Continuum Absorption at Frequency Tip

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IAEA Technical Meeting on Plasma Instability
Sep 5th, 2011
Austin, TX

Continuum Absorption

- Occurs in *inhomogeneous* plasma where Alfven velocity is a function of position **(x)**.
- When external driving frequency matches with local shear Alfven frequency, phase mixing occurs around the resonance and wave energy is dissipated to the plasma.
- The energy absorption rate is calculated by introducing an artificial growth rate to mimic the effects of viscous dissipation. (e.g., Chen and Hasegawa 1974)

Estimation of Absorption Rate

$$\frac{d}{dx}\left(\rho[\tilde{\omega}^2-\omega_A^2(x)]\frac{d\xi_x}{dx}\right)-k_\perp^2\rho[\tilde{\omega}^2-\omega_A^2(x)]\xi_x=0$$

$$\tilde{\omega}=\omega+i\gamma$$

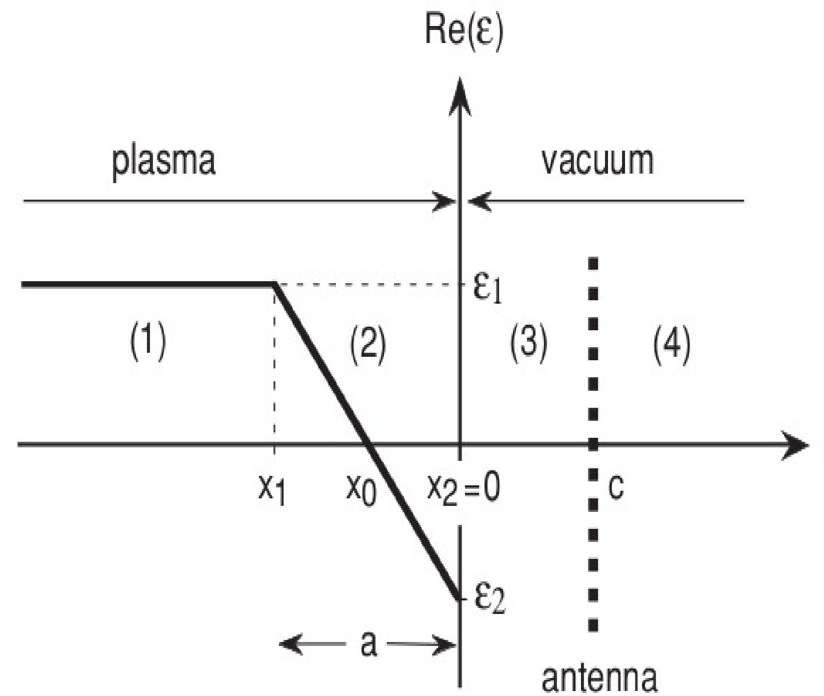
$$\frac{d}{dx}\left(\rho[\tilde{\omega}^2-\omega_A^2(x)]\frac{d\xi_x}{dx}\right)=0$$

$$\rho[\tilde{\omega}^2-\omega_A^2(x)]\frac{d\xi_x}{dx}=C$$

$$\frac{d\xi_x}{dx}=\frac{C}{\tilde{\omega}^2-\omega_A^2(x)}=\frac{C}{\omega^2-\omega_A^2(x)+2i\omega\gamma}=\frac{C}{\frac{d\omega_A^2(x)}{dx}(x-x_0)+2i\omega\gamma}$$

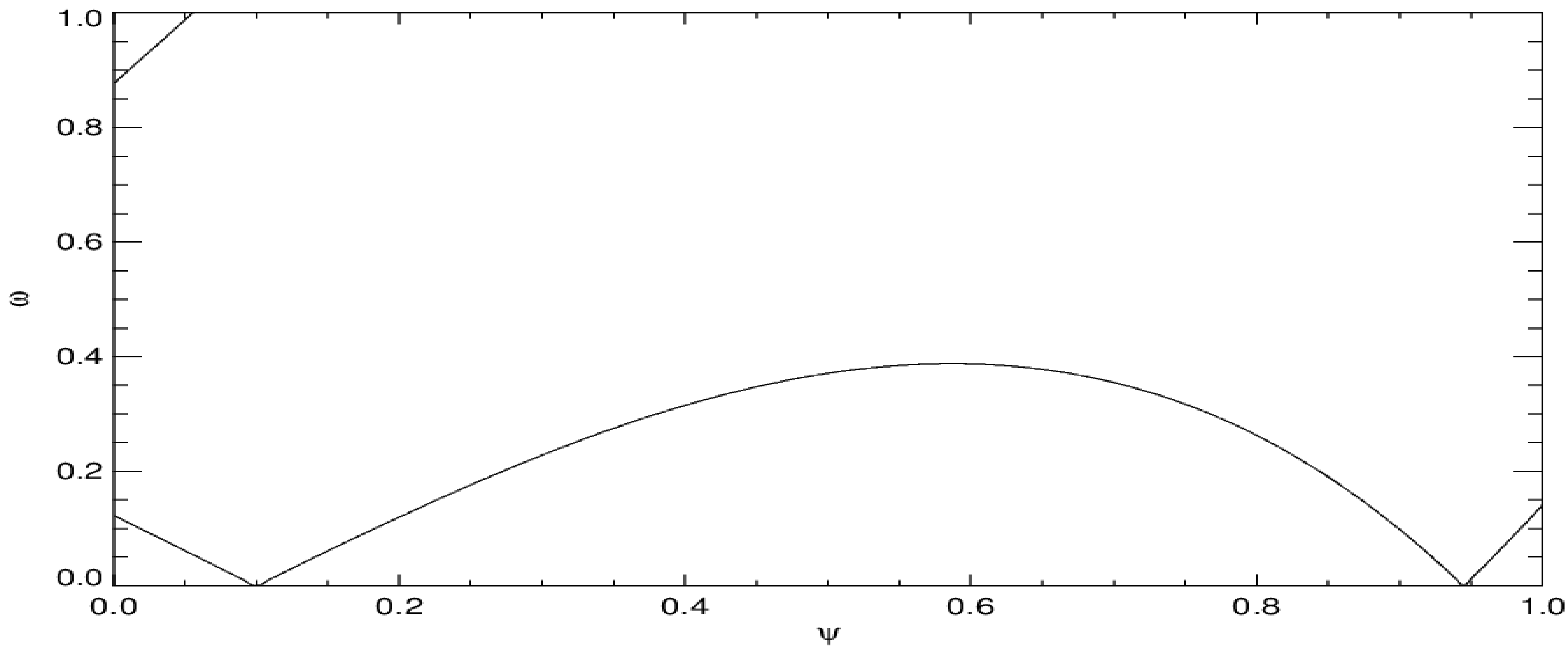
$$Q=-\int\rho\gamma v^2 dV\approx-\int\rho\gamma v_s^2 dV\approx-\int\rho\gamma\left(\frac{d\dot{\xi}_x}{dx}\right)^2 dV\approx\int\rho\gamma\omega^2\left(\frac{d\xi_x}{dx}\right)^2 dV$$

$$=\int\frac{\rho\gamma\omega^2|C|^2}{\left|\frac{d\omega_A^2(x)}{dx}(x-x_0)+2i\omega\gamma\right|^2}dV=\int\frac{\rho\gamma\omega^2|C|^2}{\left(\frac{d\omega_A^2(x)}{dx}\right)^2(x-x_0)^2+4\omega^2\gamma^2}dV\propto\left|\frac{d\,\omega_A^2(x)}{dx}\right|^{-1}$$



Q is independent of γ, proportional to $\left|\frac{d\,\omega_A^2(x)}{dx}\right|^{-1}$

Slope of the Continuum is usually not a constant...

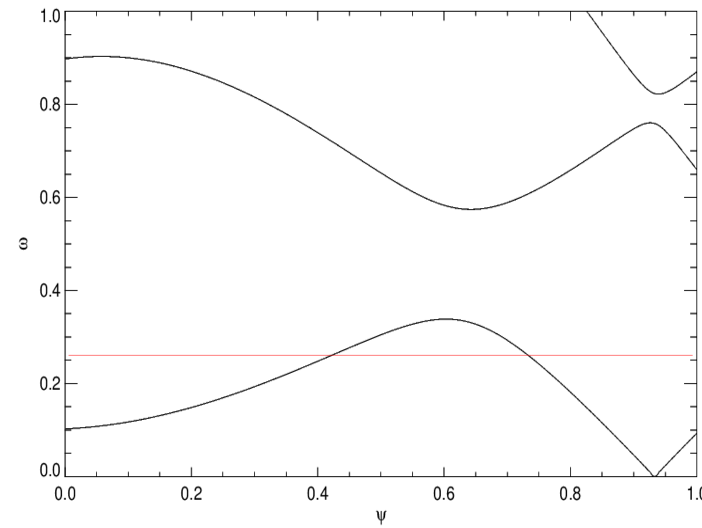


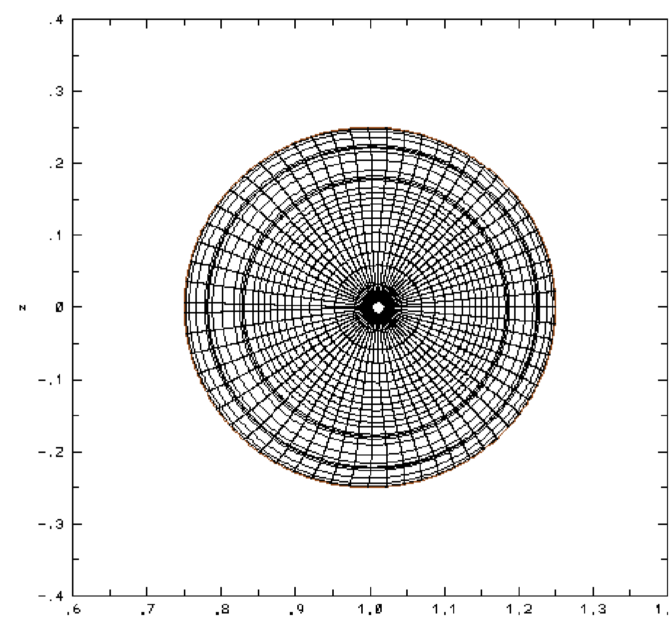
$$\left|\frac{d\,\omega_A^2(x)}{dx}\right|^{-1} \to \infty$$

What is the absorption rate as the slope approaches zero??

Numerical Investigation (I)

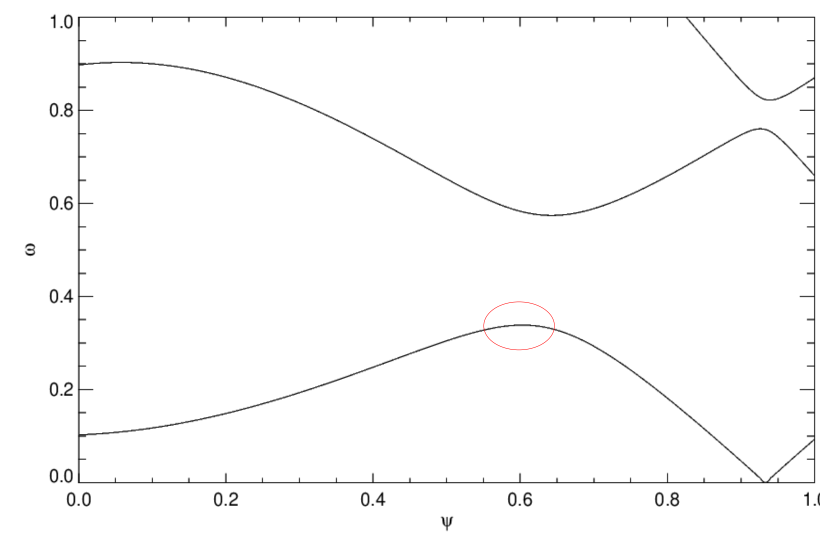
- We use a Grad-Shafranov Solver TOQ to generate the numerical equilibra. The input parameters are poloidal current flux and pressure profile.
- Our ideal MHD code AEGIS is adaptive, thus, high density of grids are placed in the vicinity of continuum.

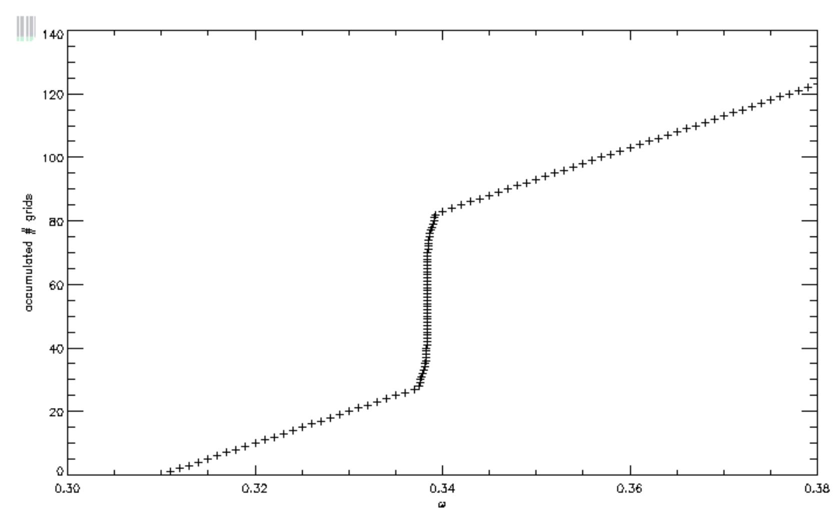




Numerical Investgiation (II)

- A poloidal current is driven at x=0.2 with a tunable frequency.
- We investigate the absorption as a function of frequency near to the frequency tip.



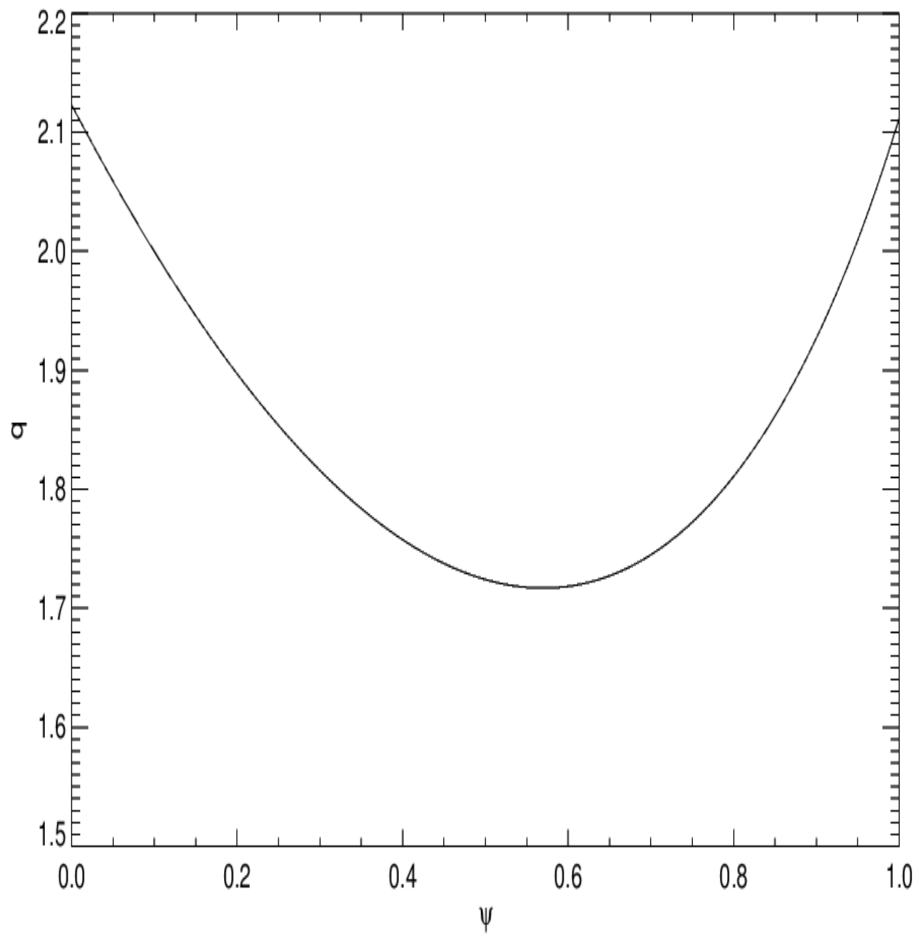


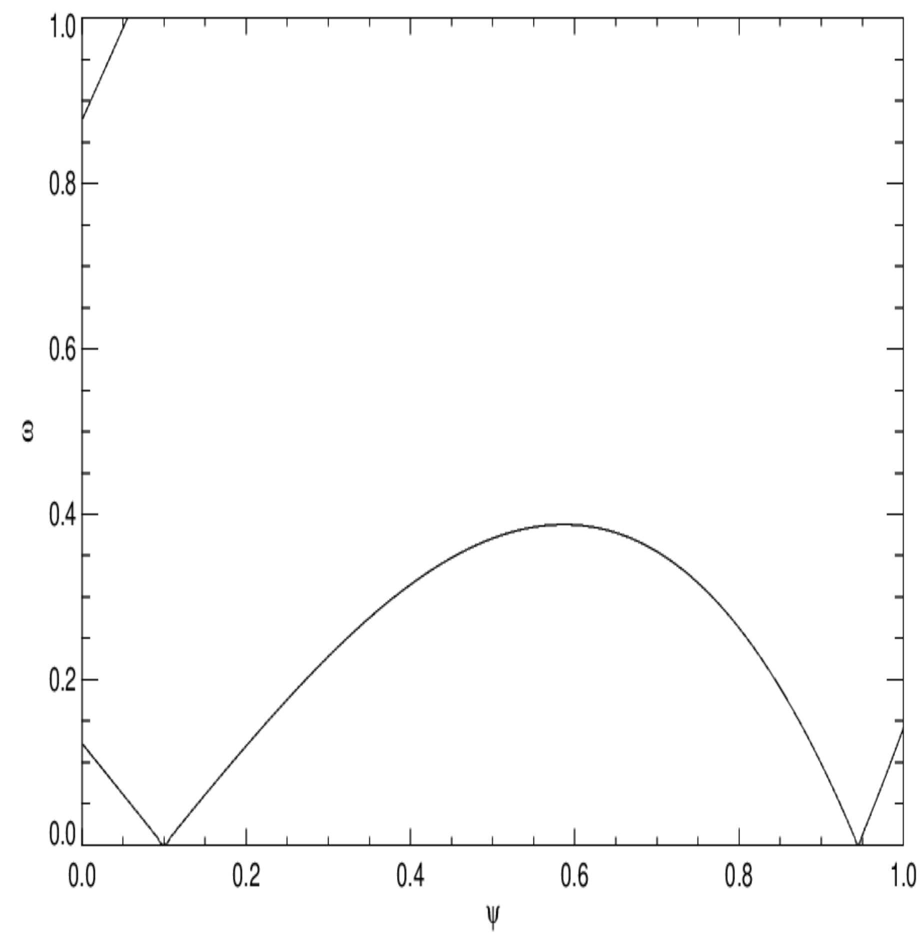
Estimation of Absorption Rate at Continuum Tip

$$Q = \int \frac{\rho\gamma\omega^2|C|^2}{\left|\frac{d\omega_A^2(x)}{dx}(x-x_0)+2i\omega\gamma\right|^2}dV = \int \frac{\rho\gamma\omega^2|C|^2}{\left(\frac{d\omega_A^2(x)}{dx}\right)^2(x-x_0)^2+4\omega^2\gamma^2}dV \propto \left|\frac{d\,\omega_A^2(x)}{dx}\right|^{-1}$$

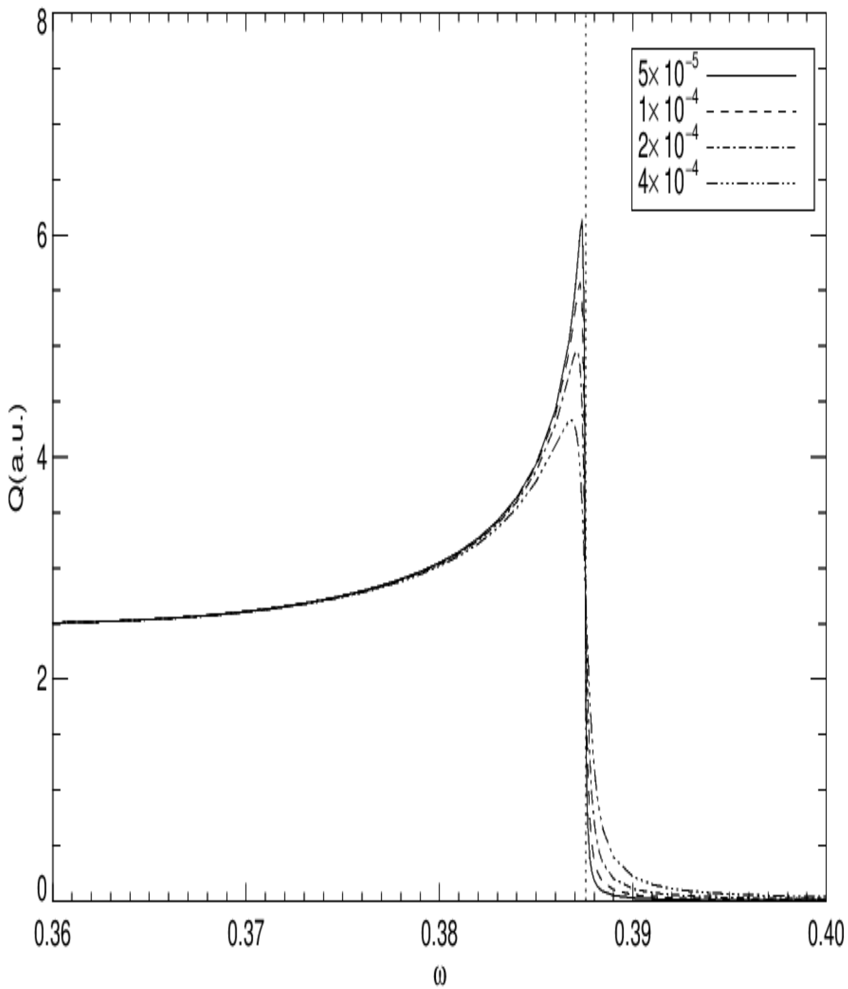
$$Q = \int \frac{\rho\gamma\omega^2|C|^2}{\left|\frac{1}{2}\frac{d^2\omega_A^2(x)}{dx^2}(x-x_0)^2+2i\omega\gamma\right|^2}dV = \int \frac{\rho\gamma\omega^2|C|^2}{\frac{1}{4}\left(\frac{d^2\omega_A^2(x)}{dx^2}\right)^2(x-x_0)^4+4\omega^2\gamma^2}dV \propto \left|\frac{d^2\,\omega_A^2(x)}{dx^2}\right|^{\frac{-1}{2}}\gamma^{\frac{-1}{2}}$$

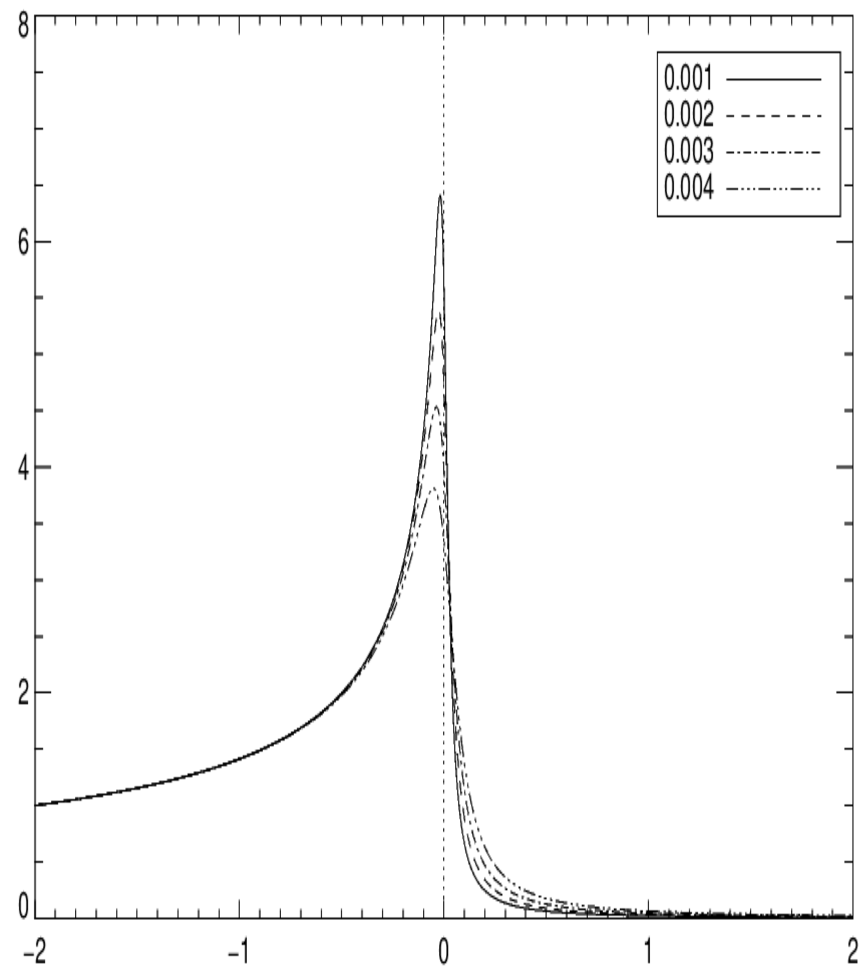
Reverse Shear Tip (I)



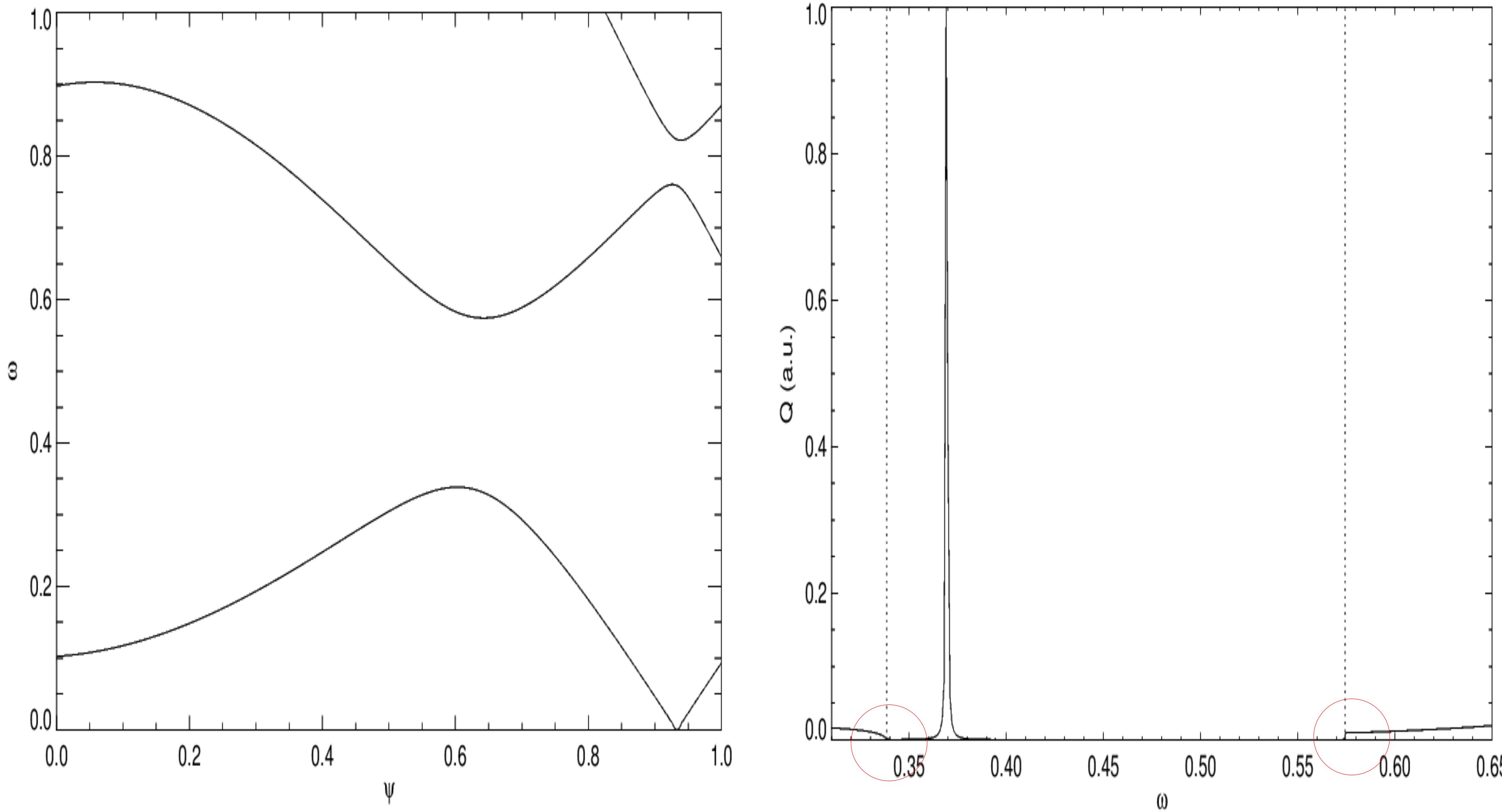


Reverse Shear Tip (II)





TAE Frequency Gap



Estimation of Continuum Absorption at TAE Tip

- The tip is formed due to the toroidal coupling of two adjacent poloidal harmonics. Both harmonics need to be taken into account.
- We applied the TAE tip model (Rosenbluth, Berk et at.; Breizman and Sharapov) to the problem.
- It can be shown that as $|g| \to 1$

$$Q \sim \sqrt{(1-g^2)}$$ where

$$g = \frac{1}{\varepsilon}\left(\frac{4q^2R^2\tilde{\omega}^2}{v_A^2} - 1\right)$$

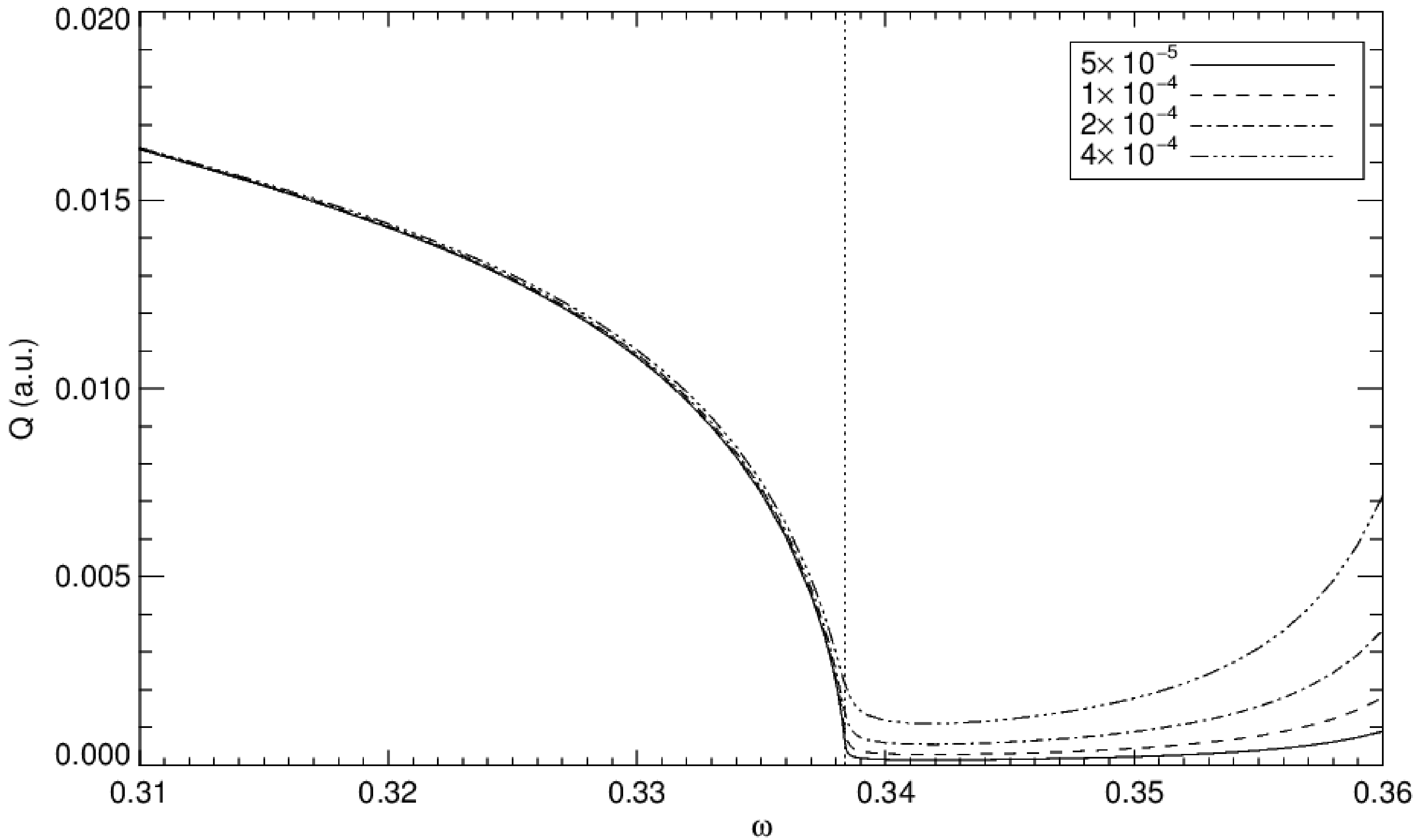
- Absorption shall vanish at the tip!

$$v_s \approx i\omega \frac{\begin{bmatrix} z-g & -1 \\ 1 & z+g \end{bmatrix}\begin{bmatrix} C_m \\ C_{m-1} \end{bmatrix}}{1-g^2+z^2}$$

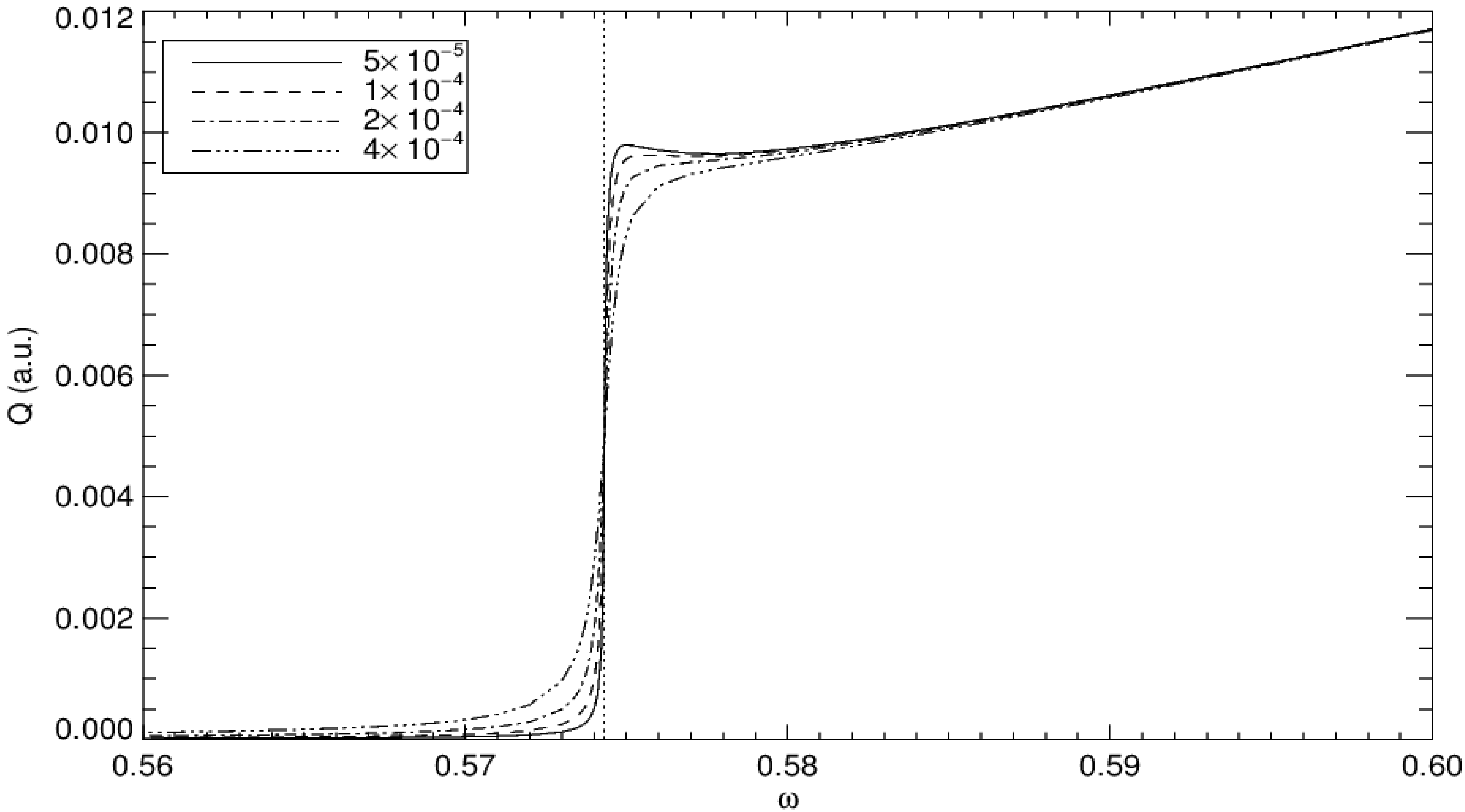
$$\begin{bmatrix} C_m \\ C_{m-1} \end{bmatrix} \approx \frac{\pi}{\sqrt{(1-g^2)}\pi^2 + \frac{g\pi^3 S}{2}}\begin{bmatrix} g\tilde{J}_m + \tilde{J}_{m-1} \\ -\tilde{J}_m - g\tilde{J}_{m-1} \end{bmatrix}$$

$$Q \sim \int \gamma\omega^2 \frac{[(1-g^2)\tilde{J}_m]^2 + [(1-g^2)\tilde{J}_{m-1}]^2}{[\sqrt{(1-g^2)}(\pi - \frac{\pi^3 S^2}{16} + \frac{g\pi^2 S}{2})](1-g^2+z^2)^2}$$

Lower TAE Tip



Upper TAE Tip

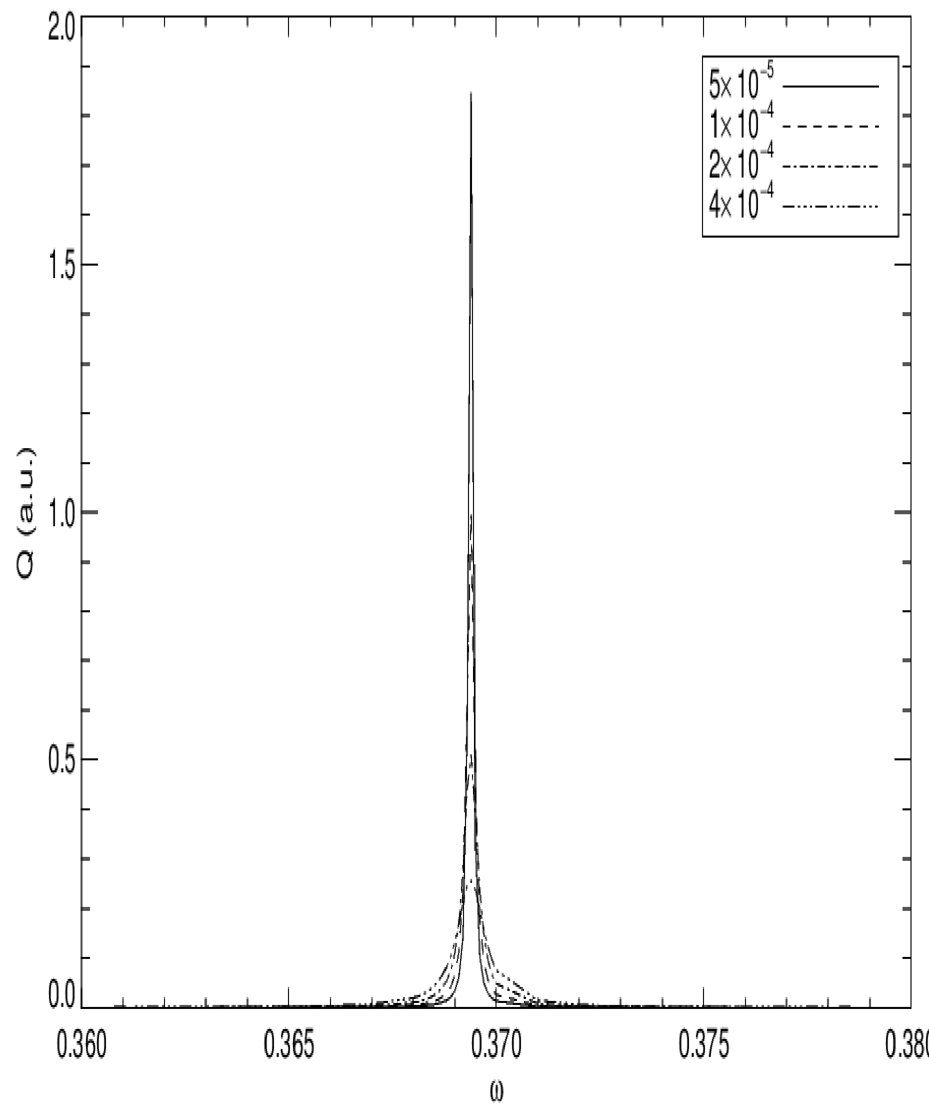


TAE Frequency

- For eigenmodes, $v \propto \dfrac{1}{\gamma}$, independent of position.

$$Q \sim \int \gamma v^2 dV \sim \gamma^{-1}$$

Numerical results show good agreement.



Summary and Conclusions

- In the case of reverse shear tip and lower TAE tip, analytic results and numerical results are in adequate agreement with each other.
- The response of upper TAE tip cannot be obtained in the TAE tip model of Rosenbluth and Berk et al. / Breizman and Sharapov.
- The numerical results around TAE frequency are in agreement with analytic model.
- We are modifying the tip model to see why the discontinuous absorption at upper TAE tip is achieved.