Coupled simulations of RF feedback stabilization of tearing modes

5th IAEA Technical Meeting on the "Theory of Plasma Instabilities"

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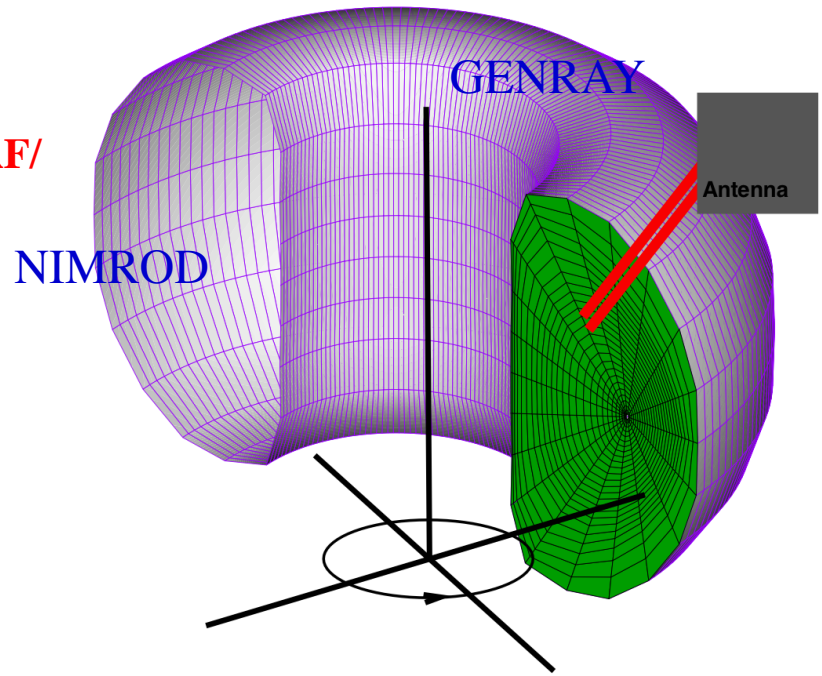
Coupled simulations of RF feedback stabilization of tearing modes

Approach to modeling ECCD in fluid simulations

- Use nonlinear, extended MHD code to model plasma dynamics
- Use ray tracing code to model RF field propagation
- Integration of RF sources into fluid model

OUTLINE:

- Formulation of multiscale problem
  - What is the right theoretical approach to RF/ MHD integration for electrons?
- Results of *ad hoc* source
  - What are the physics effects of adding a current source into fluid simulations?
- Computational approach to coupled simulations
  - How are we solving the problem?
- First results of initial simulations



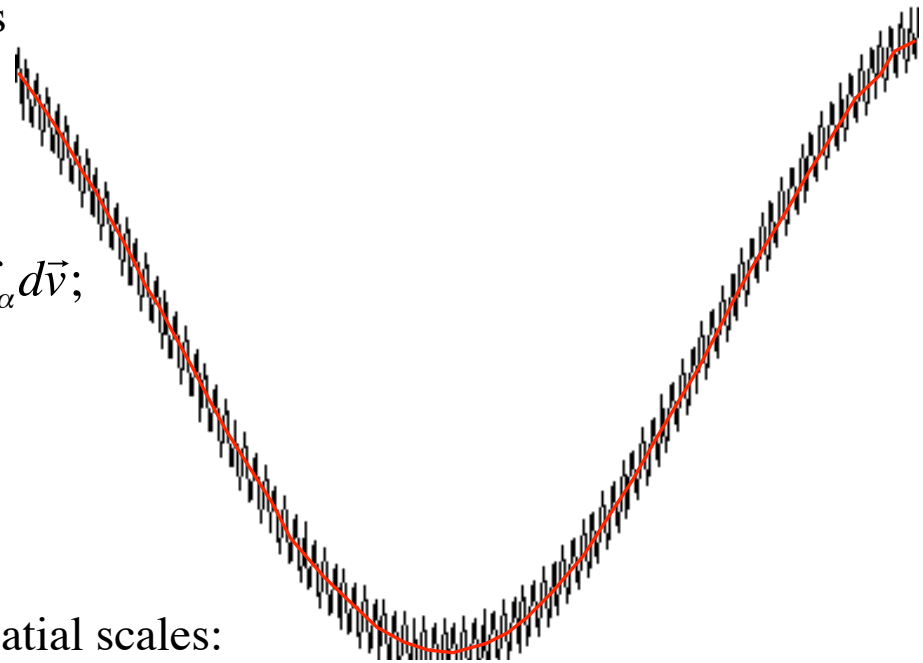
Mathematical formulation relies on separation of time and spatial scales between MHD and RF time scales

•Start with Vlasov-Maxwell system of equations

$$\frac{\partial f_\alpha}{\partial t} + \vec{v}_\alpha \bullet \nabla f_\alpha + \frac{q_\alpha}{m_\alpha}\left(\vec{E} + \vec{v}_\alpha \times \vec{B}\right) \bullet \nabla_v f_\alpha = C(f_\alpha)$$

$$\frac{\partial \vec{B}}{\partial t} = -\nabla \times \vec{E}; \qquad \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} = \nabla \times \vec{B} - \mu_0 \sum_a q_\alpha \int \vec{v} f_\alpha d\vec{v};$$

$$\nabla \bullet \vec{E} = \sum_a \frac{q_\alpha}{\varepsilon_0} \int f_\alpha d\vec{v}; \qquad \nabla \bullet \vec{B} = 0$$



•Take advantage of separation of temporal and spatial scales:

$$f_\alpha(x,v,t) = \langle f_\alpha \rangle^{MHD}(x_{MHD}, v_{MHD}, t_{MHD}) + \varepsilon f_\alpha^{RF}(x_{MHD}, t_{MHD}, x_{RF}, v_{RF}, t_{RF})$$

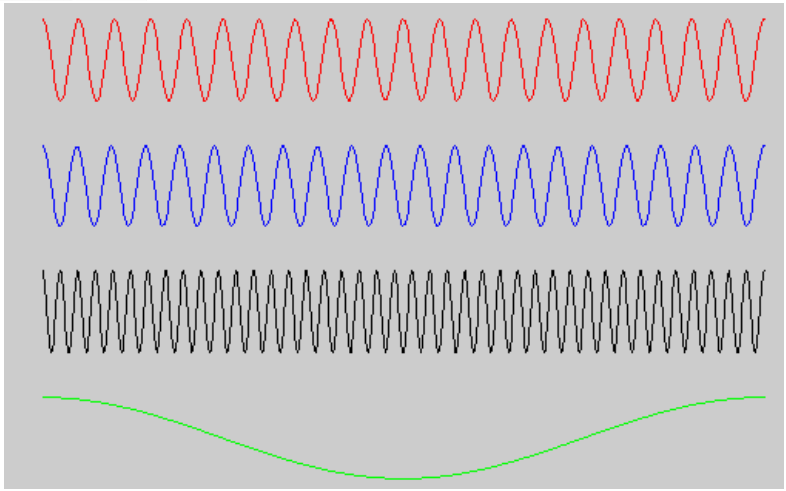
$$\vec{E}(x,v,t) = \langle \vec{E} \rangle^{MHD}(x_{MHD}, t_{MHD}) + \varepsilon \vec{E}(x_{MHD}, t_{MHD}, x_{RF}, t_{RF})$$

$$\vec{B}(x,v,t) = \langle \vec{B} \rangle^{MHD}(x_{MHD}, t_{MHD}) + \varepsilon \vec{B}(x_{MHD}, t_{MHD}, x_{RF}, t_{RF})$$

where <…> denotes the average over RF timescales

•Goal is to represent the effects of the fast motion on slow motion

RF effects the slow time scales through the introduction of a quasilinear diffusion operator



Two waves of near-identical wavelengths

beat at wavelengths $\lambda_b = (\lambda_1 \pm \lambda_2)$.

•Vlasov-Maxwell system – order ε^2 pieces enter averaged kinetic equation at the same order as ε^0 pieces because of beating:

$$\frac{\partial \langle f_\alpha \rangle_{MHD}}{\partial t} + \mathbf{v} \cdot \frac{\partial \langle f_\alpha \rangle_{MHD}}{\partial \mathbf{x}} + \frac{q_\alpha}{m_\alpha}[\langle \mathbf{E} \rangle_{MHD} + \mathbf{v} \times \langle \mathbf{B} \rangle_{MHD}] \cdot \frac{\partial \langle f_\alpha \rangle_{MHD}}{\partial \mathbf{v}}$$

$$+ \left\langle \epsilon^2 \frac{q_\alpha}{m_\alpha}[\mathbf{E}_{RF} + \mathbf{v} \times \mathbf{B}_{RF}] \cdot \frac{\partial f_{\alpha RF}}{\partial \mathbf{v}} \right\rangle = C[\langle f_\alpha \rangle_{MHD}]$$

quasilinear diffusion

so the kinetic equation effectively has an extra term from the RF;

$$\frac{\partial f_\alpha}{\partial t} + \mathbf{v} \cdot \nabla f_\alpha + \frac{q_\alpha}{m_\alpha}(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \frac{\partial f_\alpha}{\partial \mathbf{v}} = C(f_\alpha) + Q(f_\alpha)$$

This will modify “normal” MHD equations

Closures have been the key to the Slow MHD campaign

- **Issue: Need closure terms to be consistent with sources**

$$m_s n_s\left(\frac{\partial \vec{v}_s}{\partial t} + \vec{v}_s \cdot \nabla \vec{v}_s\right) = n_s q_s(\vec{E} + \vec{v}_s \times \vec{B}) - \nabla p_s - \nabla \cdot \pi_s + \vec{R}_s + \vec{F}_s^{rf},$$

$$\frac{3}{2} n_s\left(\frac{\partial T_s}{\partial t} + \vec{v}_s \cdot \nabla T_s\right) + n_s T_s \nabla \cdot \vec{v}_s = Q_s \qquad - \vec{\pi}_s : \nabla \vec{v}_s - \nabla \cdot q_s + S_s^{rf},$$

$$\vec{F}_s^{RF} = \int m_s \vec{v} Q_s d\vec{v}$$

$$S_s = \int \frac{m_s v^2}{2} Q_s d\vec{v}$$

- **Assuming a small kinetic distortion:** $f_s = f_{Ms} + F_s, \quad F_s << f_{Ms}$
- **Using a Chapman-Enskog-like approximation (Held04), form for evolution of kinetic distortions are:**

$$\frac{dF}{dt} - C(f_M + F) = \ldots + Q(f_M) - \frac{\vec{v}' \cdot \vec{F}^{rf}}{nT} f_M - \frac{2}{3}\frac{S^{rf}}{nT}\left(\frac{mv'^2}{2T} - \frac{3}{2}\right) f_M,$$

Thermodynamic drives from f_M that gives the "normal" heat flux/stress tensors (Landau damping, bootstrap current)

$$\pi_s = \int \vec{v}\vec{v} F_s d\vec{v}$$

$$\vec{q}_s = \int \vec{v} \frac{m_s v^2}{2} F_s d\vec{v}$$

$$R_s = \int \vec{v} C d\vec{v}$$

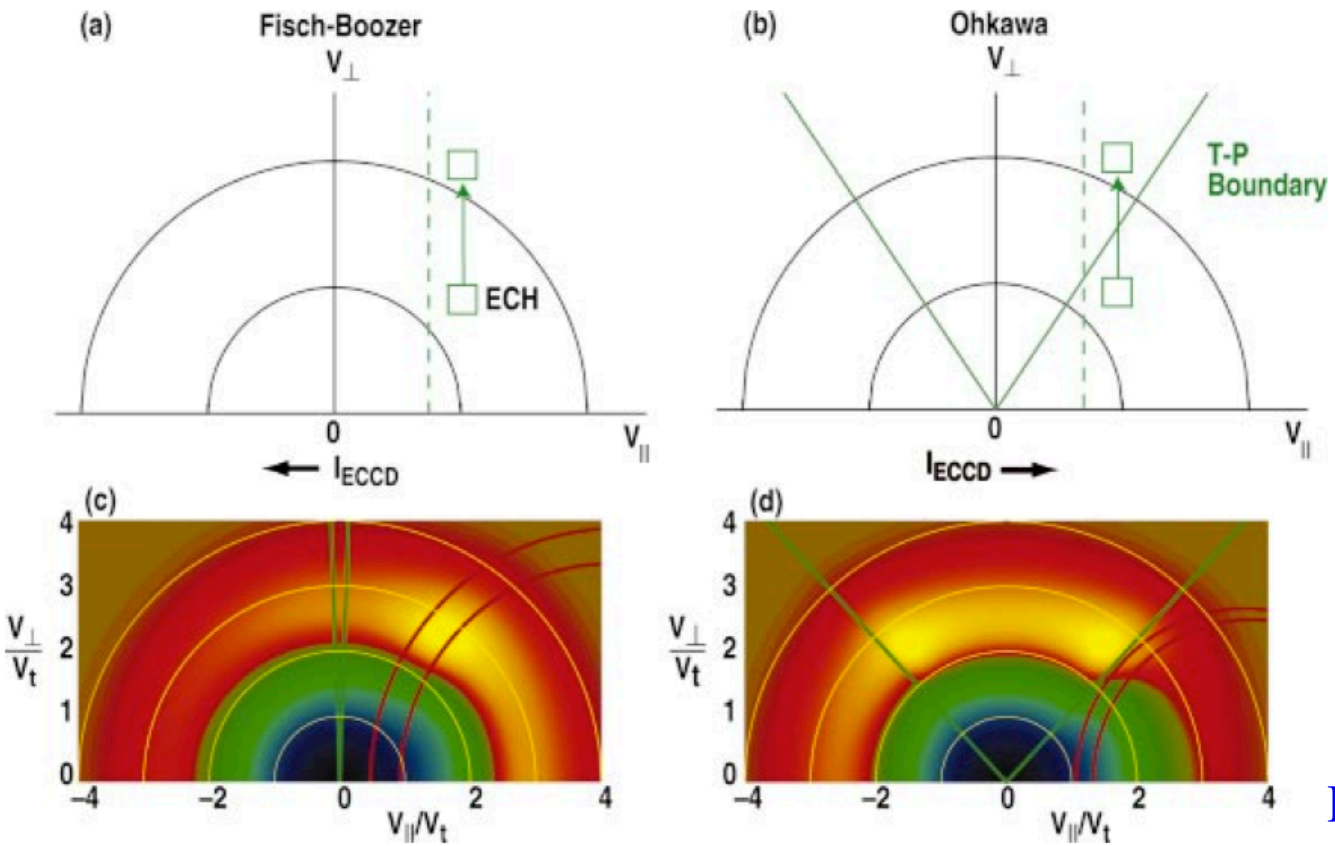
- **See:**

C.C. Hegna and J.D. Callen, Phys. Plasmas 16, 112501 (2009).

J.J. Ramos, Phys. Plasmas 15, 082106 (2008).

J.J. Ramos, Phys. Plasmas 17, 105106 (2010).

Where did the Fokker-Planck physics go?



From Prater 2004

Direct momentum drive
~1/3 of current drive

$$m_s n_s\left(\frac{\partial \vec{v}_s}{\partial t} + \vec{v}_s \cdot \nabla \vec{v}_s\right) = n_s q_s(\vec{E} + \vec{v}_s \times \vec{B}) - \nabla p_s - \nabla \cdot \pi_s + \eta\left[\vec{J} + \frac{3e\vec{q}}{5T_e}\right] + \vec{F}_s^{rf}$$

Ohkawa

Fisch-Boozer

Accurate closures needed for quantitative results

This formulation more rigorous, self-consistent then previous models

- **Giruzzi model used in** Yu et.al., PP 2000, Gianakon PP 2001 **with key discussions in** Giruzzi et.al. NF 1999

$$\vec{E} + \vec{v} \times \vec{B} = \eta(\vec{J} - \vec{J}_{rf}) + \ldots$$

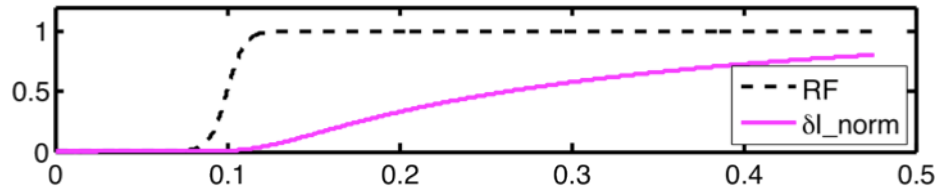
$$\frac{\partial n_{RF}}{\partial t} + \nabla \bullet \left(\chi_{\parallel} \nabla_{\parallel} n_{rf} + \chi_{\perp} \nabla_{\perp} n_{rf} \right) + \nu_{rf} (n_{rf\,source} - n) = 0.$$

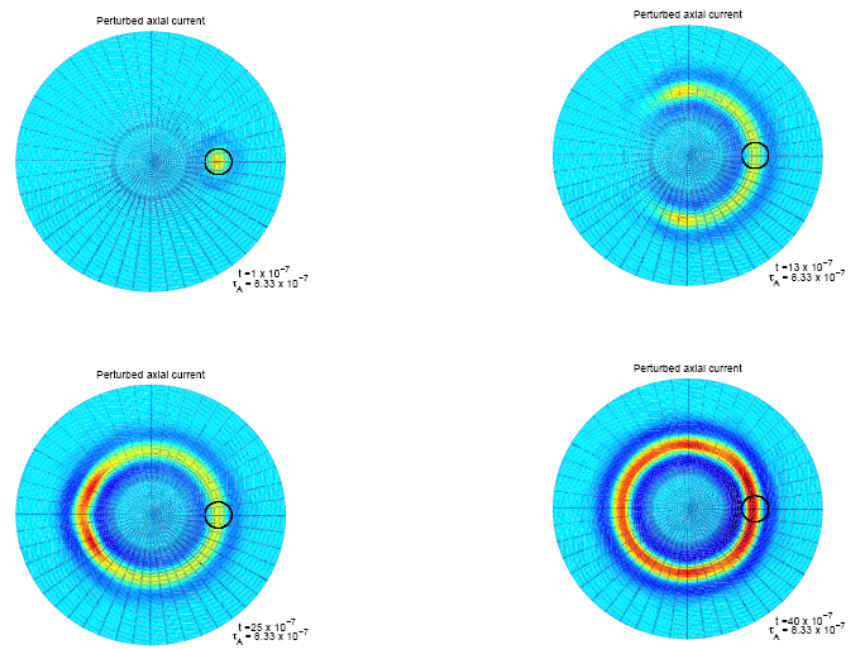
  - **J_{RF}/n_{RF} represents resonant electrons treated as a separate species**
    - **Similar approach used routinely for hot ions assuming separation of energy allows separation of distribution function**
    - **Using similar approach for ECCD which gives smaller perturbations to distribution not valid**
  - **Anisotropic diffusion approximate physics of stress tensor (momentum equilibration)**
    - **Used in reduced MHD which analytically eliminates compressional Alfven waves which would also equilibrate current**
  - **Source approximates physics parallel heat flux and momentum input**
    - **Implementations used a helical box for shape in poloidal or toroidal direction**
  - **Subsequent simulations presented:**
    - **No closures used – momentum drive term only**

Simpler model to give same physics as Giruzzi model relies on compressional Alfven waves

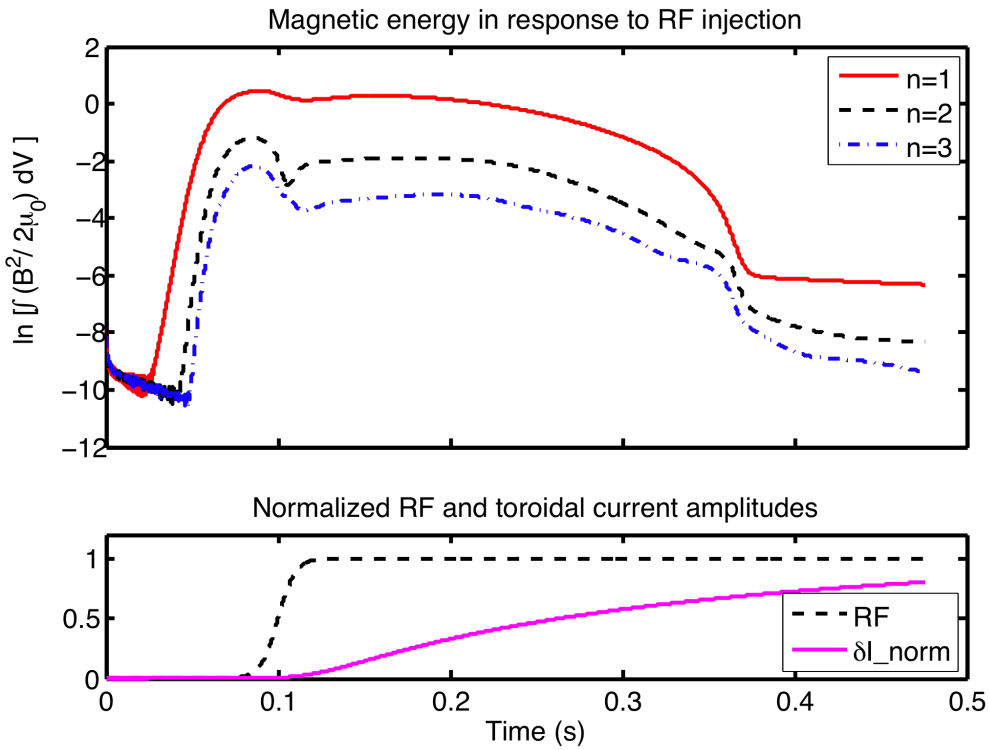
- Just specify time rise of current source as tanh function (lumps direct momentum source with Fisch-Boozer/Ohkawa currents)
- Equilibration requires compressional Alfven waves
- Test:
  - $F_{RF} = F_{RF}(R,Z)$ is specified as an analytic function.
  - Evolve only axisymmetric fields
  - Equilibration occurs over the flux surfaces after only a few Alfven times
- Flux surface averages of RF terms are unnecessary - force balance in NIMROD spreads the RF effects over the flux surfaces
- Kinetic effects also affect equilibration, but basic physics is achieved without closure
- Still achieve desired time scales:

$$\tau_{equilibrate} << \tau_{L/R}, \tau_{Rutherford} < \tau_R$$

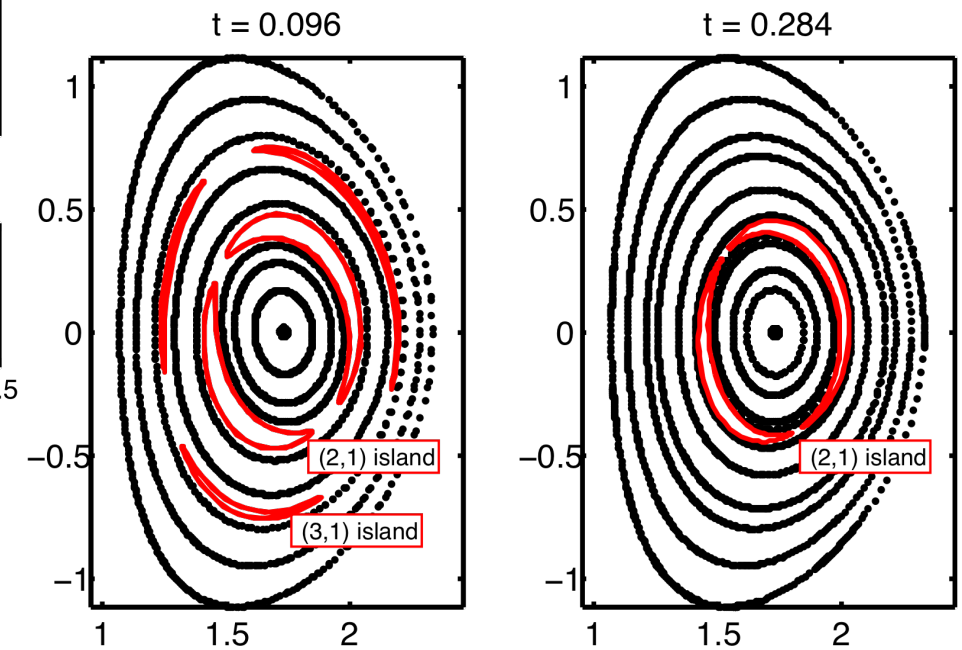




Studies with axisymmetric *ad hoc* sources used to understand computational requirements for fully coupled model



- Source is toroidally symmetric
- RF source induces a net toroidal current whose value is ~4% of the initial toroidal current.



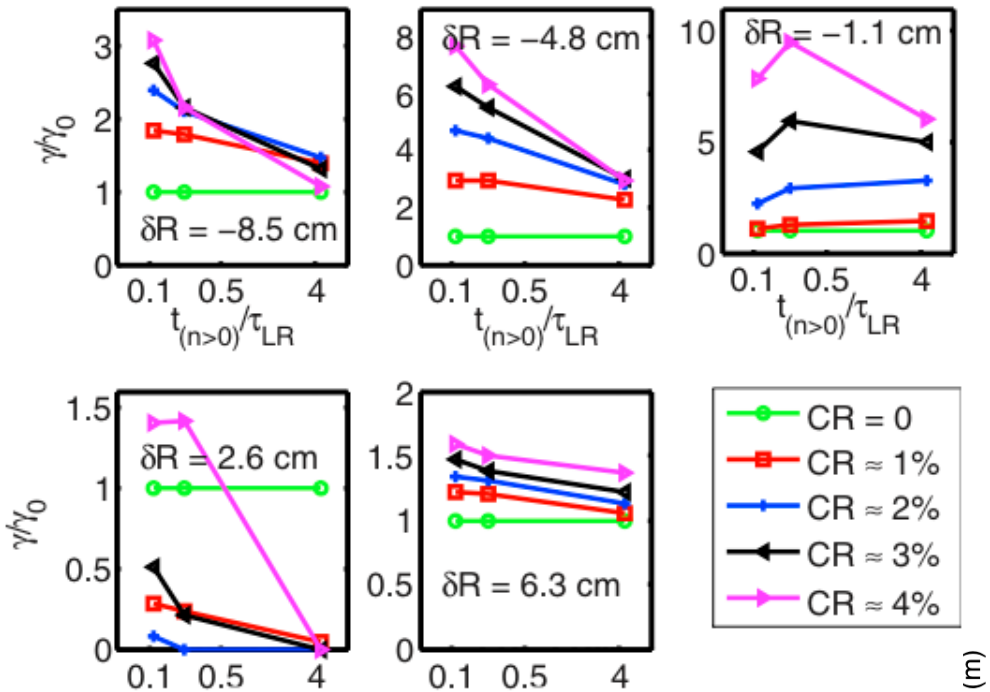
- Driven current is induced on a timescale ~ τ_R*(geometric factor related to poloidal ECCD localization). Here, $\tau_R = 0.99$ s.

- By t=0.35 s, island structures have vanished.

Time step is 10^{-6} sec
CFL number ~ 10^5

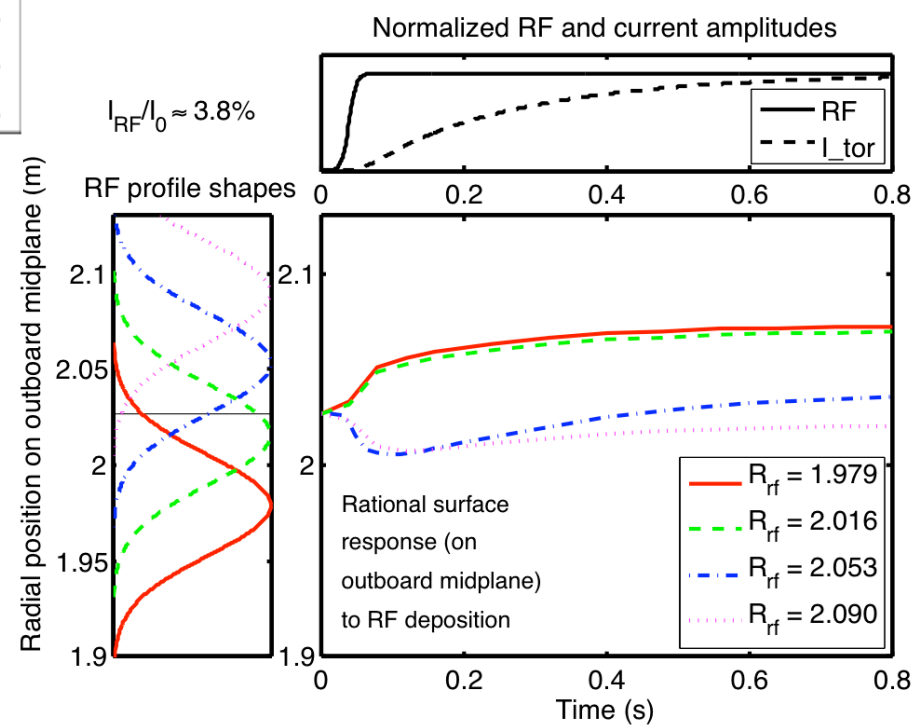
Make sure I discuss transport/sources

Rational surface motion affects mode growth dramatically for axisymmetric sources



Alignment on the outboard side gives better stabilization

Stabilization affected by rational surface movement



See:
Jenkins et.al., Phys. Plasmas ***17****, 012502 (2010)*
for complete analysis and physics results

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Conclusions of modeling with axisymmetric sources

- **Movement of rational surfaces is important**
  - **Experimental uncertainties of rational surfaces translates into**
  - **Transport is important**
- **For experimental situations where rational surfaces are not known perfectly, the situation is more complicated than the Hegna model would indicate**
  - **Experimental uncertainties in knowing O-point location**
  - **Rational surface is moving around due to ECCD source and underlying transport**

- **Eventual goal of numerical experiments is to guide plasma control system of experiments.**

To calculate the terms within the fluid equations, need to calculate the quasilinear operator

- Recall fundamental assumption of fields and functions:

$$\vec{E}(x,v,t) = \left\langle \vec{E} \right\rangle^{MHD}(x_{MHD},t_{MHD}) + \varepsilon \vec{E}^{RF}(x_{MHD},t_{MHD},x_{RF},t_{RF})$$

- ECRF fields are calculated using the eikenol approximation so dependencies are:

$$\vec{E}^{RF}(x_{MHD},t_{MHD},k_{RF},\omega_{RF})$$

- Formula for ion source is for example:

$$S_\alpha^{RF} = \frac{q_\alpha^2 n_{0\alpha}\epsilon^2 i}{2\pi L^3 T_{0\alpha}} \int\int\int\int \sum_{m=-\infty}^{\infty} \frac{\left|E_{\|RF} v_\| J_m(z) + v_\perp \left(\frac{E_k + iE_\xi}{2}\right) J_{m-1}(z) + v_\perp \left(\frac{E_k - iE_\xi}{2}\right) J_{m+1}(z)\right|^2}{\omega + i\gamma - k_\| v_\| - m\Omega} \left(\frac{m_\alpha}{2\pi T_{0\alpha}}\right)^{3/2} e^{-m_\alpha v_\|^2/2T_{0\alpha}} e^{-m_\alpha v_\perp^2/2T_{0\alpha}} k_\perp v_\perp dk_\perp dk_\| dv_\perp dv_\|$$

- General procedure is:

Obtain $E_{RF}(x_{mhd}, t_{mhd}, k_{RF}, \omega_{RF})$ from GENRAY

→ Integrate over k_{RF}

Obtain $Q(x_{MHD}, v_{MHD}, t_{mhd})$ from GENRAY

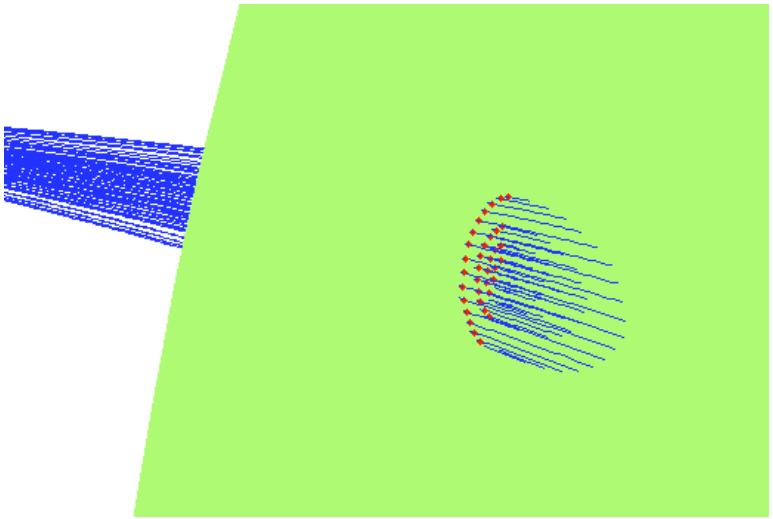
→ Integrate over v

Obtain $F(x_{mhd}, t_{mhd})$, $S(x_{mhd}, t_{mhd})$ on GENRAY grid

↓ Interpolate

Obtain $F(x_{mhd}, t_{mhd})$, $S(x_{mhd}, t_{mhd})$ on NIMROD grid

Integration over GENRAY rays requires turning rays into unstructured grid

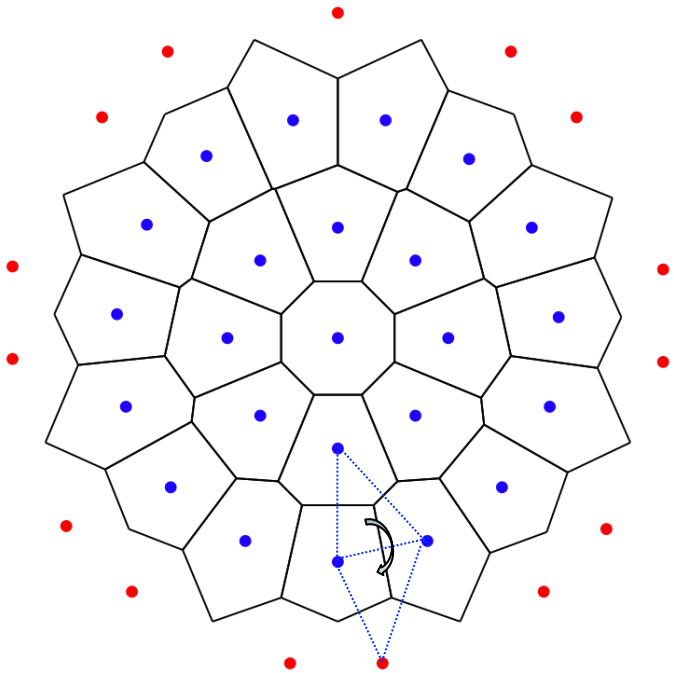


•Increased ray density ➔ lower power content and smaller area-perpendicular-to-flow for each ray.

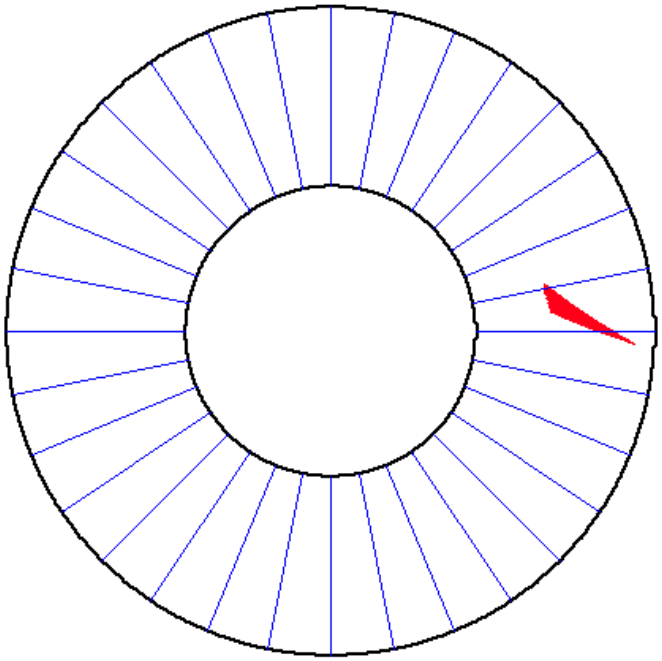
•PC/Aperp ratio appears in the quasilinear terms (➔ convergence with more rays); area must be calculated to evaluate these terms along ray paths.

•Interpolate ray data to planes along bundle path; use QHULL libraries to find areas:

-Delaunay triangulation
-Ghost points via reflection over convex hull
-Construct dual (Voronoi) mesh
-Find area of resultant polygons.



Differing scaling lengths provides computational challenges

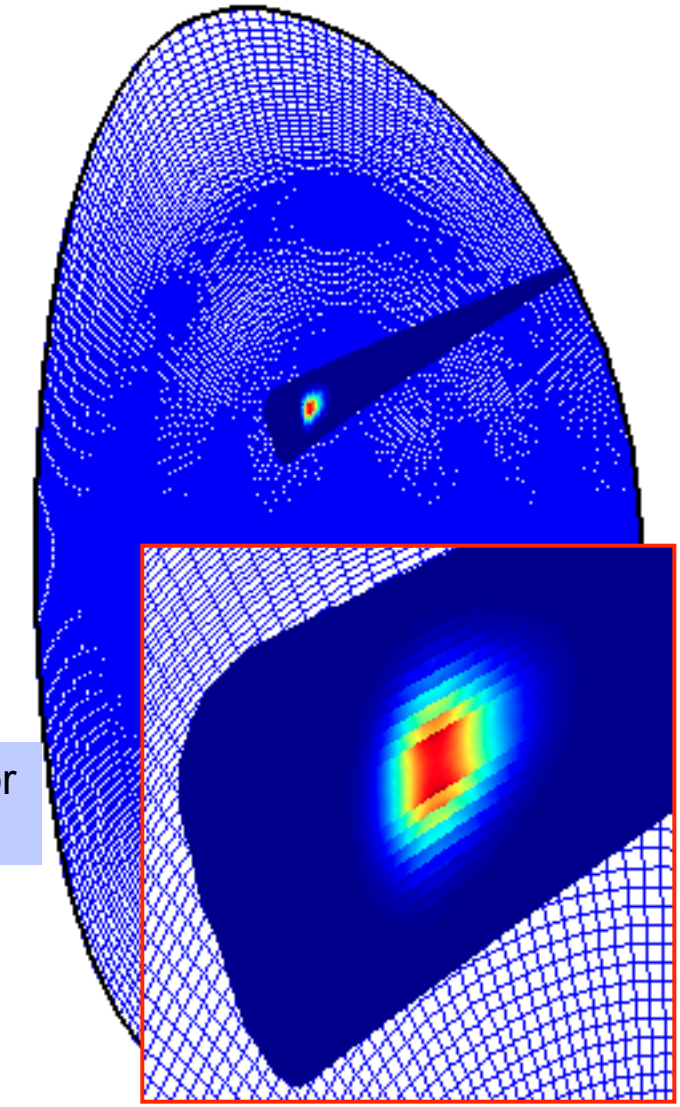


•Ray bundles are highly localized; quasilinear diffusion coefficients are only large near the electron cyclotron resonance, so we get even more localization…

$$\mathbf{F}^{\mathrm{rf}}_{\alpha 0} \sim \exp\left[-\left(\frac{\omega - n\Omega_e}{2k_{\|}v_{te}}\right)^2\right]$$

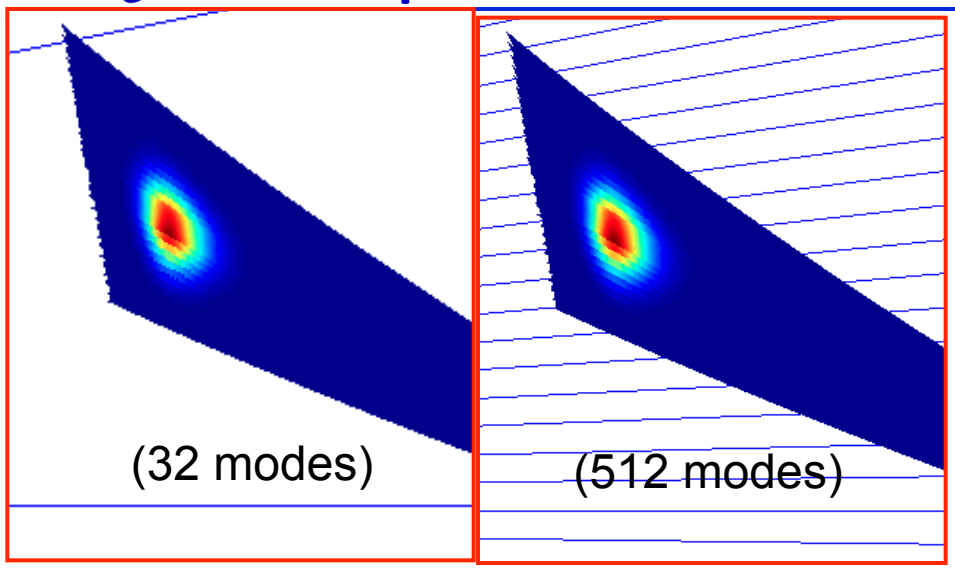
•Only a few points on the NIMROD R-Z grid are affected by the RF source.

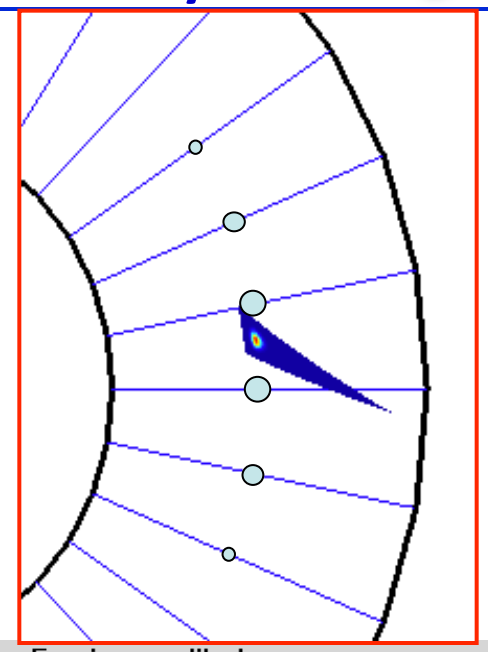
Shephard's algorithm used for Interpolation onto FE mesh



Quasilinear diffusion coefficient amplitude, R-Z projection

Toroidally resolving the RF deposition is prohibitive so source is extended toroidally



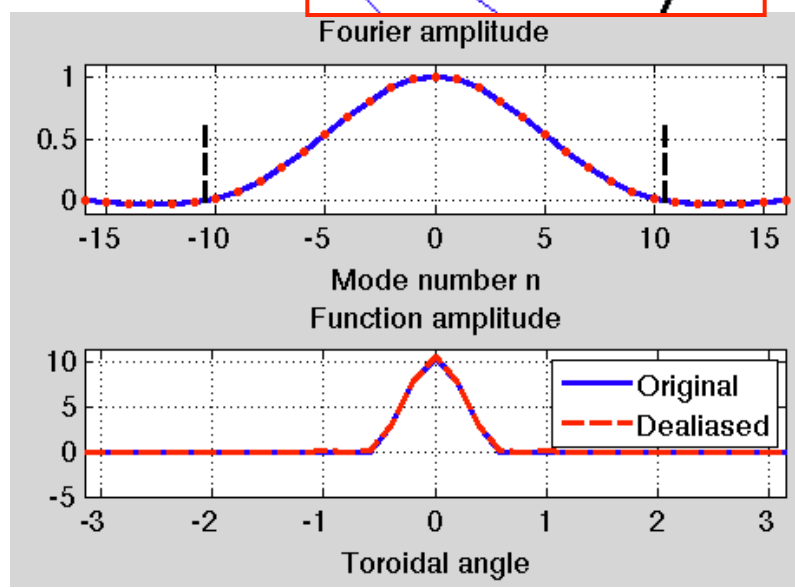


- Resolving source prohibitive

$$F^{RF}(R,Z,\phi) = \left[\frac{1}{2\pi}\oint F^{RF}(R,Z,\phi')d\phi'\right]g(\varphi)$$

for some normalized $g(\phi)$.

$$g(\phi) = \frac{2\pi}{\phi_c}\cos\left(\frac{\pi(\phi-\phi_0)}{\phi_c}\right)^2; \quad |\phi-\phi_0| \le \phi_c$$

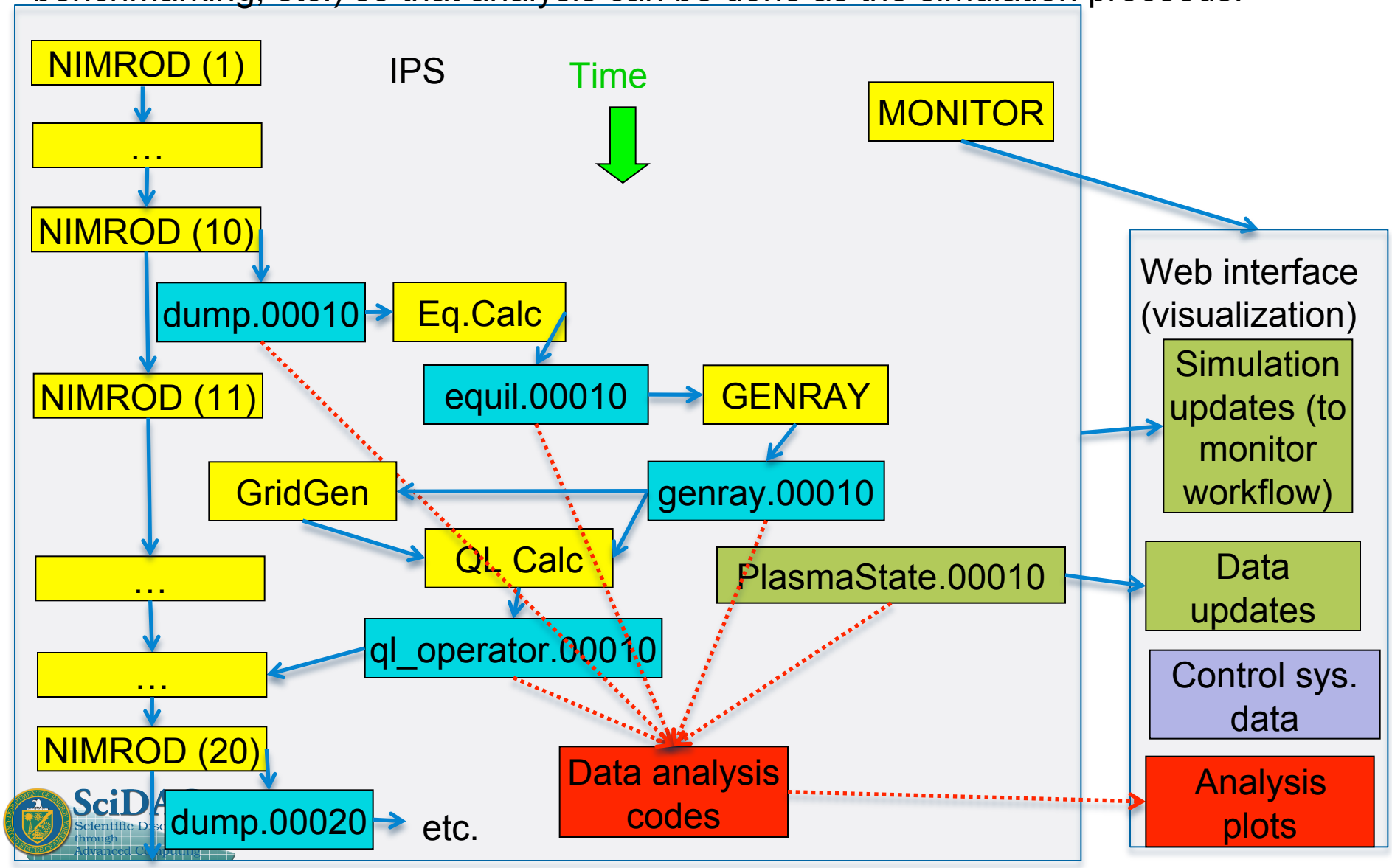


ϕ_c = 0.6; 32 modes

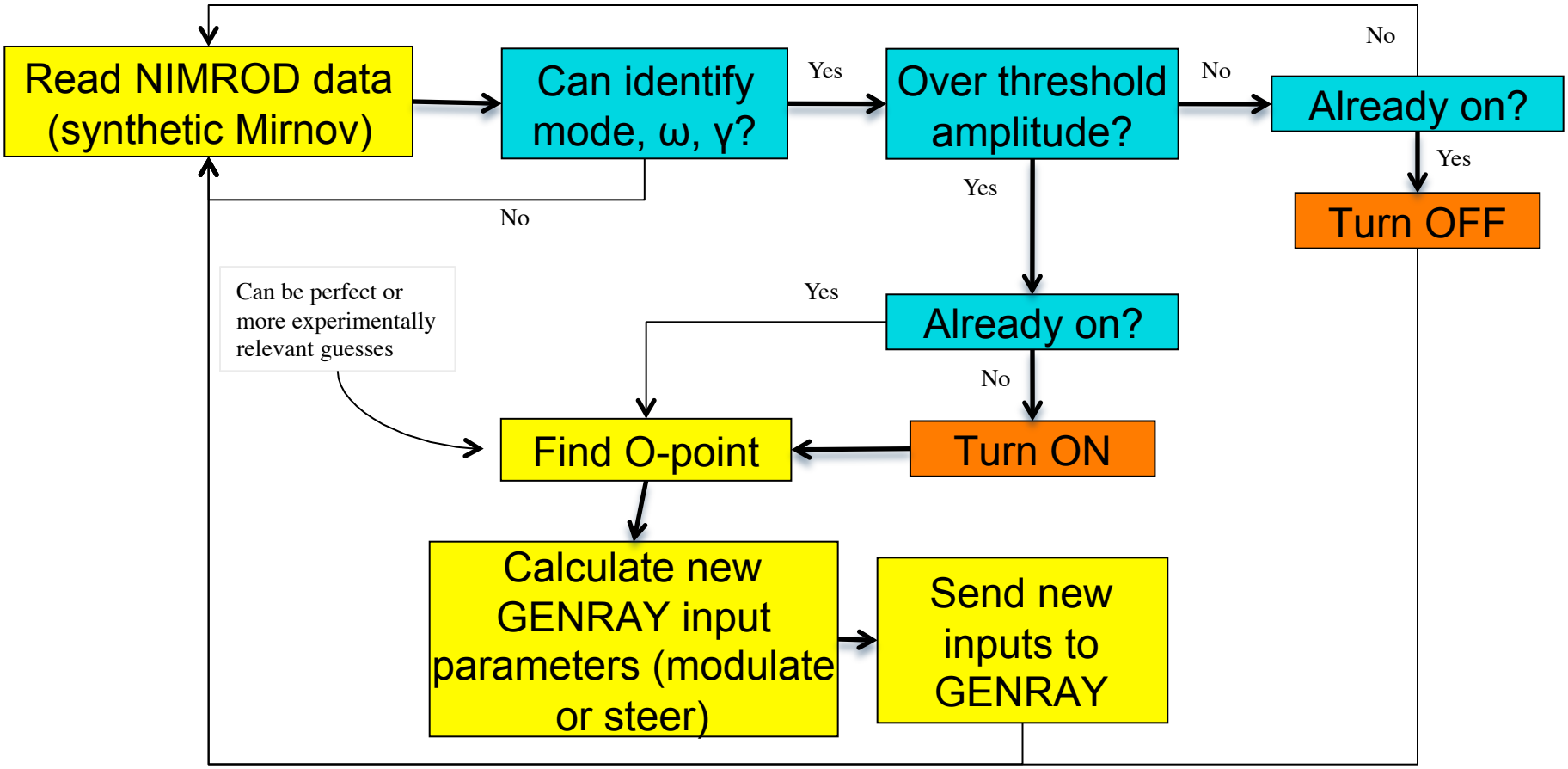
Argue , like Giruzzi, that spreading out represents effect of toroidal rotation

Time advance done with standalone executables

•Additionally, can add data analysis programs (Poincaré solver, plots, benchmarking, etc.) so that analysis can be done as the simulation proceeds.

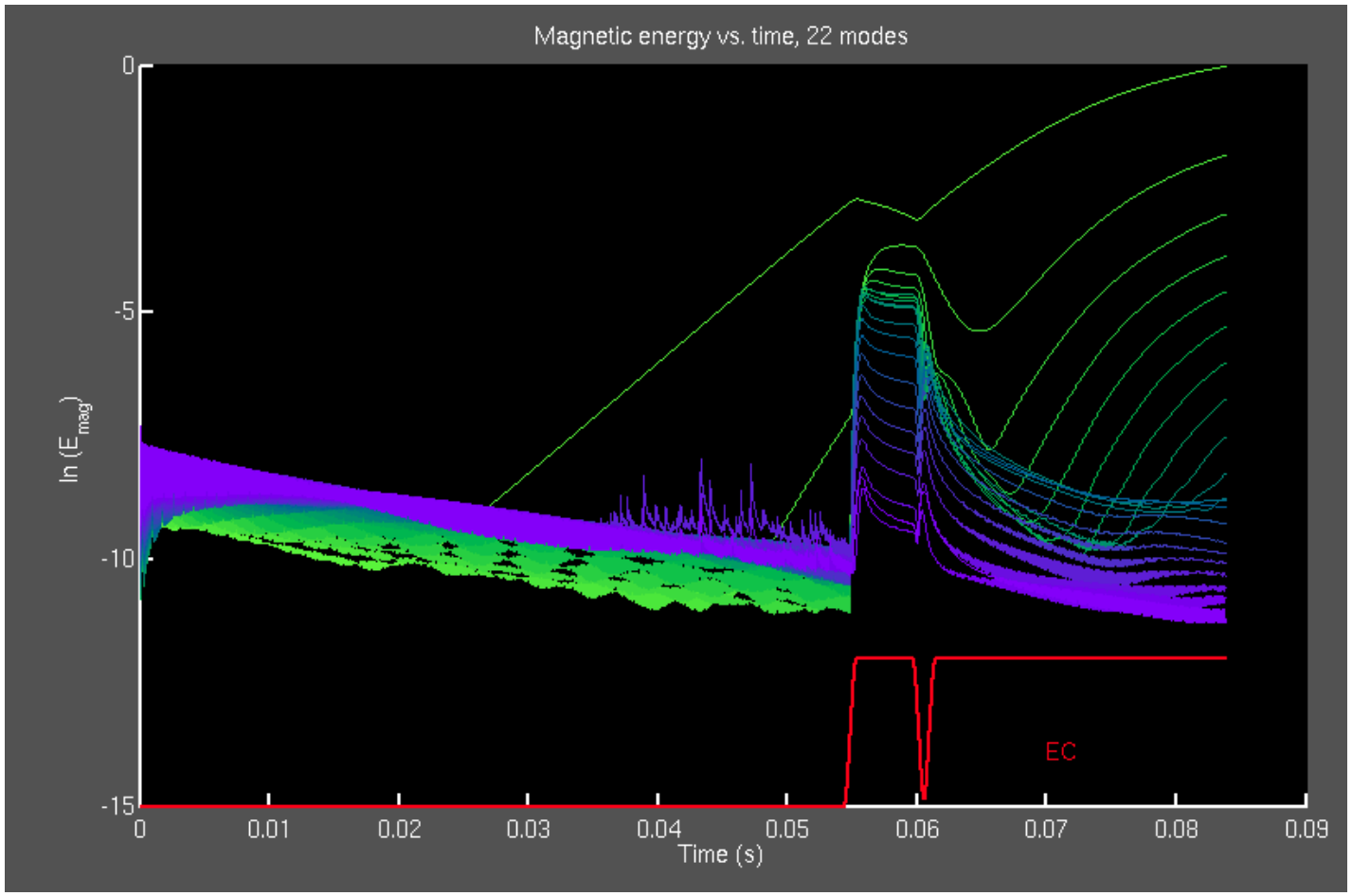


Synthetic control system relies on synthetic diagnostics to control RF sources



Can also add time modulation of RF source (for rotating plasmas). NIMROD reads ON or OFF status and ramps up RF power accordingly (next slide).

The numerical Plasma Control System can respond to changing conditions of the simulation



Modulation of the RF sources based on control system

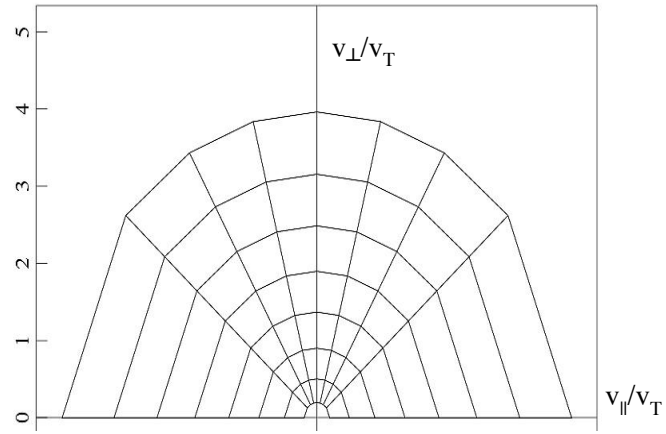
When mode amplitude exceeds threshold, PCS injects ECCD at island O-point, ➔island shrinks.

Here, toroidal rotation (period = 0.02 seconds) moves island X-point around to the injection location.

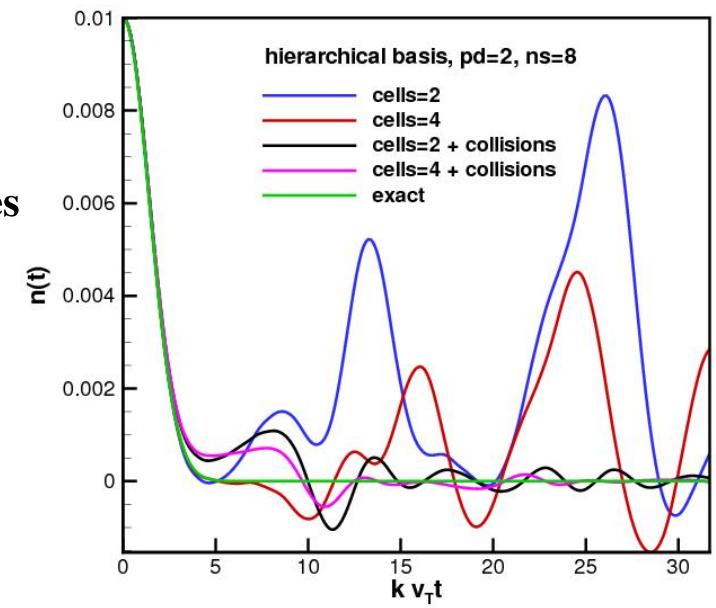
O-point location is not recalculated; mode is driven unstable by RF.

Quantitative results will rely on kinetic closure under active development

- **Continuum method for solving drift kinetic equations (DKEs) implemented in NIMROD.**
  - **Many similarities to NEO (Belli and Candy) and COGENT efforts (LLNL)**
  - **Also similar to continuum GK codes (GYRO, GS2, …)**
- **Solves for coefficients of 1D FE expansion in pitch angle, $v_{\|}/v$, on a grid in normalized speed, $s=v/v_T$:**
  $F = \quad {}_l F_l(x,s,t) \quad {}_l(v_{\|}/v)$.
- **Uses moment form (Ji, Held 2006) for full, linearized Coulomb collision operator valid for arbitrary mass and temperature ratios.**
- **-centered, implicit approach permits large time steps compared to electron bounce and transit times**
- **Currently benchmarking with the NEO code for axisymmetric calculations of the bootstrap current**



velocity grid with 4 pitch angle cells (2nd order polynomials) and 8 speed grid points, $s^2=(v_\perp/v_T)^2+(v_{\|}/v_T)^2$.



Test of phase mixing problem with and without collisions.

Summary and future work

- **Incorporation of ECCD into fluid equations is well understood**
  - **Quantitative statements require development of kinetic closures**
  - **Kinetic closures using a continuum approach to solving the CEL-form of the DKE is underway and undergoing benchmarking**
- **Computational techniques for self-consistently modeling ECCD from ray tracing is developed**
  - **Sophisticated techniques for developing code coupling methods have been used**
- **Current efforts are focused on developing synthetic control system to understand effects of alignment and control system**
  - **Two fluid terms will be added soon**

Extra slides

Experimental/Theory Comparisons Framed In Terms of Modified Rutherford Equations:

- **Rutherford equation with RF terms:**

Hegna et.al., *Phys. Plasmas* **4,** 2904 (1997)
Sauter et.al, *Phys. Plasmas* **4,** 1654 (1997)
LaHaye et.al., *Nucl. Fusion* **44,** 451 (2006)

Extended MHD Physics

$$\frac{\tau_R}{r}\frac{dW}{dt} = \Delta' r + \delta\Delta_{RF} r + a_2 \frac{J_{bs}}{J_{\|}}\frac{L_q}{w}\left(1 - \frac{W_{m\arg}^2}{3W^2} - K_1 \frac{J_{ECRF}^{hel}}{J_{bs}}\right)$$

RF Physics

Two physical effects of RF:

- Axisymmetric component of RF source (Δ_{RF})
- Helical contribution of RF source (J^{hel}_{ECRF})

Pletzer and Perkins,
Phys. Plasmas ***5,*** *(1998)*

Yu and Gunter parameters

- Tau_LR=0.001 T_A
- Delta phi ~ 1 radians which we can get with about 32 modes