1999, Kloster Seeon, Germany, Abstract P15;
Mikhailenko V.S., Mikhailenko V.V., and Stepanov K.N., Temporal evolution of linear drift waves in a collisional plasma with homogeneous shear flow. Physics of Plasmas, 2000, vol.7, N.1, P.94–100.
Mikhailenko V.S., Mikhailenko V.V., Stepanov K.N., Azarenkov N.A. Anomalous transport of particles in plasma flow with strong inhomogeneous velocity shear, Physics of Plasmas, vol. 15, 072102, 2008

We investigate the temporal evolution of drift modes in shear flow using the Hasegawa–Wakatani system of equations for the dimensionless density $n = \tilde{n}/n_e$ and potential $\phi = e\varphi/T_e$ perturbations (n_e is the electron background density, T_e is the electron temperature)

$$\rho_s^2 \left(\frac{\partial}{\partial t} + V_0' x \frac{\partial}{\partial y} - \frac{cT_e}{eB} \left(\frac{\partial \phi}{\partial y}\frac{\partial}{\partial x} - \frac{\partial \phi}{\partial x}\frac{\partial}{\partial y} \right) \right) \nabla^2 \phi = a \frac{\partial^2}{\partial z^2} (n - \phi)$$

$$\left(\frac{\partial}{\partial t} + V_0' x \frac{\partial}{\partial y} - \frac{cT_e}{eB} \left(\frac{\partial \phi}{\partial y}\frac{\partial}{\partial x} - \frac{\partial \phi}{\partial x}\frac{\partial}{\partial y} \right) \right) n + v_{de} \frac{\partial \phi}{\partial y} = a \frac{\partial^2}{\partial z^2} (n - \phi)$$

With new spatial variables ξ, η,

$$t = t, \qquad \xi = x, \qquad \eta = y - V_0' x t, \qquad z = z.$$

the Hasegawa–Wakatani system has a form

$$\rho_s^2 \left(\frac{\partial}{\partial t} - \frac{cT_e}{eB} \left(\frac{\partial \phi}{\partial \eta} \frac{\partial}{\partial \xi} - \frac{\partial \phi}{\partial \xi} \frac{\partial}{\partial \eta} \right) \right) \Delta \phi = a \frac{\partial^2}{\partial z^2} (n - \phi) \, ,$$

$$\left(\frac{\partial}{\partial t} - \frac{cT_e}{eB} \left(\frac{\partial \phi}{\partial \eta} \frac{\partial}{\partial \xi} - \frac{\partial \phi}{\partial \xi} \frac{\partial}{\partial \eta} \right) \right) n + v_{de} \frac{\partial \phi}{\partial \eta} = a \frac{\partial^2}{\partial z^2} (n - \phi) \, .$$

The Laplacian operator Δ now becomes time-dependent,

$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} = \left(\frac{\partial}{\partial \xi} - V_0' t \frac{\partial}{\partial \eta} \right) \left(\frac{\partial}{\partial \xi} - V_0' t \frac{\partial}{\partial \eta} \right) + \frac{\partial^2}{\partial \eta^2}.$$

leaving us with an initial value problem to solve.

In the normal-mode approach, the solution is sought in the form $\phi(\mathbf{r},t) = \phi(x)\exp(ik_y y - i\omega t)$, and for the modal structure $\phi(x)$ we obtain for $n = \phi$, Hasegawa-Mima equation

$$\rho_s^2 \frac{\partial^2 \phi(x)}{\partial x^2} - \left[\left(1 + \rho_s^2 k_y^2\right) - \frac{k_y v_{de}}{\omega - k_y V_0(x)} \right] \phi(x) = 0.$$

Normal-mode approach is not acceptable for time dependent velocity shear flows

With new spatial variables ξ, η,

$$t = t, \qquad \xi = x, \qquad \eta = y - V_0' x t, \qquad z = z$$

the Fourier transformed Hasegawa–Wakatani system reduces to the equation

$$\frac{1}{C} \frac{\partial^2}{\partial T^2} \left[\left(1 + T^2\right) \phi \right] + \frac{\partial}{\partial T} \left\{ \left[1 + k_y^2 \rho_s^2 \left(1 + T^2\right) \right] \phi \right\} + iSk_y \rho_s \phi = 0,$$

where a dimensionless time variable T is defined by $T = V_0' t - (k_x / k_y)$ and parameters C and S are equal respectively to $C = \dfrac{ak_z^2}{\rho_s^2 k_y^2 V_0'} = \dfrac{T_e k_z^2}{\rho_s^2 k_y^2 \, V_0' n_0 e^2 \eta_{\parallel}}$, $S = \dfrac{k_y v_{de}}{V_0' k_y \rho_s}$.

The solution for $\phi(\tau, k_x, k_y, k_z)$ for large values of the parameter $C \gg 1$ (solution to Hasegawa-Mima equation) was obtained and it is equal to

$$\phi(t, k_x, k_y, k_z) = \phi(t=0, k_x, k_y, k_z) \frac{1 + \rho_s^2 \left(k_y^2 + k_x^2\right)}{1 + \rho_s^2 k_y^2 + \rho_s^2 \left(k_y V_0' t - k_x\right)^2}$$
$$\times \exp\left\{ -i \frac{S}{\sqrt{1+\rho_s^2 k_y^2}} \left(\tan^{-1} \frac{\rho_s \left(k_y V_0' t - k_x\right)}{\sqrt{1+\rho_s^2 k_y^2}} + \tan^{-1} \frac{k_x \rho_s}{\sqrt{1+\rho_s^2 k_y^2}} \right) \right\}.$$

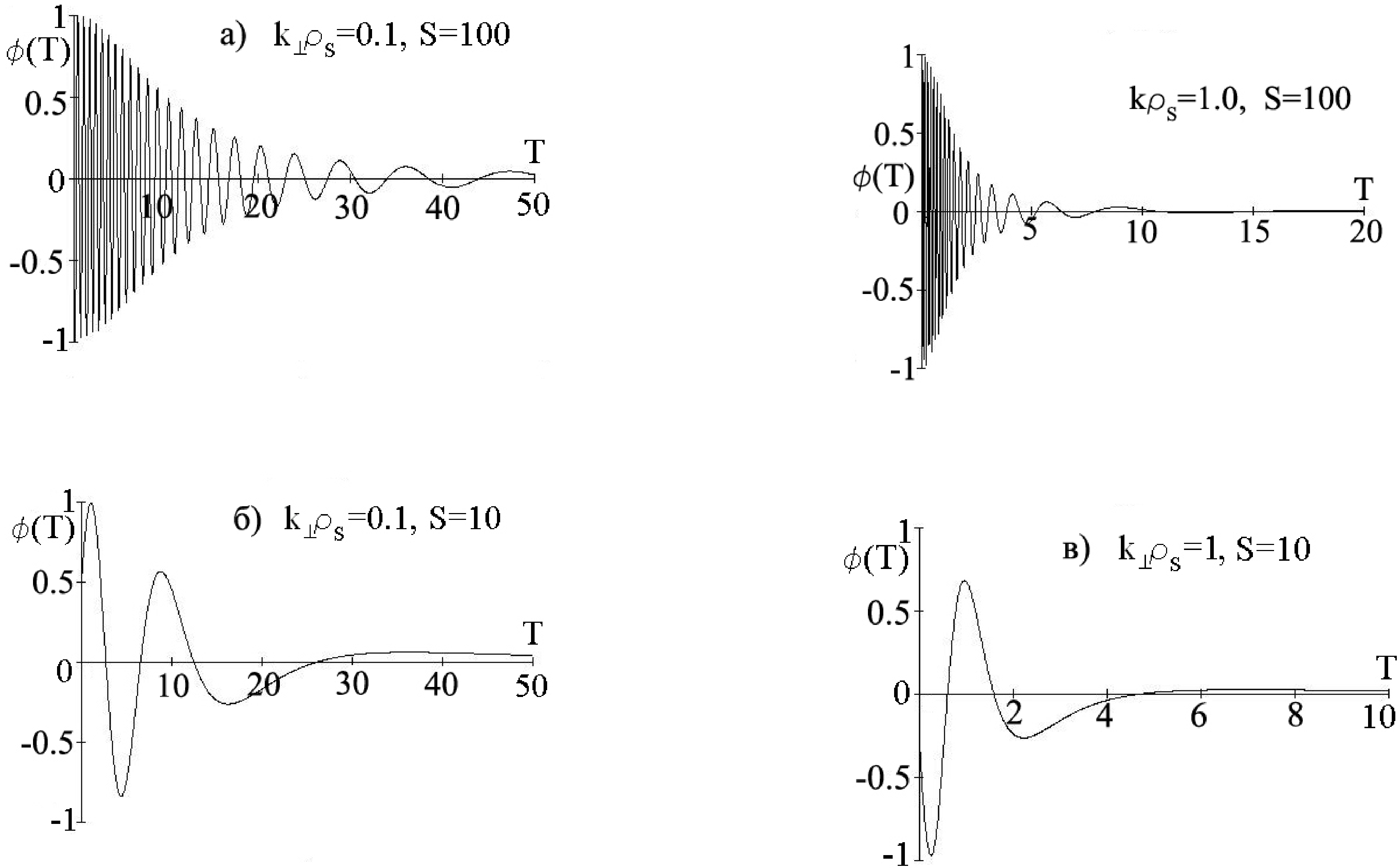
where $S = \dfrac{k_y v_{de}}{V_0' k_y \rho_s}$.

For $V_0' t < (k_y \rho_s)^{-1}$

$$\phi(t, k_x, k_y, k_z) \sim \phi(t=0, k_x, k_y, k_z) e^{i\omega_{de} t};$$

for $V_0' t > (k_y \rho_s)^{-1}$,

$$\phi(t, k_x, k_y, k_z) \sim \phi(t=0, k_x, k_y, k_z) \frac{e^{i\alpha}}{\left(k_y \rho_s V_0' t\right)^2}$$


a) $k_\perp \rho_s$=0.1, S=100
ϕ(T)
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в) $k_\perp \rho_s$=1, S=10
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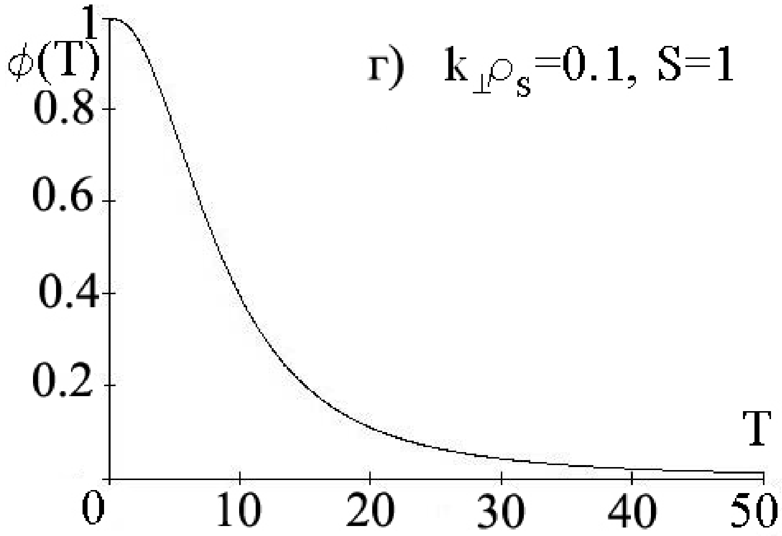

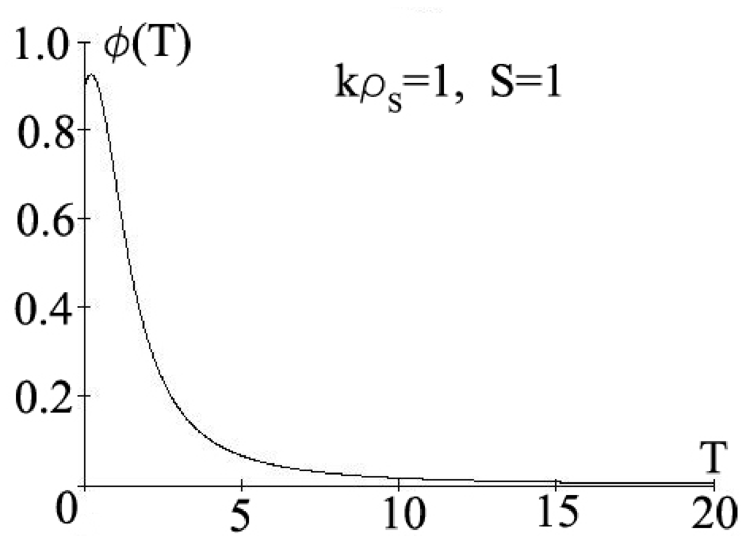

$$\phi(t) \sim \frac{1}{(V_0' t)^2}$$

The conclusions of principal importance:

- With convective variables ξ and η the system of governing equations does not contain any more the spatial dependency connected with the flow shear.

- The linear solution has a form

$$\phi(\xi,\eta,t)=\int dk_x\int dk_y\phi(k_x,k_y,0)\,g(k_x,k_y,t)\,e^{ik_x\xi+ik_y\eta}$$

where $\phi(k_x,k_y,0)$ is the initial data and $g(k_x,k_y,t)$ is the linearly unstable solution. It was shown (Mikhailenko-2000) for the case of the spatially homogeneous time-independent velocity shear, that for $\omega_d > V_0' \sim \gamma$, where γ is growth rate of the resistive drift instability in plasma without shear flow, the solution $g(k_x,k_y,t)$ in times $\left(V_0'\right)^{-1} < t \lesssim \left(V_0' k_y \rho_s\right)^{-1}$ still has an ordinary modal form,

$$g(k_x,k_y,t)=e^{-i\omega_d t+\gamma t}$$

with known frequency of drift wave $\omega_d = k_y v_{de}/\left(1+\rho_s^2\left(k_x^2+k_y^2\right)\right)$, and growth rate of the resistive drift instability γ.

- In the laboratory frame solution becomes nonseparable in space and time and therefore quite different from the normal mode assumption,

$$\phi(\mathbf{r},t) = \int dk_x \int dk_y \phi(k_x,k_y,0)\, g(k_x,k_y,t)\, e^{i(k_x - k_y V_0' t)x + ik_y y}$$
$$= \int dk_x \int dk_y \phi(k_x,k_y,0)\, e^{-i(\omega_d + k_y V_0' x)t + \gamma t + ik_x x + ik_y y}.$$

Because of the time dependence $k_{x(lab)} = k_x - k_y V_0' t$ of the wave number component along the flow shear, the modes in the laboratory frame become increasingly one-dimensional zonal-like as the perturbed $E \times B$ velocity tilts more and more closely parallel to y-axis.

- The suppression of the drift resistive instability in the case of sufficiently strong flow shear is a non–modal process, during which the initial separate spatial Fourier harmonic of the drift wave potential transformed into zero-frequency convective cell with amplitude decreasing with time as $(V_0' t)^{-2}$.

RENORMALIZED HYDRODYNAMIC THEORY FOR DRIFT MODES IN PLASMA SHEAR FLOWS

We investigate the temporal evolution of drift modes in time-dependent shear flow using the Hasegawa–Wakatani system of equations for the dimensionless density $n = \tilde{n}/n_e$ and potential $\phi = e\varphi/T_e$ perturbations (n_e is the electron background density, T_e is the electron temperature)

Mikhailenko V.S., Mikhailenko V.V., and Stepanov K.N., Renormalized theory of drift turbulence in plasma shear flows. Plasma Phys. Control. Fusion, 52, 2010 , 055007

Mikhailenko V.S., Mikhailenko V.V., and Stepanov K.N., Turbulence evolution in Plasma Shear Flows. Plasma and Fusion Research, 5, 2010 , S2015

$$\rho_s^2 \left(\frac{\partial}{\partial t} + V_0' x \frac{\partial}{\partial y} - \frac{cT_e}{eB} \left(\frac{\partial \phi}{\partial y} \frac{\partial}{\partial x} - \frac{\partial \phi}{\partial x} \frac{\partial}{\partial y} \right) \right) \nabla^2 \phi = a \frac{\partial^2}{\partial z^2} (n - \phi)$$

$$\left(\frac{\partial}{\partial t} + V_0' x \frac{\partial}{\partial y} - \frac{cT_e}{eB} \left(\frac{\partial \phi}{\partial y} \frac{\partial}{\partial x} - \frac{\partial \phi}{\partial x} \frac{\partial}{\partial y} \right) \right) n + v_{de} \frac{\partial \phi}{\partial y} = a \frac{\partial^2}{\partial z^2} (n - \phi)$$

Three steps of the renormalization procedure.

The first step.

With new spatial variables ξ, η,

$$t = t, \qquad \xi = x, \qquad \eta = y - V_0' x t, \qquad z = z.$$

the Hasegawa–Wakatani system has a form

$$\rho_s^2 \left(\frac{\partial}{\partial t} - \frac{cT_e}{eB} \left(\frac{\partial \phi}{\partial \eta} \frac{\partial}{\partial \xi} - \frac{\partial \phi}{\partial \xi} \frac{\partial}{\partial \eta} \right) \right) \Delta \phi = a \frac{\partial^2}{\partial z^2} (n - \phi) \, ,$$

$$\left(\frac{\partial}{\partial t} - \frac{cT_e}{eB} \left(\frac{\partial \phi}{\partial \eta} \frac{\partial}{\partial \xi} - \frac{\partial \phi}{\partial \xi} \frac{\partial}{\partial \eta} \right) \right) n + v_{de} \frac{\partial \phi}{\partial \eta} = a \frac{\partial^2}{\partial z^2} (n - \phi) \, .$$

The Laplacian operator Δ now becomes time-dependent,

$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} = \left(\frac{\partial}{\partial \xi} - V_0' t \frac{\partial}{\partial \eta} \right) \left(\frac{\partial}{\partial \xi} - V_0' t \frac{\partial}{\partial \eta} \right) + \frac{\partial^2}{\partial \eta^2}.$$

leaving us with an initial value problem to solve.

- THE SURPRISE IS THAT THE CONVERCTIVE NONLINEAR DERIVATIVE REMAINS THE SAME IN THE NEW CONVECTIVE COORDINATES AS FOR PLASMA WITHOUT ANY FLOWS.

Therefore, the nonlinear evolution of the resistive drift instability in times $\left(V_0'\right)^{-1} < t < \left(V_0' k_y \rho_s\right)^{-1}$ governed by H–W system will occur as in plasmas without shear flow.

The second step (is equally valid for plasmas without flows) .

With new variables ξ_1, η_1,

$$\xi_1 = \xi - \widetilde{\xi}(t) = \xi + \frac{cT_e}{eB}\int\limits_{t_0}^{t} \frac{\partial \phi}{\partial \eta} dt_1, \qquad \eta_1 = \eta - \widetilde{\eta}(t) = \eta - \frac{cT_e}{eB}\int\limits_{t_0}^{t} \frac{\partial \phi}{\partial \xi} dt_1$$

the convective nonlinearity becomes of the higher order with respect to the potential ϕ. Omitting such nonlinearity, as well as small nonlinearity of the second order in the Laplacian, resulted from the transformation to nonlinearly determined variables ξ_1, η_1, we come to linear equation with solution, where wave numbers k_x, k_y are conjugate there to coordinates ξ_1, η_1 respectively. With variables ξ and η this solution has a form

$$\phi(\xi, \eta, t) = \int dk_x \int dk_y \phi(k_x, k_y, 0)\, g(k_x, k_y, t_1)\, e^{ik_x\xi_1 + ik_y\eta_1}$$
$$= \int dk_x \int dk_y \phi(k_x, k_y, 0)\, g(k_x, k_y, t_1)\, e^{ik_x\xi + ik_y\eta - ik_x\widetilde{\xi}(t_1) - ik_y\widetilde{\eta}(t_1)},$$

This equation is in fact a nonlinear integral equation for potential ϕ, in which the effect of the total Fourier spectrum on any separate Fourier harmonic is accounted for.

The third step

Calculation the correlations of the plasma displacements in the unstable electric field of the drift turbulence.

Assuming that the displacements $\widetilde{\xi}(t)$, $\widetilde{\eta}(t)$ obey the Gaussian statistics with mean zero,

$$\left\langle \exp\left[ik_{1x}\left(\widetilde{\xi}(t_1)-\widetilde{\xi}(t_2)\right)+ik_{1y}\left(\widetilde{\eta}(t_1)-\widetilde{\eta}(t_2)\right)\right]\right\rangle$$
$$=\exp\left[-\frac{1}{2}k_x^2K_{\xi\xi}(t_1,t_2)-k_xk_yK_{\xi\eta}(t_1,t_2)-\frac{1}{2}k_y^2K_{\eta\eta}(t_1,t_2)\right]$$

we find in this case the following relation for $K_{\xi\xi}(t)$:

$$\left\langle\left(\widetilde{\xi}(t)\right)^2\right\rangle=K_{\xi\xi}(t,t_0)=K_{\xi\xi}(t)=\frac{c^2T_e^2}{e^2B^2}\int\limits_{t_0}^{t}dt_1\int\limits_{t_0}^{t}dt_2\int dk_x\int dk_y\left|\phi(k_x,k_y,0)\right|^2k_y^2$$
$$\times\exp\left(\gamma(t_1+t_2)-i\omega_d(t_1-t_2)\right)$$
$$\times\exp\left[-\frac{1}{2}k_x^2K_{\xi\xi}(t_1,t_2)-k_xk_yK_{\xi\eta}(t_1,t_2)-\frac{1}{2}k_y^2K_{\eta\eta}(t_1,t_2)\right].$$

where

$$K_{\xi\xi}(t_1,t_2)=\left\langle\left(\widetilde{\xi}(t_1)-\widetilde{\xi}(t_2)\right)^2\right\rangle,\quad K_{\eta\eta}(t_1,t_2)=\left\langle\left(\widetilde{\eta}(t_1)-\widetilde{\eta}(t_2)\right)^2\right\rangle$$
$$K_{\xi\eta}(t_1,t_2)=\left\langle\left(\widetilde{\xi}(t_1)-\widetilde{\xi}(t_2)\right)\left(\widetilde{\eta}(t_1)-\widetilde{\eta}(t_2)\right)\right\rangle,$$

The two-time scale procedure with time variables $\tau = t_1 - t_2$, $\widehat{t} = (t_1 + t_2)/2$ of the calculation of the dispersion tensor of random displacements of the plasma is developed.

A general equation,

$$k^{2x} K_{\xi\xi}(t) + 2k_x k_y K_{\xi\eta}(t) + k_y^2 K_{\eta\eta}(t)$$

$$= \frac{T_e^2 c^2}{e^2 B^2} \int dk_{1x} \int dk_{1y} \int_0^t d\widehat{t} \left| \phi\left(k_{1x}, k_{1y}, \widehat{t}\,\right) \right|^2 \left| [\mathbf{k}_\perp \times \mathbf{k}_{1\perp}] \right|^2 \frac{C\left(k_{1x}, k_{1y}, \widehat{t}\right)}{\omega_d^2\left(k_{1x}, k_{1y}\right)}$$

$$= 2 \int_{t_0}^t d\widehat{t}\, C\left(k_x, k_y, \widehat{t}\right)$$

where $\left| \phi\left(k_{1x}, k_{1y}, \widehat{t}\,\right) \right|^2 = \left| \phi\left(k_{1x}, k_{1y}, 0\,\right) \right|^2 e^{2\gamma\left(k_{1x}, k_{1y}\right)\widehat{t}}$.

The renormalized form of the potential, in which the average effect of the random convection is accounted for,

$$\phi(\xi,\eta,t)=\int dk_x \int dk_y \phi(k_x,k_y,0)$$
$$\times \exp\left[-i\omega_d t+\gamma t-\int_{t_0}^{t} d\widehat{t}\, C\left(k_x,\, k_y,\, \widehat{t}\right)+ik_x\xi+ik_y\eta\right].$$

The saturation of the instability occurs when $\partial\left(\phi(\xi,\eta,t)\right)^2/\partial t=0$, i.e. when

$$\gamma(k_x,k_y)=C(k_x,k_y,t)$$
$$=\frac{T_e^2c^2}{e^2B^2}\int dk_{1x}\int dk_{1y}\left|\phi(k_{1x},k_{1y},t)\right|^2\left|[\mathbf{k}_\perp\times\mathbf{k}_{1\perp}]\right|^2\frac{C(k_{1x},\,k_{1y},\,t\,)}{\omega_d^2(k_{1x},k_{1y})}$$

$$\gamma\left(k_x,k_y\right)=\frac{T_e^2c^2}{e^2B^2}\int dk_{1x}\int dk_{1y}\left|\phi\left(k_{1x},k_{1y},t\right)\right|^2\left|\left[\mathbf{k}_\perp\times\mathbf{k}_{1\perp}\right]\right|^2\frac{\gamma\left(k_{1x},k_{1y}\right)}{\omega_d^2\left(k_{1x},k_{1y}\right)}$$

The sought–for value is a time t_{sat} at which the balance of the linear growth and nonlinear damping occurs for given initial disturbance $\phi\left(k_{1x},k_{1y},0\right)$ and dispersion. With obtained t_{sat} the saturation level will be equal to $\left|\phi\left(t_{sat}\right)\right|^2$.

The well known order of value estimate for the potential ϕ in the saturation state is obtained easily

$$\frac{e\phi}{T_e}\sim\frac{1}{k_\perp L_n}$$

for times $\left(V_0'\right)^{-1}<t<\left(V_0'k_y\rho_s\right)^{-1}$.

Obtained results show that the nonlinearity of the Hasegawa–Wakatani system of equations in variables ξ and η does not display any effects of the enhanced decorrelations provided by flow shear.

In the laboratory frame of reference such spatial Fourier modes are observed as a sheared modes with time dependent component of the wave number $k_{x(lab)} = k_x - k_y V_0' t$, directed along the velocity shear,

$$\phi\left(\mathbf{r}, t\right) = \int dk_x \int dk_y \phi\left(k_x, k_y, 0\right) e^{i(k_x - k_y V_0' t)x + ik_y y - i\omega_d t + \gamma t - ik_x \widetilde{\xi}(t_1) - ik_y \widetilde{\eta}(t_1)}$$

The displacements $\widetilde{\xi}\left(t\right)$ and $\widetilde{\eta}\left(t\right)$ are observed in the laboratory frame as the displacements $\widetilde{x}\left(t\right)$ and $\widetilde{y}\left(t\right)$ which are equal to

$$\widetilde{x}\left(t\right) = \widetilde{\xi}\left(t\right)$$

and

$$\widetilde{y}\left(t\right) = \int\limits_{t_0}^{t} \widetilde{v}_y\left(t_1\right) dt_1 = \frac{cT_e}{eB} \int\limits_{t_0}^{t} dt_1 \frac{\partial \phi}{\partial x} + \int\limits_{t_0}^{t} dt_1 \frac{\partial V_0\left(x, t_1\right)}{\partial x} \widetilde{x}\left(t_1\right)$$

$$= i\frac{c}{B} \int dk_x \int dk_y \int\limits_{t_0}^{t} dt_1 \phi\left(k_x, k_y, 0\right) \left(k_x - k_y V_0'\left(t - t_1\right)\right)$$

$$\times \exp\left(-i\omega_d t_1 + \gamma t_1 + i(k_x - k_y V_0' t_1)x + ik_y y - ik_x \widetilde{\xi}\left(t_1\right) - ik_y \widetilde{\eta}\left(t_1\right)\right)$$

The correlation $K_{yy}(t)$ is

$$K_{yy}(t) = \frac{c^2 T_e^2}{e^2 B^2} Re \int dk_x \int dk_y \left|\phi\left(k_x, k_y, 0\right)\right|^2 \frac{C_1\left(k_x, k_y, \widehat{t}, x\right)}{\left(\omega_d\left(k_x, k_y\right) + k_y V_0' x\right)^2}$$
$$\times \left(\frac{2}{3}\left(k_y V_0'\right)^2 t^3 - 2k_x k_y V_0' t^2 + 2k_x^2 t\right),$$

It displays the effect of the anisotropic dispersion conditioned by flow shear, observed in the laboratory frame of reference - dispersion increases much faster along flow than in the direction of the flow shear.

The ”enhanced decorrelation by flow shear” have nothing in common with ”enhanced suppression” of turbulence in shear flows . The ”universal rule $V_0' \simeq \gamma$” can't be considered as a condition for the suppression of turbulence by shear flow. Under that condition the perturbation, considered in convected set as a separate spatial Fourier mode with definite frequency and growth rate, becomes observed in laboratory set in time $t \simeq \gamma^{-1}$ as a sheared mode with time dependent wave number. Only for the perturbations with $k_y \rho_i \simeq 1$ that time coincides with time $t \leq \left(V_0' k_y \rho_s\right)^{-1}$, at which strong nonmodal suppression of the drift turbulence occurs.

NON–MODAL KINETIC THEORY OF PLASMA SHEAR FLOWS

Kinetic effects, such as finite Larmor radius effects, Landau and cyclotron damping and the numerous resulting kinetic instabilities, which are naturally not involved in the fluid description of plasma shear flows, require the development of a kinetic description of plasma shear flows.

$$\frac{\partial F_\alpha}{\partial t} + \mathbf{v}\frac{\partial F_\alpha}{\partial \mathbf{r}} + \frac{e_\alpha}{m_\alpha}\left(\mathbf{E}_0\left(\mathbf{r},t\right) + \frac{1}{c}\left[\mathbf{v}\times\mathbf{B}\right] - \nabla\varphi\left(\mathbf{r},t\right)\right)\frac{\partial F_\alpha}{\partial \mathbf{v}} = 0.$$

The starting point of the derivation of the basic equations of the nonmodal kinetic theory is the transformation of Vlasov-Poisson system to convective (sheared) coordinates in velocity and configuration spaces.

We transform in Vlasov equation the variables $(t, \mathbf{r}, \mathbf{v})$ onto new spatial variables $(t_c, \mathbf{r}_\alpha, \mathbf{v}_\alpha)$, connected by the relations

$$t = t_c, \qquad \mathbf{v} = \mathbf{v}_\alpha + \mathbf{U}_\alpha\left(\mathbf{r}_\alpha, t_c\right), \qquad \mathbf{r} = \mathbf{r}_\alpha + \int\limits_{t_{(0)}}^{t_c} \mathbf{U}_\alpha\left(\mathbf{r}_\alpha, t_{1c}\right) dt_{1c},$$

or

$$\mathbf{v}_\alpha = \mathbf{v} - \mathbf{V}_\alpha\left(\mathbf{r}_\alpha, t\right), \qquad \mathbf{r}_\alpha = \mathbf{r} - \int\limits_{t_{(0)}}^{t} \mathbf{V}_\alpha\left(\mathbf{r}, t_1\right) dt_1,$$

with set of reference moving with velocity $\mathbf{V}_\alpha\left(\mathbf{r}, t\right) = \mathbf{U}_\alpha\left(\mathbf{r}_\alpha, t_c\right)$. $t_{(0)}$ denotes the time at which shear flow emerges.

For homogeneous stationary velocity shear $\left(\mathbf{E}_0(\mathbf{r}) = E_0' x \mathbf{e}_x\right)$ (Mikhailenko V.S., Mikhailenko V.V., and Stepanov K.N., Renormalized non-modal theory of the kinetic drift instability of plasma shear flows, Physics of Plasmas 18, 062103, 2011)

$$\mathbf{v}_\alpha = \mathbf{v} - V_0' x \mathbf{e}_y, \qquad \mathbf{r}_\alpha = \mathbf{r} - \int_0^t \mathbf{V}_0(\mathbf{r}, t)\, dt = \mathbf{r} - V_0' x_\alpha t \mathbf{e}_y,$$

$$\frac{\partial F_\alpha}{\partial t} + v_{\alpha x}\frac{\partial F_\alpha}{\partial x_\alpha} - \left(v_{\alpha y} - v_{\alpha x} V_0' t\right)\frac{\partial F_\alpha}{\partial y_\alpha} + \omega v_{\alpha x}\frac{\partial F_\alpha}{\partial v_{\alpha x}} - \left(\omega_{c\alpha} + V_0'\right) v_{\alpha x}\frac{\partial F_\alpha}{\partial v_{\alpha y}}$$

$$-\frac{e_\alpha}{m_\alpha}\left(\frac{\partial \varphi}{\partial x_\alpha} - V_0' t \frac{\partial \varphi}{\partial y_\alpha}\right)\frac{\partial F_\alpha}{\partial v_{\alpha x}} + v_{\alpha z}\frac{\partial F_\alpha}{\partial z_\alpha} - \frac{e_\alpha}{m_\alpha}\frac{\partial \varphi}{\partial y_\alpha}\frac{\partial F_\alpha}{\partial v_{\alpha y}} - \frac{e_\alpha}{m_\alpha}\frac{\partial \varphi}{\partial z_\alpha}\frac{\partial F_\alpha}{\partial v_{\alpha z}} = 0$$

With leading center coordinates

$$v_{\alpha x} = v_\perp \cos\phi, \qquad v_{\alpha y} = \sqrt{\eta}\, v_\perp \sin\phi \qquad \eta = 1 + V_0'/\omega_{c\alpha}$$

$$X_\alpha = x_\alpha + \frac{v_\perp}{\sqrt{\eta}\,\omega_{c\alpha}} \sin\left(\phi_1 - \sqrt{\eta}\,\omega_{c\alpha} t\right)$$

$$Y_\alpha = y_\alpha - \frac{v_\perp}{\eta\,\omega_{c\alpha}} \cos\left(\phi_1 - \sqrt{\eta}\,\omega_{c\alpha} t\right) - V_0' t\, \frac{v_\perp}{\sqrt{\eta}\,\omega_{c\alpha}} \sin\left(\phi_1 - \sqrt{\eta}\,\omega_{c\alpha} t\right)$$

$$= y_\alpha - \frac{v_\perp}{\eta\,\omega_{c\alpha}} \cos\left(\phi_1 - \sqrt{\eta}\,\omega_{c\alpha} t\right) - V_0' t\, \left(X_\alpha - x_\alpha\right).$$

Vlasov equation becomes

$$\frac{\partial F_\alpha}{\partial t} + \frac{e_\alpha}{m_\alpha}\frac{\sqrt{\eta}\,\omega_{c\alpha}}{v_\perp}\left(\frac{\partial \varphi}{\partial \phi}\frac{\partial F_\alpha}{\partial v_\alpha} - \frac{\partial \phi}{\partial v_\perp}\frac{\partial F_\alpha}{\partial \phi}\right) + \frac{e_\alpha}{m_\alpha}\frac{1}{\eta\,\omega_{c\alpha}}\left(\frac{\partial \varphi}{\partial X_\alpha}\frac{\partial F_\alpha}{\partial Y_\alpha} - \frac{\partial \phi}{\partial Y_\alpha}\frac{\partial F_\alpha}{\partial X_\alpha}\right) - \frac{e_\alpha}{m_\alpha}\frac{\partial \varphi}{\partial z_\alpha}\frac{\partial F_\alpha}{\partial v_{\alpha z}} = 0$$

$$\varphi(\mathbf{r}_\alpha, t) = \int \varphi(t, k_x, k_y, k_z) \exp(ik_x x_\alpha + ik_y y_\alpha + ik_z z_\alpha)$$

$$= \int \varphi(t, k_x, k_y, k_z) \exp(ik_x X + ik_y Y + ik_z z)$$

$$\times \exp\left[-\frac{ik_\perp(t) v_\perp}{\sqrt{\eta}\omega_c} \sin\left(\phi_1 - \sqrt{\eta}\omega_c t - \theta(t)\right)\right] dk_x dk_y dk_z$$

$$f_\alpha(t, X, Y, v_\perp, \phi, v_z, z_1)$$

$$= \frac{e_\alpha}{m_\alpha} \int^t \left[\frac{1}{\eta\omega_c} \frac{\partial\varphi}{\partial Y} \frac{\partial F_\alpha}{\partial X} - \frac{\sqrt{\eta}\omega_c}{v_\perp} \frac{\partial\varphi}{\partial\phi} \frac{\partial F_\alpha}{\partial v_\perp} + \frac{\partial\varphi}{\partial z_1} \frac{\partial F_\alpha}{\partial v_z}\right] dt_1$$

$$= \frac{ie_\alpha}{m_\alpha} \sum_{n=-\infty}^{\infty} \sum_{n_1=-\infty}^{\infty} \int^t dt_1 \varphi(t_1, k_x, k_y, k_z)$$

$$\times \exp\Big(-ik_z v_z (t - t_1) + in\left(\phi_1 - \sqrt{\eta}\omega_c t - \theta(t)\right)$$

$$-in_1\left(\phi_1 - \sqrt{\eta}\omega_c t_1 - \theta(t)\right)\Big)$$

$$\times J_n\left(\frac{k_\perp(t) v_\perp}{\sqrt{\eta}\omega_c}\right) J_{n_1}\left(\frac{k_\perp(t_1) v_\perp}{\sqrt{\eta}\omega_c}\right) \left[\frac{k_y}{\eta\omega_{c\alpha}} \frac{\partial F_\alpha}{\partial X_\alpha} + \frac{\sqrt{\eta}\omega_c n_1}{v_\perp} \frac{\partial F_\alpha}{\partial v_\perp} + k_{1z} \frac{\partial F_\alpha}{\partial v_z}\right]$$

where $k_\perp^2(t) = \left(k_x - V_0' t k_y\right)^2 + k_y^2$ and $\tan\theta = k_y/(k_x - V_0' t k_y)$.

The Poisson equation for the separate spatial Fourier harmonic gives the governing integral equation

$$\left[\left(k_x - V_0' t k_y\right)^2 + k_y^2 + k_z^2\right]\varphi(\mathbf{k},t) =$$

$$= \sum_\alpha \frac{i}{\lambda_{D\alpha}^2} \sum_{n=-\infty}^{\infty} \int_{t_0}^{t} dt_1 \varphi(\mathbf{k},t_1) I_n\left(k_\perp(t)\, k_\perp(t_1)\, \rho_\alpha^2\right) e^{-\frac{1}{2}\rho_\alpha^2\left(k_\perp^2(t)+k_\perp^2(t_1)\right)}$$

$$\times e^{-\frac{1}{2}k_z^2 v_{T\alpha}^2 (t-t_1)^2 - in\sqrt{\eta}\omega_{c\alpha}(t-t_1) - in(\theta(t)-\theta(t_1))}$$

$$\times \left[\frac{k_y v_{d\alpha}}{\sqrt{\eta}} - n\sqrt{\eta}\omega_{c\alpha} + i k_z^2 v_{T\alpha}^2 (t-t_1)\right] + \sum_\alpha e_\alpha \delta n_\alpha(\mathbf{k},t_0)$$

For low frequency perturbations

$$\left[\left(k_x - V_0' t k_y\right)^2 + k_y^2 + k_z^2\right]\varphi(\mathbf{k},t) =$$

$$= \sum_\alpha \frac{i}{\lambda_{D\alpha}^2} \int_{t_0}^{t} dt_1 \varphi(\mathbf{k},t_1) I_0\left(k_\perp(t)\, k_\perp(t_1)\, \rho_\alpha^2\right) e^{-\frac{1}{2}\rho_\alpha^2\left(k_\perp^2(t)+k_\perp^2(t_1)\right)} e^{-\frac{1}{2}k_z^2 v_{T\alpha}^2 (t-t_1)^2}$$

$$\times \left[\frac{k_y v_{d\alpha}}{\sqrt{\eta}} + i k_z^2 v_{T\alpha}^2 (t-t_1)\right] + \sum_\alpha e_\alpha \delta n_\alpha(\mathbf{k},t_0)$$

In drift kinetic equation shear flow effects are omitted completely!

$$
\begin{aligned}
&I_0\left(k_\perp(t)\,k_\perp(t_1)\,\rho_i^2\right)e^{-\frac{1}{2}\rho_i^2\left(k_\perp^2(t)+k_\perp^2(t_1)\right)} \\
&\approx 1-k_\perp^2\rho_i^2+\left(k_xk_yV_0'\rho_i^2\,(t+t_1)-\frac{1}{2}k_y^2\rho_i^2\left(V_0'\right)^2\left(t^2+t_1^2\right)\right)\Theta(t)\,, \qquad (1)
\end{aligned}
$$

$\Phi(\mathbf{k},t)=\varphi(\mathbf{k},t)\,\Theta(t-t_0)$, where $\Theta(t-t_0)$ is the unit-step Heaviside function

$$
\begin{aligned}
&\int_{t_0}^{t}dt_1\left(\frac{d\Phi(\mathbf{k},t_1)}{dt_1}+i\omega(\mathbf{k})\,\Phi(\mathbf{k},t_1)\right) \\
&=-\frac{b_i}{a_i}\int_{t_0}^{t}dt_1\left(\frac{d\Phi(\mathbf{k},t_1)}{dt_1}+ik_yv_{di}\Phi(\mathbf{k},t_1)\right)\left(1-e^{-\frac{1}{2}k_z^2v_{Ti}^2(t-t_1)^2}\right) \\
&+\frac{b_i}{a_i}\int_{0}^{t}dt_1\left(\frac{d\Phi(\mathbf{k},t_1)}{dt_1}+ik_yv_{di}\Phi(\mathbf{k},t_1)\right)\left(\frac{k_x}{k_y}\frac{(t+t_1)}{a_iV_0't_s^2}-\frac{\left(t^2+t_1^2\right)}{2a_it_s^2}\right) \\
&+\int_{0}^{t}dt_1\Phi(\mathbf{k},t_1)\,\frac{1}{a_iV_0't_s^2}\left(\frac{k_x}{k_y}-V_0't\right) \\
&+\frac{\tau}{a_i}\int_{t_0}^{t}dt_1\left(\frac{d\Phi(\mathbf{k},t_1)}{dt_1}+ik_yv_{de}\Phi(\mathbf{k},t_1)\right)e^{-\frac{1}{2}k_z^2v_{Te}^2(t-t_0)^2},
\end{aligned}
$$

where $b_i = 1 - k_\perp^2 \rho_i^2$, $a_i = \tau + k_\perp^2 \rho_i^2$ and $\omega(\mathbf{k}) = -\frac{b_i}{a_i} k_y v_{di}$.

$$\Phi(\mathbf{k}, t) = \Phi_0 \exp\left[-i\omega(\mathbf{k})\, t \left(1 - \frac{1+\tau}{a_i b_i} \frac{t^2}{3t_s^2}\right) + iRe\delta\omega(\mathbf{k})\, t + \gamma(\mathbf{k})t - \frac{t^2 \Theta(t)}{2a_i t_s^2}\right]$$

where

$$\delta\omega(\mathbf{k}) = i\frac{\omega(\mathbf{k})\left(\omega(\mathbf{k}) - k_y v_{di}\right)}{k_z v_{Ti}} \frac{b_i}{a_i} \sqrt{\frac{\pi}{2}} W\left(\frac{\omega(\mathbf{k})}{\sqrt{2} k_z v_{Ti}}\right)$$
$$+ i\tau \frac{\omega(\mathbf{k})\left(\omega(\mathbf{k}) - k_y v_{de}\right)}{a_i k_z v_{Te}} \sqrt{\frac{\pi}{2}} W\left(\frac{\omega(\mathbf{k})}{\sqrt{2} k_z v_{Te}}\right)$$

The non-modal effects, which reveal in non-modal reduction of the frequency and growth rate, are negligible at $t \ll t_s$ and become dominant at $t \sim t_s$. Note, that for $\tau \gg k_\perp^2 \rho_i^2$ the time $a_i^{1/2} t_s$ is approximately equal to time $\left(V_0' k_\perp \rho_s\right)^{-1}$ of the transition to strongly non-modal regime in the fluid theory of the drift turbulence of the plasma shear flow

For times $t \geq \left(V_0' k_y \rho_i\right)^{-1}$

$$\Phi(\mathbf{k}, t) = \varphi_0 \exp\left(\frac{1}{\sqrt{2\pi} V_0' k_y \rho_i t} + i\frac{k_y v_{di}}{2t\left(V_0' k_y \rho_i\right)^2}\right)$$

which gradually becomes a zero-frequency cell-like perturbation.

RENORMALIZED NON-MODAL KINETIC THEORY

OF DRIFT TURBULENCE

OF THE NON-STATIONARY SHEAR FLOWS

In what following we adopt the ordering $\omega_{ci}T \gg 1$, where ω_{ci} is a ion cyclotron frequency and T is characteristic time of flow velocity $V_i(\mathbf{r}, t)$ variations, and assume, that $V_0' \ll \omega_{ci}$. In that case

$$\mathbf{V}_\alpha(\mathbf{r}, t) = \mathbf{V}_0(x, t) = -\frac{c}{B}\frac{\partial E_0(x, t)}{\partial x}x\mathbf{e}_y = \frac{\partial V_0(x, t)}{\partial x}x\mathbf{e}_y.$$

Here we consider the case of the spatially homogeneous, but time dependent velocity shear,

$$\frac{\partial V_0(x, t)}{\partial x} = V_0'\frac{da(t)}{dt}$$

where V_0' is a parameter with dimension of the velocity shear and $a(t)$ is a function with dimension of time. For this case, the transformations have a form

$$t = t_c, \qquad v_x = v_{\alpha x}, \qquad v_y = v_{\alpha y} + V_0'\frac{da(t)}{dt}x_\alpha, \qquad v_z = v_{z\alpha},$$

$$x = x_\alpha, \qquad y = y_\alpha + V_0' a(t)\, x_\alpha, \qquad z = z_\alpha$$

where $t_{(0)} = 0$ was assumed. With new variables Vlasov equation has a form

$$\frac{\partial F_\alpha}{\partial t} + v_{\alpha x}\frac{\partial F_\alpha}{\partial x} - \left(v_{\alpha y} - v_{\alpha x}V_\alpha' a(t)\right)\frac{\partial F_\alpha}{\partial y} - \frac{e_\alpha}{m_\alpha}\left(\frac{\partial \varphi}{\partial x} - V_\alpha' a(t)\frac{\partial \varphi}{\partial y}\right)\frac{\partial F_\alpha}{\partial v_{\alpha x}} - \frac{e_\alpha}{m_\alpha}\frac{\partial \varphi}{\partial y}\frac{\partial F_\alpha}{\partial v_{\alpha y}}$$

$$+\omega_{c\alpha}v_{\alpha y}\frac{\partial F_\alpha}{\partial v_{\alpha x}} - \left(\omega_{c\alpha} + V_\alpha'\frac{da(t)}{dt}\right)v_{\alpha x}\frac{\partial F_\alpha}{\partial v_{\alpha y}} - \frac{e_\alpha}{m_\alpha}\frac{\partial \varphi}{\partial z_\alpha}\frac{\partial F_\alpha}{\partial v_{\alpha z}} = 0.$$

The transformation to coordinates convected with shear flow eliminates the explicit spatial dependence related to shear flow from the convective derivative. However, since this equation has time-dependent coefficients it no longer describes normal modes.

With guiding center coordinates,

$$X_\alpha = x_\alpha + \frac{v_\perp}{\sqrt{\eta_\alpha(t)}\omega_{c\alpha}} \sin\phi,$$

$$Y_\alpha = y_\alpha - \frac{v_\perp}{\eta_\alpha(t)\,\omega_{c\alpha}} \cos\phi - V_0' a(t)\ (X_\alpha - x_\alpha), \qquad z_1 = z - v_z t,$$

and velocity space coordinates

$$v_{\alpha x} = v_\perp \cos\phi, \qquad v_{\alpha y} = \sqrt{\eta_\alpha} v_\perp \sin\phi, \qquad \phi = \phi_1 - \omega_{c\alpha}\mu(t), \qquad v_z = v_{\alpha z},$$

where η_α is the orbit squezing factor,

$$\eta_\alpha(t) = 1 + \frac{V_0'}{\omega_{c\alpha}} \frac{da(t)}{dt}, \qquad \mu_\alpha(t) = \int_0^t \sqrt{\eta_\alpha(t_1)} dt_1$$

Vlasov equation becomes

$$\frac{\partial F_\alpha}{\partial t} + \frac{e_\alpha}{m_\alpha \sqrt{\eta_\alpha(t)}\omega_{c\alpha}} \left(\frac{\partial \varphi}{\partial X_\alpha} \frac{\partial F_\alpha}{\partial Y_\alpha} - \frac{\partial \varphi}{\partial Y_\alpha} \frac{\partial F_\alpha}{\partial X_\alpha} \right)$$

$$+ \frac{e_\alpha}{m_\alpha} \frac{\sqrt{\eta_\alpha(t)}\omega_{c\alpha}}{v_\perp} \left(\frac{\partial \varphi}{\partial \phi_1} \frac{\partial F_\alpha}{\partial v_\perp} - \frac{\partial \varphi}{\partial v_\perp} \frac{\partial F_\alpha}{\partial \phi_1} \right) - \frac{e_\alpha}{m_\alpha} \frac{\partial \varphi}{\partial z_\alpha} \frac{\partial F_\alpha}{\partial v_{z\alpha}} = 0.$$

The Vlasov equation for the perturbation $f_i = F_i - F_{0i}$ of the ion distribution function F_i

$$\frac{\partial f_i}{\partial t} + \frac{e}{m_i\omega_{ci}}\left(\frac{\partial \varphi}{\partial X_i}\frac{\partial f_i}{\partial Y_i} - \frac{\partial \varphi}{\partial Y_i}\frac{\partial f_i}{\partial X_i}\right) + \frac{e}{m_i}\frac{\omega_{ci}}{v_\perp}\left(\frac{\partial \varphi}{\partial \phi_1}\frac{\partial f_i}{\partial v_\perp} - \frac{\partial \varphi}{\partial v_\perp}\frac{\partial f_i}{\partial \phi_1}\right) - \frac{e}{m_i}\frac{\partial \varphi}{\partial z_i}\frac{\partial f_i}{\partial v_{zi}}$$
$$= \frac{e}{m_i\omega_{ci}}\frac{\partial \varphi}{\partial Y_i}\frac{\partial F_{0i}}{\partial X_i} - \frac{e}{m_i}\frac{\omega_{ci}}{v_\perp}\frac{\partial \varphi}{\partial \phi_1}\frac{\partial F_{0i}}{\partial v_\perp} + \frac{e}{m_i}\frac{\partial \varphi}{\partial z_i}\frac{\partial F_{0i}}{\partial v_{zi}}$$

The perturbed electrostatic potential $\varphi(\mathbf{r}, t)$ is determined in the form

$$\varphi(x_\alpha, y_\alpha, z_\alpha, t) = \int \varphi(k_x, k_y, k_z, t)\, e^{ik_x x_\alpha + ik_y y_\alpha + ik_z z_\alpha} dk_x dk_y dk_z$$
$$= \int \varphi(k_x, k_y, k_z, t) \exp\Big[ik_x X_\alpha + ik_y Y_\alpha + ik_z z_\alpha$$
$$-i\frac{k_\perp(t) v_\perp}{\omega_{c\alpha}} \sin(\phi - \omega_{c\alpha}\mu(t) - \theta(t))\Big] dk_x dk_y dk_z,$$

where

$$k_\perp^2(t) = \left(k_x - V_0' a(t) k_y\right)^2 + k_y^2$$

and $\tan\theta = k_y/(k_x - V_0' a(t) k_y)$. (For the stationary velocity shear $a(t) = t$).

$$dt = \frac{dX_i}{-\frac{e}{m_i\omega_{ci}}\frac{\partial \varphi}{\partial Y_i}} = \frac{dY_i}{\frac{e}{m_i\omega_{ci}}\frac{\partial \varphi}{\partial X_i}} = \frac{dv_\perp}{\frac{e}{m_i}\frac{\omega_{ci}}{v_\perp}\frac{\partial \varphi}{\partial \phi_1}} = \frac{d\phi_1}{-\frac{e}{m_i}\frac{\omega_{ci}}{v_\perp}\frac{\partial \varphi}{\partial v_\perp}} = \frac{dv_z}{-\frac{e}{m_i}\frac{\partial \varphi}{\partial z_1}}$$
$$= \frac{df_i}{\frac{e}{m_i\omega_{ci}}\frac{\partial \varphi}{\partial Y_i}\frac{\partial F_{i0}}{\partial X_i} - \frac{e}{m_i}\frac{\omega_{ci}}{v_\perp}\frac{\partial \varphi}{\partial \phi_1}\frac{\partial F_{i0}}{\partial v_\perp} + \frac{e}{m_i}\frac{\partial \varphi}{\partial z_1}\frac{\partial F_{i0}}{\partial v_z}},$$

Last equation gives nonlinear solution for the perturbation of the ion distribution function f_i with known F_{i0},

$$f_i = \frac{e}{m}\int^{t}\left[\frac{1}{\omega_{ci}}\frac{\partial\varphi}{\partial Y_i}\frac{\partial F_{i0}}{\partial X_i} - \frac{\omega_{ci}}{v_\perp}\frac{\partial\varphi}{\partial\phi_1}\frac{\partial F_{i0}}{\partial v_\perp} + \frac{\partial\varphi}{\partial z_1}\frac{\partial F_{i0}}{\partial v_z}\right]dt'.$$

The averaged over the time $t \gg \omega_{ci}^{-1}$ and over initial phases of the drift perturbations solution for f_i in drift frequency range has a form

$$\begin{gathered} f_i = i\frac{e}{m_i}\int^{t} dt_1\varphi\left(t_1, k_x, k_y, k_z\right) \\ \times\exp\Bigg(-ik_z v_z\left(t - t_1\right) - \frac{1}{2}\left\langle\left(\mathbf{k}\left(t\right)\delta\mathbf{r}\left(t\right) - \mathbf{k}\left(t_1\right)\delta\mathbf{r}\left(t_1\right)\right)^2\right\rangle\Bigg) \\ \times J_0\left(\frac{k_\perp\left(t\right)v_\perp}{\omega_{ci}}\right)J_0\left(\frac{k_\perp\left(t_1\right)v_\perp}{\omega_{ci}}\right)\left[\frac{k_y}{\omega_{ci}}\frac{\partial F_{0i}}{\partial X_i} + k_{1z}\frac{\partial F_{0i}}{\partial v_z}\right] \\ + f_i\left(t = t_0, k_x, k_y, k_z, v_\perp\phi, v_z\right) \end{gathered}$$

The equilibrium distribution function F_{i0} is a Maxwellian,

$$F_{0\alpha} = \frac{n_{0\alpha}\left(X_\alpha\right)}{\left(2\pi v_{T\alpha}^2\right)^{3/2}}\exp\left(-\frac{v_\perp^2 + v_z^2}{v_{T\alpha}^2}\right).$$

and the inhomogeneity of the density of plasma shear flow species on coordinate X_α is assumed.

With Poisson equation for the potential $\varphi\left(\mathbf{r}_{\alpha},t\right)$,

$$\triangle\,\varphi\left(\mathbf{r}_{\alpha},t\right)=-4\pi\sum_{\alpha=i,e}e_{\alpha}\int f_{\alpha}\left(\mathbf{v},\mathbf{r}_{\alpha},t\right)d\mathbf{v}_{\alpha}, \tag{2}$$

we obtain integral equation for separate spatial Fourier harmonic $\varphi\left(\mathbf{k},t\right)$, in which effect of the turbulent scattering of ions on sheared drift modes of random phases is accounted for before the averaging over time $t\gg\omega_{ci}^{-1}$,

$$k^{2}\left(t\right)\varphi\left(\mathbf{k},t\right)=\sum_{\alpha=i,e}\frac{i}{\lambda_{D\alpha}^{2}v_{T\alpha}^{2}}\int\limits_{t_0}^{t}dt_{1}\varphi\left(\mathbf{k},t_{1}\right)\int\limits_{0}^{\infty}dv_{\perp}v_{\perp}\exp\left(-\frac{v_{\perp}^{2}}{v_{T\alpha}^{2}}\right)$$
$$\times J_{0}\left(\frac{k_{\perp}\left(t\right)v_{\perp}}{\omega_{c}}\right)J_{0}\left(\frac{k_{\perp}\left(t_{1}\right)v_{\perp}}{\omega_{c}}\right)\exp\left(-\frac{1}{2}k_{z}^{2}v_{T\alpha}^{2}\left(t-t_{1}\right)^{2}-\frac{1}{2}\left\langle\left(\mathbf{k}\left(t\right)\delta\mathbf{r}\left(t\right)-\mathbf{k}\left(t_{1}\right)\delta\mathbf{r}\left(t_{1}\right)\right)^{2}\right\rangle\right)$$
$$\times\left[k_{y}v_{d\alpha}+ik_{z}^{2}v_{T\alpha}^{2}\left(t-t_{1}\right)\right]-4\pi\sum_{\alpha=i,e}e_{\alpha}\delta n_{\alpha}\left(\mathbf{k},t,t_{0}\right),$$

The electric potential is assumed to consist of a randomly phased waves, and coordinates X_i, Y_i, $v_\perp$, ϕ are $X_i = \bar{X}_i + \delta X_i$, $Y_i = \bar{Y}_i + \delta Y_i$, $\phi = \bar{\phi} + \delta\phi$, where $\bar{X}_i$, $\bar{Y}_i$, are the guiding center coordinates averaged over the turbulent pulsations, and $\delta X_i(t)$, $\delta Y_i(t)$, $\delta\phi$ are random ion orbit disturbances due to their scattering by electrostatic turbulence. The disturbances are assumed sufficiently small and, after the averaging over the times $t \gg (\omega_{ci})^{-1}$, they are determined by the equations

$$\delta X_i = -\frac{e}{m_i\omega_{ci}}\int_{t_0}^{t}\frac{\partial\varphi}{\partial\bar{Y}_i}dt_1 = -\frac{c}{B}\int_{t_0}^{t}dt_1\int d\mathbf{k}\varphi\left(\mathbf{k},t_1\right)k_y J_0\left(\frac{k_\perp\left(t_1\right)v_\perp}{\omega_{ci}}\right)e^{i\Psi}$$

$$\delta Y_i = \frac{e}{m_i\omega_{ci}}\int_{t_0}^{t}\frac{\partial\varphi}{\partial\bar{X}_i}dt_1 = \frac{c}{B}\int_{t_0}^{t}dt_1\int d\mathbf{k}\varphi\left(\mathbf{k},t_1\right)k_x J_0\left(\frac{k_\perp\left(t_1\right)v_\perp}{\omega_{ci}}\right)e^{i\Psi}$$

$$\delta\phi = -\frac{e}{m_i}\frac{\omega_{ci}}{v_\perp}\int_{t_0}^{t}\frac{\partial\varphi}{\partial\bar{v}_\perp}dt_1 = \frac{e}{mv_\perp}\int_{t_0}^{t}dt_1\int d\mathbf{k}\varphi\left(\mathbf{k},t_1\right)k_\perp\left(t_1\right)J_1\left(\frac{k_\perp\left(t_1\right)v_\perp}{\omega_{ci}}\right)e^{i\Psi},$$

and $\delta v_\perp = 0$, $\Psi = k_x X_i + k_y Y_i + k_z z + \mathbf{k}\left(t_1\right)\delta\mathbf{r}\left(t_1\right)$ and $i\mathbf{k}\left(t\right)\delta\mathbf{r}\left(t\right)$ denotes the phase shift resulted from perturbations of the ions orbits due to random waves-ion interactions,

$$\mathbf{k}\left(t\right)\delta\mathbf{r}\left(t\right) = k_x\delta X_i\left(t\right) + k_y\delta Y_i\left(t\right) - \frac{k_\perp\left(t\right)\bar{v}_\perp}{\omega_{ci}}\cos\left(\phi-\theta\right)\delta\phi\left(t\right),$$

For the times $a(t) < \left(V_0'\right)^{-1}$ the main effect, which determines the nonlinear scattering of ions by long wavelength drift turbulence with $k_\perp \rho_i < 1$ is the scattering of the leading center coordinates, δX and δY. The non-modal effects are negligible at this time. At times $a(t) > \left(V_0'\right)^{-1}$ the non-modal effects determine the nonlinear evolution of drift turbulence with dominant breakdown of phase of the potential due to scattering of the phase angle $\delta\phi$ in velocity space.

For times $\left(V_0'\right)^{-1} < a(t) < \left(V_0' k_y \rho_i\right)^{-1}$

$$k_\perp \rho_i \delta\phi / k_x \delta X \sim k_y \rho_i \left(V_0' a(t)\right)^3 \gg 1,$$

and for times $a(t) > \left(V_0' k_y \rho_i\right)^{-1}$

$$k_\perp \rho_i \delta\phi / k_x \delta X \sim \left(V_0' a(t)\right)^2 \gg 1.$$

$$\left\langle \left(\mathbf{k}\left(t\right)\delta\mathbf{r}\left(t\right) - \mathbf{k}\left(t_1\right)\delta\mathbf{r}\left(t_1\right)\right)^2 \right\rangle \approx \frac{v_\perp^2}{2\omega_{ci}^2} \left\langle \left(\mathbf{k}\left(t\right)\delta\phi\left(t\right) - \mathbf{k}\left(t_1\right)\delta\phi\left(t_1\right)\right)^2 \right\rangle$$

$$\approx 2a^2\left(t\right) \int\limits_{t_1}^{t} d\hat{t} \frac{C\left(\mathbf{k}, \hat{t}\right)}{a^2\left(\hat{t}\right)}.$$

$$C\left(\mathbf{k}_1, \hat{t}\right) = \frac{c^2 k_{1y}^2 \left(V_0' a\left(\hat{t}\right)\right)^6 v_\perp^2}{8B^2\omega_{ci}^2} \int d\mathbf{k}_2 k_{2y}^4 \left|\varphi\left(\mathbf{k}_2, t_0\right)\right|^2 e^{2\gamma(\mathbf{k}_2)\hat{t}} \frac{C\left(\mathbf{k}_2, \hat{t}\right)}{\omega^2\left(\mathbf{k}_2\right)}.$$

For the stationary velocity shear $a\left(t\right) = t$.

$$\int_{t_0}^{t} dt_1 \left(\frac{d\Phi(\mathbf{k}, t_1)}{dt_1} + i\omega(\mathbf{k}) \Phi(\mathbf{k}, t_1) \right)$$

$$= -\frac{b_i}{T + k_\perp^2 \rho_i^2} \int_{t_0}^{t} dt_1 \left(\frac{d\Phi(\mathbf{k}, t_1)}{dt_1} + ik_y v_{di} \Phi(\mathbf{k}, t_1) \right) \left(1 - e^{-\frac{1}{2} k_z^2 v_{Ti}^2 (t - t_1)^2} \right)$$

$$- \frac{b_i}{T + k_\perp^2 \rho_i^2} \int_{t_0}^{t} dt_1 \left(\frac{d\Phi(\mathbf{k}, t_1)}{dt_1} + ik_y v_{di} \Phi(\mathbf{k}, t_1) \right) \left(1 - e^{-a^2(t) \int_{t_1}^{t} d\hat{t} \frac{C(\mathbf{k}, \hat{t})}{a^2(\hat{t})}} \right)$$

$$+ \frac{1}{T + k_\perp^2 \rho_i^2} \int_{0}^{t} dt_1 \left(\frac{d\Phi(\mathbf{k}, t_1)}{dt_1} + ik_y v_{di} \Phi(\mathbf{k}, t_1) \right) \left(\frac{k_x}{k_y} \frac{(a(t) + a(t_1))}{V_0' t_s^2} - \frac{\left(a^2(t) + a^2(t_1)\right)}{2 t_s^2} \right)$$

$$+ \frac{1}{T + k_\perp^2 \rho_i^2} \int_{0}^{t} dt_1 \Phi(\mathbf{k}, t_1) \frac{1}{V_0' t_s^2} \left(\frac{k_x}{k_y} - V_0' a(t) \right)$$

$$+ \frac{T}{T + k_\perp^2 \rho_i^2} \int_{t_0}^{t} dt_1 \left(\frac{d\Phi(\mathbf{k}, t_1)}{dt_1} + ik_y v_{de} \Phi(\mathbf{k}, t_1) \right) e^{-\frac{1}{2} k_z^2 v_{Te}^2 (t - t_1)^2},$$

where $T = T_i / T_e$ and $\omega(\mathbf{k})$ is

$$\omega(\mathbf{k}) = -\frac{1 - k_\perp^2 \rho_i^2}{T + k_\perp^2 \rho_i^2} k_y v_{di}.$$

$$\Phi(\mathbf{k},t) = \varphi_0 \exp\left[-i\omega(\mathbf{k})t\left(1 - \frac{(1+\tau)}{a_i b_i}\frac{t^2}{3t_s^2}\Theta(t)\right)t + iRe\delta\omega\left(\mathbf{k}\right)t \right.$$
$$\left. +\gamma(\mathbf{k})t - \frac{t^2\Theta(t)}{2a_i t_s^2} - \int\limits_0^t C(\mathbf{k},t_1)dt_1\right].$$

$C(\mathbf{k},t)$ is determined by the equation

$$C(\mathbf{k},t) = \frac{c^2}{B^2}k_y^2\rho_i^2\frac{\left(V_0' t\right)^6}{8}\int d\mathbf{k}_1 \left|\Phi(\mathbf{k}_1,t)\right|^2 C(\mathbf{k}_1,t)\frac{k_{1y}^4}{\omega^2(\vec{k}_1)}$$

The potential ceases to grow, $\partial\Phi/\partial t = 0$, when $\gamma(\mathbf{k}) = C(\mathbf{k},t)$,

$$\frac{\gamma(\mathbf{k})}{\left(V_0' t\right)^6} = \frac{c^2}{8B^2}k_y^2\rho_i^2\int d\mathbf{k}_1 \left|\Phi(\mathbf{k}_1,t)\right|^2 \gamma(\mathbf{k}_1)\frac{k_{1y}^4}{\omega^2(\mathbf{k}_1)}$$

Renormalized non-modal quasilinear theory of drift turbulence of time-dependent shear flows

$$\frac{\partial F_i}{\partial t} = \frac{e^2}{m_i^2}\int_{t_0}^{t} dt_1 \int d\mathbf{k}\left(\frac{k_y}{\omega_{ci}}\frac{\partial}{\partial X_i} + k_z\frac{\partial}{\partial v_z}\right)$$

$$\times\left(\frac{k_y}{\omega_{ci}}\frac{\partial F_i}{\partial X_i} + k_z\frac{\partial F_i}{\partial v_z}\right)\left|\Phi\left(\mathbf{k},t\right)\right|^2 \frac{\left|\gamma\left(\mathbf{k}\right) - C\left(\mathbf{k},t\right)\right|}{\Omega^2\left(\mathbf{k}\right) + \left(\gamma\left(\mathbf{k}\right) - C\left(\mathbf{k},t\right)\right)^2},$$

where

$$\left|\varphi\left(\mathbf{k},t\right)\right|^2 = \left|\Phi\left(\mathbf{k},t_0\right)\right|^2 e^{\left(2\gamma(\mathbf{k})t - 2\int_0^t C(\mathbf{k},t_1)dt_1\right)}.$$

$$C(\mathbf{k},t) = \frac{c^2}{B^2}k_y^2\rho_i^2\frac{\left(V_0' a(t)\right)^6}{8}\int d\vec{k}_1\left|\Phi(\mathbf{k}_1,t)\right|^2 C(\mathbf{k}_1,t)\frac{k_{1y}^4}{\omega^2(\mathbf{k}_1)}$$

CONCLUSIONS

We develop the non-modal renormalized kinetic theory of the drift turbulence in plasma shear flow, directed across the homogeneous magnetic field, with time dependent spatially homogeneous velocity shear. We find, that shear flow introduces two different effects into Vlasov-Poisson (VP) system of equations.

The first effect is the inhomogeneous Doppler shift. This term displays the shearing of waves patterns in shear flow, observed in laboratory frame. This effect, in the case of flows with equal flow velocity for all plasma components, may be excluded from VP system by transformation to convective variables (coordinates) and has nothing in common with suppression or development of plasma instabilities. Effect of the enhanced decorrelation is of this type. It easily excludes from governing equations and it has not anything common with enhanced suppression or even quenching of instabilities in shear flow.

The second effect manifests itself as a time-dependent finite ion Larmor radius effect and is of principal importance for turbulence evolution in plasma shear flows. It consist in the interaction of ions undergoing cyclotron motion in inhomogeneous electric field with sheared modes, which due to their stretching by shear flows have time dependent wave number. That effect is unavoidable from VP system by any transforms of the variables. We obtain, that right this effect is a source of the enhanced suppression of the instabilities by shear flow. In linear non-modal kinetic theory, it reveals in the non-modal reduction with time the frequency and the growth rate of the unstable perturbations. In the renormalized nonlinear theory, which accounts for the effect of the turbulent scattering of ions by the ensemble of the sheared modes with randomly distributed initial phases, this effect reveals as the process of the extremely strong suppression of turbulence by shear flow, which seems as a rapid reduction of the growth rate with time as $\gamma / \left(V_0' t\right)^{-6}$. It is the strongest effect of the suppression of turbulence by flow shear among known to date. It is important to note, that effect of the scattering of ion gyration phase is the dominant process in the formation of the turbulent shift of the phase of the electric potential. This effect, however is absent in nonlinear gyrokinetic theory because of the primordial gyrophase averaging. It is important to note also, that ion drift kinetic equation, which does not contain finite ion Larmor effects, senseless to apply to the analysis of the instabilities and turbulence evolution in plasma shear flows.