Reversed Shear Alfenic Eigenmodes in Internal Transport Barriers

Deng Zhou

Institute of Plasma Physics, CAS, China

5th IAEA technical meeting on “Theory of Plasma Instabilities”,

Austin USA, Sept. 5-7, 2011

Outline

- Introduction
- Local equilibrium theory
- The Reversed Shear Alfenic Eigenmodes in ITB
- Conclusion

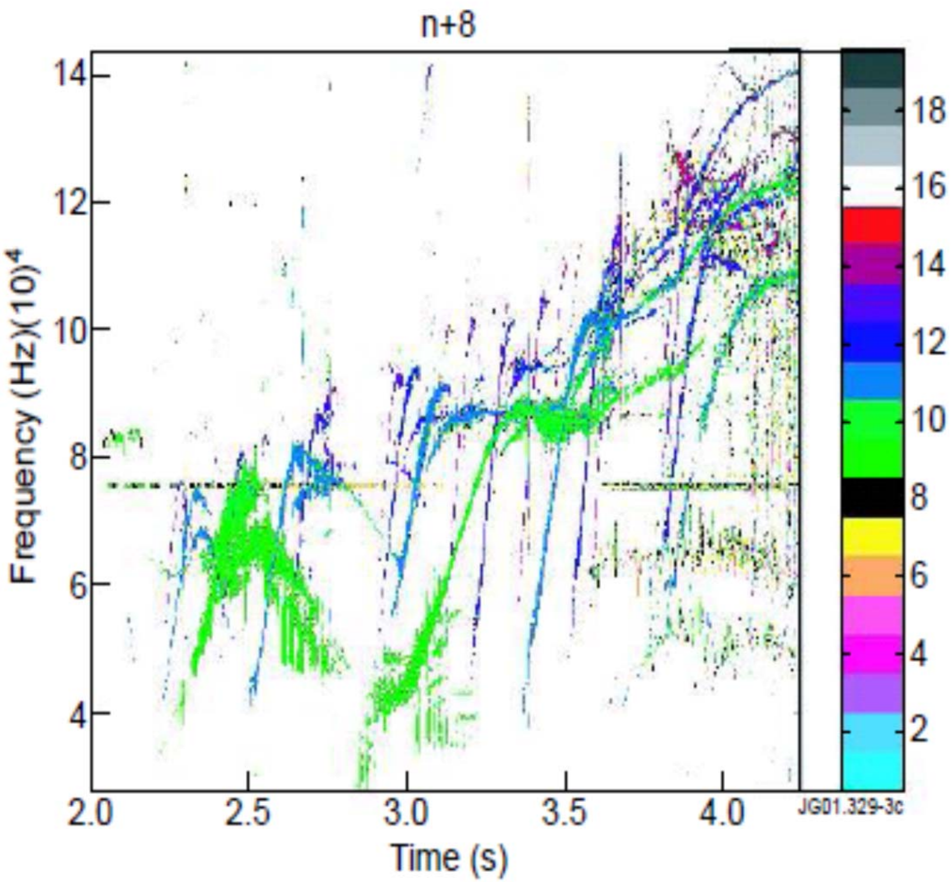
INTRODUCTION

Introduction

- Reversed shear Alfenic Eigenmode (RSAEs) or Alfenic Cascades (Acs) appear around the flux surface where the magnetic shear is 0.

(Kimura98, Sharapov02, Snipes05,…..)

- The frequencies have an upward sweeping as the q_{min} value decreases in time.



Berk, *et al.*, PRL01

Introduction(2)

- The first theoretical explanation of ACs was given by Berk *et al*.(PRL 01).
- They are sheared Alfvenic eigenmodes localized at the q_{min} surface, usually destabilized by in fast ions
- The toroidal coupling effect (Breizman03) and the finite pressure gradient effect (Fu06; Breizman05; Gorelenkov06)) were taken into account.

Introduction(3)

• Usually, experiments find that an internal transport barrier (ITB) may appear at the $q' = 0$ surface when the safety factor is close to a low order rational number (Joffrin NF03; Razumova PPCF06).

• Previous theoretical works did not consider the effect of such a large pressure gradient; How to consider the ACs in this situation is an open question.

• The normal large aspect ratio shifted circular equilbrium model, i. e., *s*-*α*model, was adopted in previous theory, the mode is only applicable for low beta circular plasma, not suitable for ITB region

Introduction(4)

- For the analysis of ACs in ITB, the key work is to derive an equilibrium suitable for ITB region
- It is difficult to give a global analytical equilibrium model with ITB
- Measurements show that the ACs are localized at the region of q_{min}, so the local equilibrium solution might be appropriate

Local equilirium solution at ITB

- The local equilibrium theory was first developed by Mercier et al. (1974),

--- The local equilibrium is completely determined by a referenced flux surface and the poloidal magnetic field on this surface.

- For analytical application, we have to apply some constraints on the poloidal magnetic field and the referenced flux, but keep the essential feature sought.

- The referenced surface at $q' = 0$ is circular

$$R = R_0 + \rho_0 \cos(\theta)$$

$$Z = \rho_0 \sin(\theta)$$

- The coordinate surfaces are

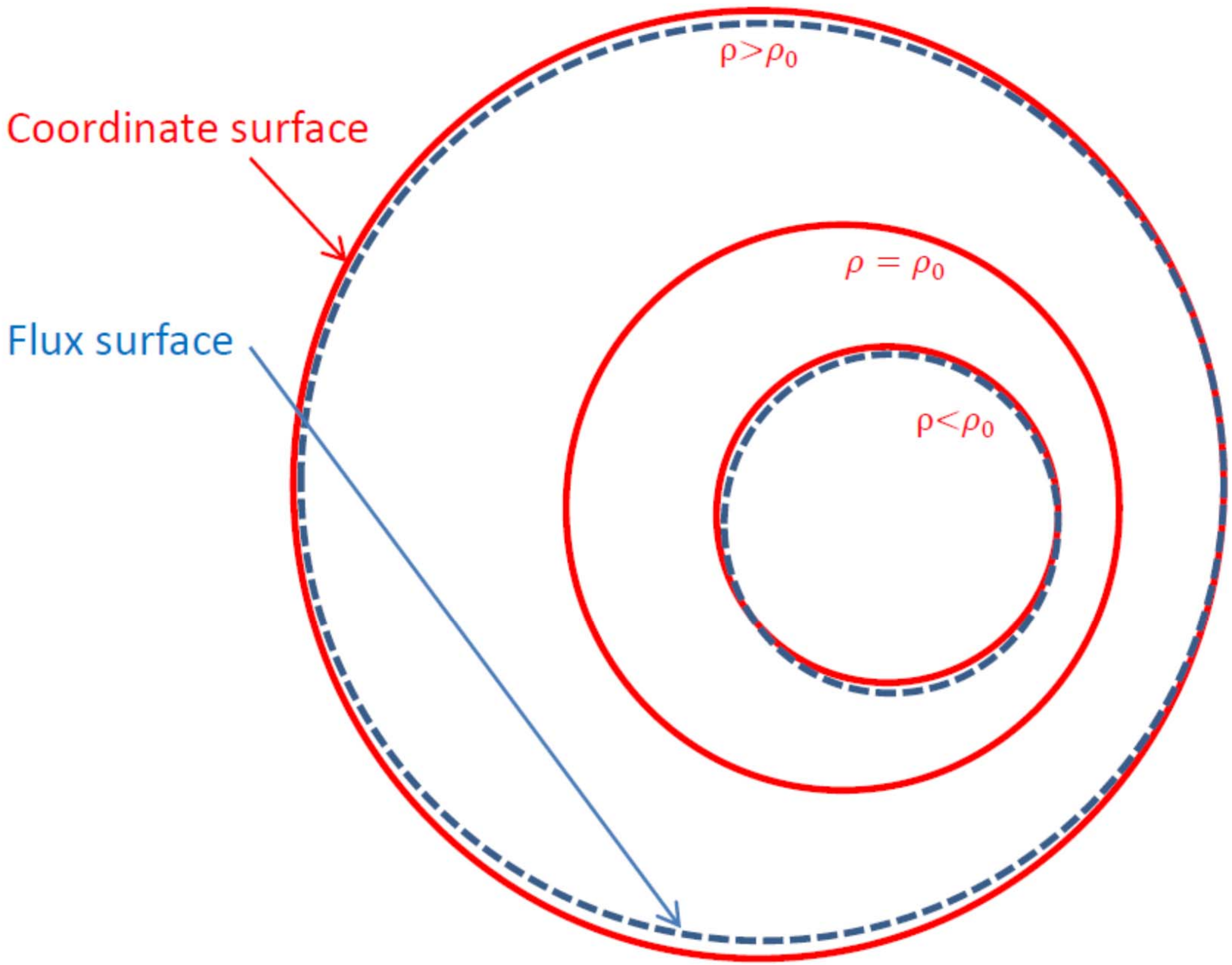
$$R = R_0 - \Delta(\rho) + \rho\cos(\theta)$$

$$Z = \rho\sin(\theta)$$

$$\Delta(\rho) = \Delta'(\rho - \rho_0)$$

$\Delta' < 1$ is a constant to be determined

- (ρ, θ, φ) forms a curvilinear coordinate system


Coordinate surface
Flux surface
ρ>ρ0
ρ = ρ0
ρ<ρ0

The metric tensors

$$g^{\rho\rho} = \frac{1}{(1-\Delta'\cos\theta)^2} \qquad g^{\rho\theta} = \frac{-\Delta'\sin\theta}{\rho(1-\Delta'\cos\theta)^2}$$

$$g^{\theta\theta} = \frac{1-2\Delta'\cos\theta+\Delta'^2}{\rho^2(1-\Delta'\cos\theta)^2} \qquad g^{\varphi\varphi} = \frac{1}{R^2}$$

The Jacobi

$$J = (\nabla\rho\times\nabla\theta\cdot\nabla\varphi)^{-1} = \rho R(1-\Delta'\cos\theta)$$

Some constraints

1. Large aspect ratio

$$\varepsilon = \rho_0 / R_0 << 1$$

2. Normal low Beta

$$\beta \sim \varepsilon^2$$

3. Higher pressure gradient

$$\alpha = q^2 R_0 (d\beta / dr) \sim 1$$

4. Given q and α, at the referenced surface

$$q' = 0$$

Expanding the flux

$$\psi = \psi_0 + \psi_1 x + \psi_2(\theta) x^2 + \cdots$$

$$x = \rho - \rho_0$$

Iteratively , we derive

$$\rho = \rho_0 + r - \frac{\psi_2}{\psi_1} r^2 + 2\left(\frac{\psi_2}{\psi_1}\right)^2 r^3 + \cdots$$

$$r = \frac{\psi - \psi_0}{\psi_1}$$

We have to solve for the following

ψ_i , i = 1, 2.... f' Δ'

From the definition of *q*

$$q(\psi_0) = \frac{1}{2\pi}\oint \frac{\mathbf{B}\cdot\nabla\varphi}{\mathbf{B}\cdot\nabla\theta} d\theta = \frac{1}{2\pi}\oint \frac{\rho_0(1-\Delta'\cos\theta) f}{R\psi_1} d\theta$$

$$\frac{\psi_1}{f} = \frac{1}{2\pi q(\psi_0)}\oint \frac{\rho_0(1-\Delta'\cos\theta)}{R} d\theta \cong \frac{\varepsilon}{q}\left[1 + \frac{1}{2}\varepsilon(\varepsilon + \Delta')\right]$$

$$\mathbf{B} = \nabla\varphi \times \nabla\psi + f\nabla\varphi$$

Substitute the flux expansion into Grad-Shafrarov equ. to derive all other $\psi_i(\theta)$

$$R^2\nabla\cdot\left(\frac{\nabla\psi}{R^2}\right) = -\mu_0 R^2 p'(\psi) - ff'(\psi)$$

Taylor expanding G-S at $\rho = \rho_0$, from the $O(r^0)$ terms, yields

$$\psi_2(\theta) = -\frac{1-\Delta'\cos\theta}{2}\left[1-\frac{\rho_0}{R}\cos\theta+\frac{\Delta'^2\sin^2\theta}{(1-\Delta'\cos\theta)^2}+(1-\Delta'\cos\theta)(\mu_0 R^2 p'+ff')\right]$$

From the $O(r^i)$ term, yields

$$\psi_{i+2}(\theta) = F(\psi_i, \psi_{i+1})$$

Two constraints

(1) $$\frac{dq}{d\psi}\bigg|_{\psi=\psi_0} = 0$$

(2) $$\psi_2(0) = \psi_2(\pi)$$

From constraint (1) $\frac{dq}{d\psi}|_{\psi=\psi_0} = 0$

$$B\left(2+\Delta'^2\right)+q\left(1+\frac{3}{2}\Delta'^2\right)\left(\mu_0 R^2 p' + ff'\right)-\frac{3}{2}q\Delta'\varepsilon\left(1+\frac{1}{4}\Delta'^2\right)\left(\mu_0 R^2 p' - ff'\right)=0$$

$$ff' \sim \ -\mu_0 R^2 p' \sim \frac{B}{2q\varepsilon}\alpha \qquad \alpha = q^2 R_0 \frac{d\beta}{dr} \sim 1$$

$$ff' \cong -\mu_0 R^2 p' - \frac{2\left(2+\Delta'^2\right)}{q\left(2+3\Delta'^2\right)}B$$

From constraint (2) $\psi_2(0)=\psi_2(\pi)$

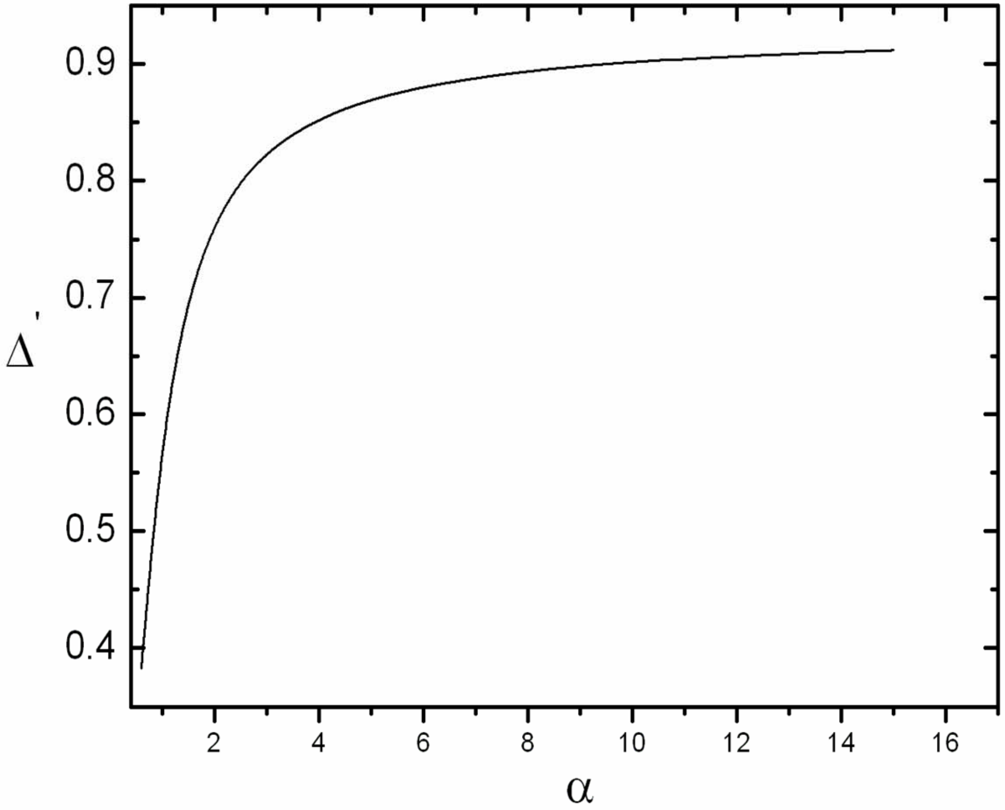
$$\varepsilon\left(\frac{1}{\Delta'}+\Delta'\right)\mu_0 R^2 p' - \left(\mu_0 R^2 p' + ff'\right) - \frac{1}{2}\left(1+\frac{\varepsilon}{\Delta'}\right)\frac{B}{q} = 0$$

Use

$$ff' \cong -\mu_0 R^2 p' - \frac{2\left(2+\Delta'^2\right)}{q\left(2+3\Delta'^2\right)}B$$

Obtain

$$\frac{3}{4}\Delta'^4 - \frac{1}{2\alpha}\Delta'^3 + \frac{1}{2}\Delta'^2 + \frac{3}{2\alpha}\Delta' - 1 = 0$$



Δ' is of order $O(\varepsilon^0)$, $\Delta' \sim 0.932$ when $\alpha \to \infty$

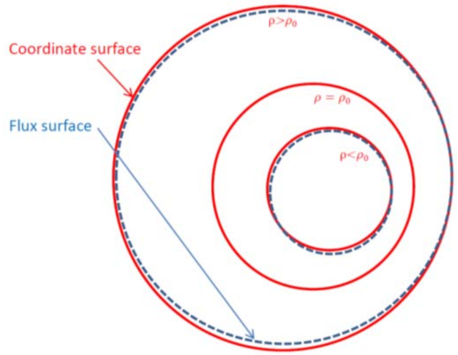
Some discussions

1. The equilibrium at ITB is solved, the only assumption is that the flux surfaces are close to circles.

2. Δ' is not the derivation of Shafranov shift, but since it holds that

$$\psi_2(0) = \psi_2(\pi)$$

$$\rho = \rho_0 + r - \frac{\psi_2}{\psi_1} r^2 + 2\left(\frac{\psi_2}{\psi_1}\right)^2 r^3 + \cdots$$



can be treated as the derivation of Shafranov shift of flux surfaces.

Reversed Shear Alfenic Eigenmodes in ITB

- The local equilibrium solution is derived at ITB, in principle, we can analyze the RSAEs using this equilibrium.

- The simplest approach is to treat the shifted circles with the given Δ' as flux surfaces, then we have constant Δ' , and

$\Delta'' = 0$ then we can straightforwardly take the result from previous works, like Fu *et al.*, POP06.

- The standard form of the eigenvalue equation governing RSAEs is (Berk01, Fu06, Breizman03, 05, ….)

$$\frac{d}{dx}\left(S + x^2\right)\frac{d}{dx}\Phi_m + \left(Q - S - x^2\right)\Phi_m = 0$$

$$S = \frac{mq_0{}^2}{(-k_{m0})\rho_0{}^2 q_0{}''}\left(\overline{\varpi}^2 - k_{m0}{}^2\right)$$

$$k_{m0} = n - m/q_0$$

$$\overline{\varpi} = \omega/(V_A/R_0)$$

$$x = \frac{mr}{\rho_0}$$

- The existing condition for RSAEs is

$$S > 0 \quad \text{and} \quad Q > 1/4$$

- The general form of Q in shifted circle configuration is

$$Q = Q_h + Q_{tor} + Q_{pres}$$

$$Q_{tor} = \frac{2mq_0^{\ 2}(-k_{m0})}{\rho_0^{\ 2} q_0''} \frac{\varepsilon(\varepsilon + 2\Delta'\varepsilon)}{1 - 4k_{m0}^{\ 2} q_0^{\ 2}}$$

$$Q_{pres} = \frac{mq_0^{\ 2}}{(-k_{m0})\rho_0^{\ 2} q_0''} \left[\frac{4\Delta'\overline{\varpi}^2\alpha - \alpha^2 / 2q_0^{\ 2}}{1 - 4k_{m0}^{\ 2} q_0^{\ 2}} + \frac{1}{q_0^{\ 2}} \overline{\kappa}_r^{\ 2}\alpha \right]$$

- In the solved equilibrium configuration, the total contribution to Q from toroidal and the pressure effect is

$$Q = \frac{m q_0^2}{(-k_{m0})\rho_0^2 q_0''}\tilde{Q}$$

$$\tilde{Q} = \left[\frac{4\Delta'\overline{\omega}^2\alpha - \alpha^2/2q_0^2 + 2k_{m0}^2\varepsilon(\varepsilon + 2\Delta')}{1 - 4k_{m0}^2 q_0^2} + \frac{1}{q_0^2}\overline{\kappa}_r^2 R_0\alpha\right]$$

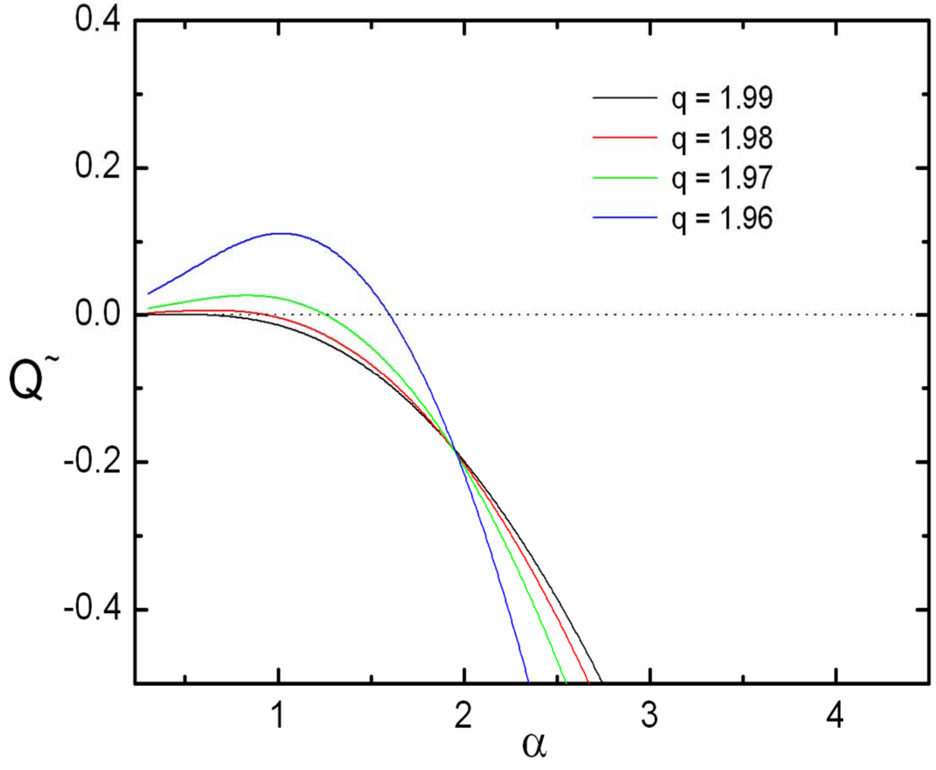
$$\overline{\kappa}_r = \frac{\Delta'}{2R_0} + \frac{\varepsilon}{q_0^2 R_0}\left[\frac{H}{2} + \frac{\varepsilon(1-L)}{\Delta'}\right]$$

$$H = \frac{1}{2\pi}\oint\frac{\cos\theta}{(1-\Delta'\cos\theta)^2}d\theta$$

$$L = \frac{1}{2\pi}\oint\frac{1}{1-\Delta'\cos\theta}d\theta$$

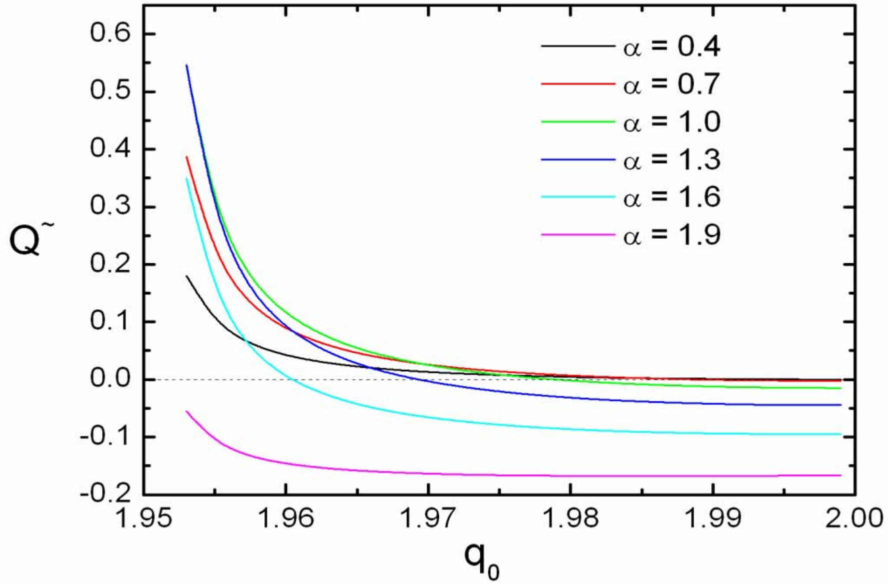
- Easy to have a numerical estimate of Q~
- The q_{min} value drops from above 2.0 to 1.95.
- Mode number *m/n* = 20/10, ε= 0.125

$$\overline{\omega}^2 = \varepsilon^2 + k_{m0}{}^2$$



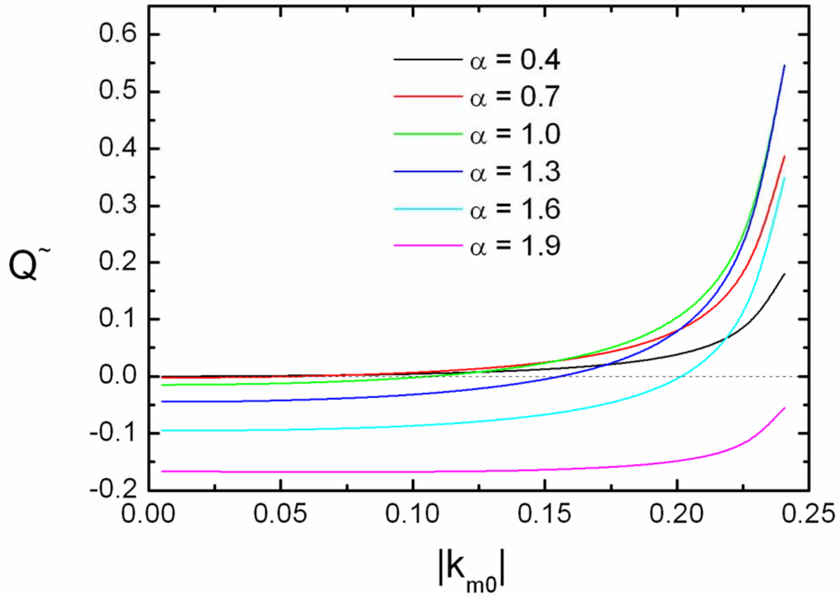
There exists a critical α_{crit} , above which no RSAEs may be present

in this $\alpha_{crit} \cong 1.75$



For a $\alpha < \alpha_{crit}$, the mode may exist only when $q_{\min}$ drops quite enough from m/n, or

K_{m0}^2 is larger than a critical value.



CONCLUSIONS

1. The local equilibrium is solved at the minimum *q* surface for ITB case, the Shafranov shift is of order $O(\varepsilon^0)$

2. The existence of RSAEs is analyzed for at ITB. There is a critical α , above which no RSAEs exist ; below the critical value, the (m, n) mode exists only when the *q* value drops low enough from m/n.

Thank You