DISSIPATIVE INSTABILITY IN CURRENT CARRYING PLASMA

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MICROINSTABILITIES

Magnetically confined plasma cannot be in equilibrium.

Potential for instability always exists.

Not all instabilities can be described by single fluid MHD model.

If one takes into account

- relative motion of plasma components
- kinetic effects
- Finite Larmor radius of plasma particles

he concludes that

$\Rightarrow$ additional instabilities -

microinstabilities

ROLE of MICROINSTABILITIES

In fusion plasma microinstabilities play an important role. They are accompanied by

- generation of fine scale plasma turbulence
- anomalous transport
- anomalous resistance
- plasma heating
- diffusion

other

MICROINSTABILITIES SREAMING TYPE

The most commonly occurring microinstabilities are that of streaming type. They are due to relative drift of plasma components e.g.

a group of fast electrons (beam) relative to another (*e-e* type)

well-known conventional BPI

- *motion of plasma electrons relative to ions i.e. current in plasma* (*e-i* type)

Buneman instability

$u << v_T$ ion-acoustic instability

$u >> v_T$ Buneman itself

O.Buneman. Phys Rev v.**115**, 503, (1959)

Role of Buneman instability

- Buneman Instability –relative motion of electron against ions
- Electrons and ions respond differently to EM perturbations

m / M Play an important role

The difference is due to their desperate masses

Drive and damping

$\mathrm{Im}\,\omega$ $\mathrm{Re}\,\omega$

Basic Properties of Buneman Instability

$$1-\frac{\omega_{Le}^2}{(\omega-\mathbf{ku})^2}-\frac{\omega_{Li}^2}{\omega^2}=0$$

1. Conditions of instability development $\quad \mathbf{k}_{\parallel} \le \frac{\boldsymbol{\omega}_{Le}}{u}$

2. Achieve max of growth rate at $\quad \mathbf{k}_{\parallel} \approx \frac{\boldsymbol{\omega}_{Le}}{\mathbf{u}}$

3. Growth rate ~ frequency $\quad \sim \omega_{Le}(m/M)^{1/3}$

Low frequency

strongly growing $\quad \mathrm{Im}\,\omega \sim \mathrm{Re}\,\omega$

In dense fusion plasma
both become smaller than $\quad \nu_{ei}$

DISSIPATIVE BEAM INSTABILITY AND ITS ROLE

One more important variety of streaming instabilities

Dissipative beam instability **(DBI)**

This type of streaming instabilities is due to presence of NEW *in stream*

Dissipation never suppresses streaming instability completely regardless on its level. It serves as a channel for energy withdrawal. This leads to excitation of NEW.

Properties. Relatively low growth rate

Relatively low energy of excited oscillations

The Role of DI in fusion devices

DSI have been widely discussed in context of explanation various phenomena in fusion plasma. In fusion devices **DSI** play an important role. They are accompanied by

- Plasma Heating
- Diffusion
- **DI** can greatly enhance plasma transport across magnetic fields line

Dissipative Instability in Current Carrying Plasma

In this report current carrying plasma in conditions

$$\nu_{ei} >> \operatorname{Im}\omega \sim \operatorname{Re}\omega$$

It is shown the conditions result in **dissipative instability** It is obtained

1. Growth rate
2. Gradual transition of the well known Bun inst to that of dissipative type with increase in level of dissipation
3. Evolution of initial perturbation →PDE of third order

Content of the Report

1. Introduction
2. Statement of the problem
3. Solution of kinetic equation with LCI
4. DR for potential oscillations and its comparison with HD
5. Dissipative instability in current carrying plasma its growth rate
6. Gradual transition of Buneman instability (no dissipative) to dissipative instability with increase in level of dissipation
7. Dynamics of DI development in CC plasma

Collisions in plasma

ei and *ee* collisions. if ionization level *Z* >>1, *ei* collisions are predominant (Lorentzian plasma)

$$\sigma_{ee} \sim N_e \left(e^2\right)^2 \quad ; \quad \sigma_{ei} \sim N_i \left(Ze^2\right)^2 = N_e \left(Ze^2\right)^2 >> \sigma_{ee}$$

Collisions in fully ionized plasma are described by Landau Collision Integral (LCI). This is

- complex integro-differential formation
- complicate kernel

⇒Set of Integro-Diff eqs for *e* and *i coupled with according LCI* describes the problem

Conditions of real solution

The LCI in kin eqs complicate the problem
Real solution is possible with significant simplifications only

1 Ions are immobile
2. Linear approx for electrons
3. Neglecting terms ~m/M

Unperturbed functions for electrons and ions are

$$f_e(\mathbf{v}) = C\exp\left[-\frac{m_e(\mathbf{v}-\mathbf{u})^2}{2T_e}\right]$$

$$f_i(\mathbf{v}) \sim \delta(\mathbf{v})$$

$$f_e = f_e^{(0)} + f_1$$

$$f_1 << f_e^{(0)}$$

This reduces problem to one eq

for electrons with LCI

Kinetic equation with LCI

$$\frac{\partial f_{e,i}}{\partial t} + \mathbf{v}\frac{\partial f_{e,i}}{\partial \mathbf{r}} + e\mathbf{E}\frac{\partial f_{e,i}}{\partial \mathbf{p}} = \left(\mathrm{St}f\right)_L^{(ei)}$$

$$\left(\mathrm{St}f\right)_L^{(ei)} = \frac{\partial}{\partial p_{e\alpha}}\int d\mathbf{p}_{ie} Q_{\alpha\beta}\left[f_i\frac{\partial f_e}{\partial p_{e\beta}} - f_e\frac{\partial f_i}{\partial p_{i\beta}}\right]$$

$$Q_{\alpha\beta} = \frac{1}{2}Q\left[\frac{\delta_{ik}u^2 - u_i u_k}{u^2}\right]$$

$$u = \left|\mathbf{v}_e - \mathbf{v}_i\right|$$

Q depends on σ - Rutherford's cross-section

u-relative velocity

Solution of kinetic equation

Simplify LCI using $f_i(\mathbf{v}) \sim \delta(\mathbf{v})$

$$(\mathrm{St} f)_L^{(ei)} \to \frac{N_i}{m^2} \frac{\partial}{\partial v_{e\alpha}} Q_{\alpha\beta} \frac{\partial f_e}{\partial v_{e\beta}}$$

$$v_i = 0$$

$$Q(v_e - v_i) \to Q(v_e)$$

$$\frac{\partial f_e}{\partial t} + \mathbf{v} \frac{\partial f_e}{\partial \mathbf{r}} + e\mathbf{E} \frac{\partial f_e}{\partial \mathbf{p}} = \frac{N_i}{m^2} \frac{\partial}{\partial v_{e\alpha}} Q_{\alpha\beta} \frac{\partial f_e}{\partial v_{e\beta}}$$

$Q_{\alpha\beta}$ → out of integrand

Integrals actually fall out

Linearization

$$f_1 << f_{0e}$$

$$\frac{\partial f_1}{\partial t} + \mathbf{v}\frac{\partial f_1}{\partial \mathbf{r}} + e\mathbf{E}\frac{\partial f_{0e}}{\partial \mathbf{p}} = \frac{N_i}{m^2}\frac{\partial}{\partial \upsilon_{e\alpha}} Q_{\alpha\beta}\frac{\partial}{\partial \upsilon_{e\beta}} f_1$$

Solution of kin equation in linear approximation

$$f_1 = \frac{\mathbf{v}\mathbf{f}_2}{|\mathbf{v}|} \equiv \frac{\mathbf{v}\mathbf{f}_2}{\upsilon}$$

$$\frac{\partial}{\partial p_{e\alpha}} Q_{\alpha\beta} \frac{\partial}{\partial p_{e\beta}} f_1 \rightarrow -\nu(\upsilon) f_1$$

$$|\mathbf{v}| = \upsilon$$

$$\nu(\upsilon) = \frac{N_i 2\pi (e_1 e_2)^2 L}{m^2 \upsilon^3} \sim \upsilon^{-3}$$

$$\left(\delta_{\alpha\beta}\upsilon^2 - \upsilon_\alpha \upsilon_\beta\right)\upsilon_\beta = 0$$

Differences from standard procedure of plasma physics

1. $\nu = \nu(v)$ $\qquad$ $\nu = const \to 0$

2. $\nu(v) > \omega$ but $\mathbf{ku} >> |\nu(v)|$

$$f \sim \exp\{-i\omega t + i\mathbf{kr}\}$$

$$-i\omega f_e^{(1)} + i\mathbf{kv} f_e^{(1)} + e\mathbf{E}\frac{\partial}{\partial \mathbf{p}} f_e^{(0)} = -\nu(v) f_e^{(1)}$$

$$f_e^{(1)} = -i\frac{e\mathbf{E}\frac{\partial}{\partial \mathbf{p}} f_e^{(0)}}{(\omega - \mathbf{kv} + i\nu(v))}$$

Dispersion relation for potential oscillations

$$\mathbf{E} = -i\mathbf{k}\varphi$$

$$k^2\varphi = 4\pi e N_e^{'} + 4\pi e_i N_i$$

$$1 + \frac{4\pi e^2}{mk^2}\int \frac{\mathbf{k}\frac{\partial}{\partial \mathbf{v}} f_e^{(0)}}{\left(\omega - \mathbf{kv} + i\nu(\upsilon)\right)} d\mathbf{v} - \frac{\omega_{Li}^2}{\omega^2} = 0$$

$$f_e(\mathbf{v}) = N_e\left(\frac{m}{2\pi T_e}\right)^{\frac{3}{2}} \exp\left[-\frac{m_e(\mathbf{v}-\mathbf{u})^2}{2T_e}\right]$$

Transformation of the DR to convenient form 1

Change variable $\mathbf{w} = \frac{(\mathbf{v}-\mathbf{u})}{\sqrt{2}v_{Te}}$

$$\Delta\varepsilon^{(l)} = 1 + \Delta\varepsilon_e + \Delta\varepsilon_i$$

$$\Delta\varepsilon_e = \frac{\omega_{Le}^2}{k^2 v_{Te}^2}\frac{1}{\pi^{3/2}}\int_{-\infty}^{\infty} dw_x e^{-w_x^2}\int_{-\infty}^{\infty} dw_y e^{-w_y^2} J$$

$$J = J\left(\frac{\mathbf{ku} + i\nu(v) - \omega}{\sqrt{2}kv_T}\right) = \int_{-\infty}^{\infty}\frac{w_z e^{-aw_z^2}dw_z}{w_z + \dfrac{\mathbf{ku} + i\nu(v) - \omega}{\sqrt{2}kv_T}}$$

Transformation of the DR to convenient form 2

We split the integral J into two part using

$$\mathbf{ku} >> |\nu(\upsilon)|$$

$$J = \left\{ \int_{-\infty}^{\infty} \frac{w_z dw_z e^{-w_z^2}}{(w_z + z)} - i \frac{C_\nu}{z^2 \sqrt{2} k_z \upsilon_T} \int_{-\infty}^{\infty} \frac{w dw e^{-w_z^2}}{\upsilon^3} \right\} \approx J_{pl}(z) - i \frac{C_\nu}{(\mathbf{ku})^2} \frac{\sqrt{2} k_z \upsilon_T}{2u^3}$$

$$z = \frac{\mathbf{ku}}{\sqrt{2} k_z \upsilon_T} \left(1 - \frac{\omega}{\mathbf{ku}} \right)$$

$$\nu = \nu(\upsilon) = \frac{C_\nu}{\upsilon^3}$$

$$J_{\text{pl}} = \int_{-\infty}^{\infty} \frac{w_z dw_z e^{-w_z^2}}{(w_z + z)}$$

$$J_{\text{pl}} = \sqrt{\pi} \{ 1 + F(z) \} \rightarrow -\frac{\sqrt{\pi}}{2z^2}$$

$$u >> \upsilon_{Te}$$

Intermediate synthesis

Differeces with standard procedure of plasma physics

$$1.\quad \nu = \nu(\upsilon) \qquad\qquad \nu = const$$

$$2.\quad \nu >> \omega \qquad\qquad \nu \rightarrow 0$$

$$J \approx -\sqrt{\pi}\,\frac{k^2 \upsilon_T^2}{(\mathbf{ku})^2}\left\{1 + 2\frac{\omega}{\mathbf{ku}} + i\frac{\nu_{eff}}{\mathbf{ku}}\right\}$$

The DR $$1 - \frac{\omega_{Le}^2}{(\mathbf{ku})^2}\left\{1 + 2\frac{\omega}{\mathbf{ku}} + i\frac{\nu_{eff}}{\mathbf{ku}}\right\} - \frac{\omega_{Li}^2}{\omega^2} = 0$$

Comparison of the DR with HD

Kinetics

Hydrodynamics

$$J \approx -\sqrt{\pi}\,\frac{k^2 v_T^2}{(\mathbf{ku})^2}\left\{1 + 2\frac{\omega}{\mathbf{ku}} + i\frac{\nu_{eff}}{\mathbf{ku}}\right\}$$

$$1 - \frac{\omega_{Le}^2}{(\omega - \mathbf{ku})(\omega - \mathbf{ku})} - \frac{\omega_{Li}^2}{\omega^2} = 0$$

$$1 - \frac{\omega_{Le}^2}{(\mathbf{ku})^2}\left\{1 + 2\frac{\omega}{\mathbf{ku}} + i\frac{\nu_{eff}}{\mathbf{ku}}\right\} - \frac{\omega_{Li}^2}{\omega^2} = 0$$

$$1 - \frac{\omega_{Le}^2}{(\omega - \mathbf{ku})(\omega - \mathbf{ku} + i\nu)} - \frac{\omega_{Li}^2}{\omega^2} = 0$$

$$1 - \frac{\omega_{Le}^2}{(\mathbf{ku})^2}\left(1 + 2\frac{\omega}{\mathbf{ku}} + \frac{i\nu}{\mathbf{ku}}\right) - \frac{\omega_{Li}^2}{\omega^2} = 0$$

$$\nu_{ei}(\upsilon) \rightarrow \nu_{ei}(\upsilon_{Te}) = \nu_{eff} = \sqrt{2\pi}\,\frac{N_i Z^2 e^4 L}{\sqrt{T_e}\, m^{3/2} u^2}$$

Solution of the DR
Growth rate

$$1-\frac{\omega_{Le}^2}{(\mathbf{ku})^2}\left(1+2\frac{\omega}{\mathbf{ku}}-\frac{i\nu}{\mathbf{ku}}\right)-\frac{\omega_{Li}^2}{\omega^2}=0$$

$\mathbf{ku} < \omega_{Le}$

$$\omega=\frac{\omega_{Li}^2}{\sqrt{1-\frac{\omega_{Le}^2}{(\mathbf{ku})^2}\left(1+2\frac{\omega}{\mathbf{ku}}-\frac{i\nu_{eff}}{\mathbf{ku}}\right)}}=\mathrm{Re}\,\omega+i\,\mathrm{Im}\,\omega$$

Resonance $\omega_{Le}=\mathbf{ku}$

$$2\frac{\omega}{\mathbf{ku}}+\frac{i\nu_{eff}}{\mathbf{ku}}+\frac{\omega_{Li}^2}{\omega^2}=0$$

$$\omega \gg \nu_{eff}$$

$$\mathrm{Im}\,\omega=\frac{\sqrt{3}}{2}\omega_{Le}\left(\frac{m}{2M}\right)^{\frac{1}{3}}$$

Transition to dissipative instability

Opposite case $\omega << \nu_{eff}$

$$\operatorname{Im}\omega = \frac{1}{\sqrt{2}}\delta_0\sqrt{\frac{2\delta_0}{\nu_{eff}}} = \delta_0\sqrt{\frac{2\delta_0}{\nu_{eff}}} = \frac{\delta_0}{\sqrt{\lambda_0}}$$

$$\lambda_0 = \nu_{eff}/2\delta_0$$

Genaral case

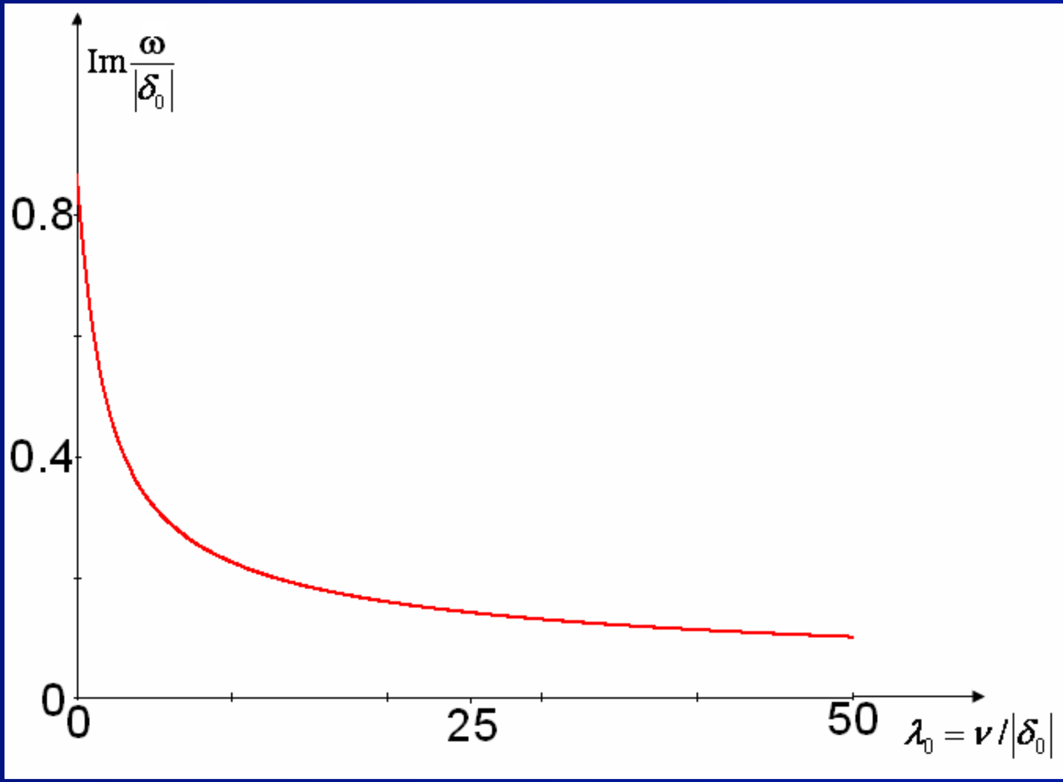
$$2\frac{\omega}{\mathbf{ku}} + \frac{i\nu_{eff}}{\mathbf{ku}} + \frac{\omega_{Li}^2}{\omega^2} = 0$$

$$\omega^3 + i\frac{\nu}{2}\omega^2 + |\delta_0|^3 = 0$$

$$y^3 + i\lambda_0 y^2 + 1 = 0$$

$$y = \frac{\omega}{|\delta_0|}$$

DEPENDENCE OF THE GROWTH RATE ON DISSIPATION LEVEL



Dynamics of dissipative instability development

For full understanding many aspects of physics of any instability e.g the manner in which

- the instability modifies the given equilibrium state of the beam into the nonlinear state
- calculate the level of the turbulence saturation
- etc

it is important to investigate space-time evolution of initial perturbation during given instability development

Space-time evolution of initial perturbation

- one of most widespread problem and not only in plasma physics.
- *Classical way* -is the method based on the integral representation of the problem with a DR in the denominator of the integrand.
- The Green's function gives overall view
- on the character of instability (abs,convect)
- The approach leads to final result without refs to any model only for homogeneous media

Development of initial perturbation. Approach

In present investigation the approach is modified. The modification relates to streaming instab.

1. Field is represented as wave train with SVA
2. Intermediate equation for SVA is obtained. It describes evolution of initial perturbation in space and time
3. The equation is solved by Laplace transform
4. Inverse Laplace transf is carried out by analogy to classical method

Results may be applied to streaming instab of various types (Cherenkov, cyclotron etc) independently on geometry, specific parameters, etc

Dynamics of dissipative instability development 1

Proceed from DR

$$1-\frac{\omega_{Le}^{2}}{(\omega-\mathbf{ku})(\omega-\mathbf{ku}+i\nu)}-\frac{\omega_{Li}^{2}}{\omega^{2}}=0$$

We actually have DR consisting of 2 parts. Main part and small addition. Main part has stable solutions.

As $\omega_{Li}^{2} << \omega_{Le}^{2}$

The parts may be equal only if $\omega_{Li} >> \omega \to 0$

instability appears only if one takes into account the small additionn

Dynamics of dissipative instability development 2

$$D_0(\omega,\mathbf{k}) = 1 - \frac{\omega_{Le}^2}{(\omega - \mathbf{ku})(\omega - \mathbf{ku} + i\nu)}$$

Main part

$$\Delta D(\omega,\mathbf{k}) = \frac{\omega_{Li}^2}{\omega^2}$$

Small addition

Ions interact with proper beam oscillations and the interaction causes instability

THE EQUATION FOR SLOWLY VARYING AMPLITUDE

The instability reveals itself most effectively if two conditions satisfy

1. frequency is close to solutions of the main part i.e. to those of the beam space charge density wave

And when right-hand side plays a role i.e. the addition increases $\omega \to 0$

$D_0(\omega, k) = 0$ $\omega_{Li} >> \omega \to 0$ but $\omega \neq 0$

Equation SVA

It would appear reasonable to represent the growing field in following form

$$E(z,t) = E_0(z,t)\exp(-i\omega_0 t + ikz)$$

where t is the time, z-longitudinal coordinate and the amplitude $E_0(z,t)$ is slowly varying i.e.

$$\left|\frac{\partial E_0}{\partial t}\right| << \omega_0 E_0 \qquad \left|\frac{\partial E_0}{\partial z}\right| << k_0 E_0$$

Using formal substitutions

$$k = k_0 - i\frac{\partial}{\partial t} \qquad \omega = \omega_0 + i\frac{\partial}{\partial t}$$

THE EQUATION FOR SLOWLY VARYING AMPLITUDE

- One can obtain from the DR

$$\frac{\partial^2}{\partial t^2}\left(\frac{\partial}{\partial t}+\mathrm{v}_0\frac{\partial}{\partial z}+\nu^*\right)E_0(z,t)=i|\delta_0|^3 E_0(z,t)$$

$$|\delta_0|^3=\left\{\omega_{Li}^2\Big/\left(\partial D_0\big/\partial\omega\right)\right\}_{\substack{\omega=\omega_0\\k=k_0}}$$

$$\mathrm{v}_0=-\left\{\frac{\left(\partial D_0\big/\partial k\right)}{\left(\partial D_0\big/\partial\omega\right)}\right\}_{\substack{\omega=\omega_0\\k=k_0}}=u$$

$$\nu^*=\left\{\mathrm{Im}\,D_0\Big/\left(\partial D_0\big/\partial\omega\right)\right\}_{\substack{\omega=\omega_0\\k=k_0}}=\frac{\nu}{2}$$

Solution of equation for SVA

Solution is based on Laplace transformation

The transformed version is

$$\left\{\omega^2\left(\omega - k\mathrm{v}_0 + i\nu^*\right) - \left|\delta_0\right|^3\right\}E_0(\omega,k) = J(\omega,k)$$

Inverse transformation

$$E_0(z,t) = \frac{1}{(2\pi)^2}\iint \frac{d\omega dk \quad J(\omega,k)e^{-i\omega t + ikz}}{\omega^2\left(\omega - k\mathrm{v}_0 + i\nu^*\right) - \left|\delta_0\right|^3}$$

The problem reduced to integration

Integration

- First integration – over *k* - by residue method. The pole is

$$k_0 = \frac{1}{v_0}\left(\omega + i\nu^* - |\delta_0|^3/\omega^2\right)$$

- Second integration - over ω by steepest descent method. The saddle point is

$$\omega_s = \left(2|\delta_0|^3 \frac{z}{u\tau}\right)^{1/3} e^{\frac{2\pi}{3}i}$$

Solution of equation for SVA

The dynamics of initial perturbation development is given by the envelope of the wave train

$$E_0(z,t) = \frac{J_0}{\sqrt{2\pi}} \frac{e^{\chi - \nu^* \frac{z}{u}} e^{i\left(\frac{\chi}{\sqrt{3}} - \frac{\pi}{6}\right)}}{\left(6uz|\delta_0|^3\right)^{1/2}}$$

$$\chi = \frac{3\sqrt{3}}{4}|\delta_0|\left\{\frac{2}{u} z\tau^2\right\}^{1/3} - \nu \frac{z}{2u}$$

$$\tau = t - z/u$$

Analysis of the growing waveform

The field is mainly given by the factor

$$\exp\left(\chi - \nu^* \frac{z}{u}\right)$$

$\nu = 0$ Max is obtained from the condition $\frac{\partial}{\partial z}\chi = 0$

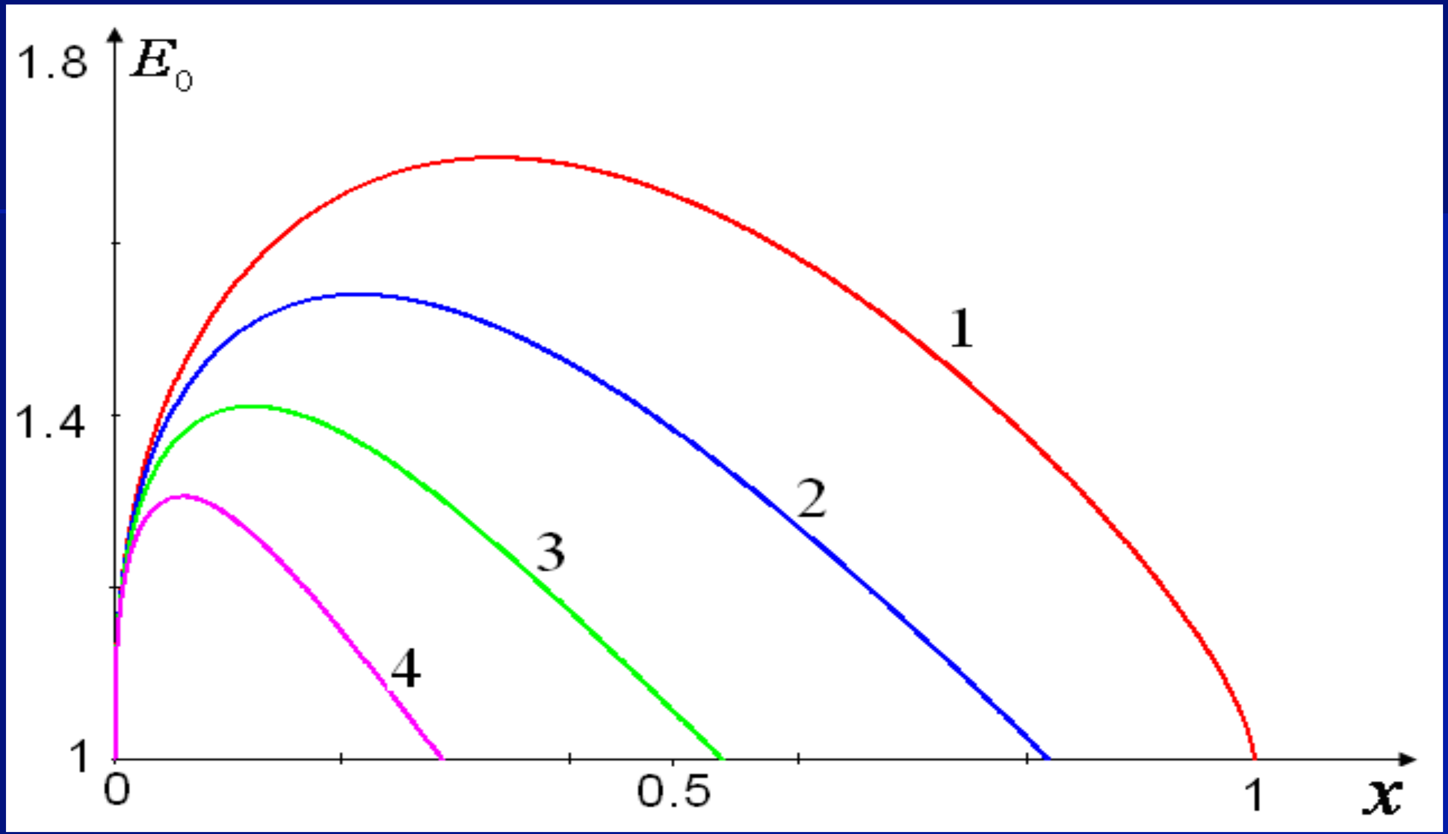
$$z = \frac{ut}{3}$$

$$E_0 \sim \exp \frac{\sqrt{3}}{2}\delta_0 t \approx \exp \delta_{\max} t$$

$$\delta_{\max} = \frac{\sqrt{3}}{2}\left|\delta_0\right|$$

I.e. the maximal growth rate, which usually describes given instability, actually is growth rate in the peak of induced wave train.

$\nu \neq 0$



Shapes of Buneman instability development depend on longitudinal coordinate (x=z/ut) for various level of dissipation: Curve 1 corresponds to $\lambda=\nu/\delta=0$; curve 2 – to $\lambda=1,2$; curve 3 – to $\lambda=3$; curve 4 – to $\lambda=6$.

δ_0 / ν_{ei}

Conclusion

In **CC** plasma with high level of *e-i* collisions transform Buneman instability to that of dissipative type with growth rate

$$\operatorname{Im}\omega \equiv \delta_\nu = \delta_0 \sqrt{\frac{\delta_0}{\nu_{ei}}} << \delta_0$$

$$\operatorname{Re}\omega \approx \operatorname{Im}\omega$$

$$\nu_{ei}(v) \rightarrow \nu_{ei}(v_{Te}) = \nu_{eff} = \sqrt{2\pi}\,\frac{N_i Z^2 e^4 L}{\sqrt{T_e}\, m^{3/2} u^2}$$

$$Z >> 1$$

The growth rate is also obtained for arbitrary level of the ratio

$$\nu / \delta_0$$

Dissipation suppresses fastest perturbations

THANK YOU