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Infernal Alfvén Eigenmodes in Low-Shear Tokamaks

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Outline

- Introduction
- Motivation of the present work
- Low frequency IAE in hybrids with trapped energetic ions
- Alfvén cascades with downward frequency sweeping
- Summary

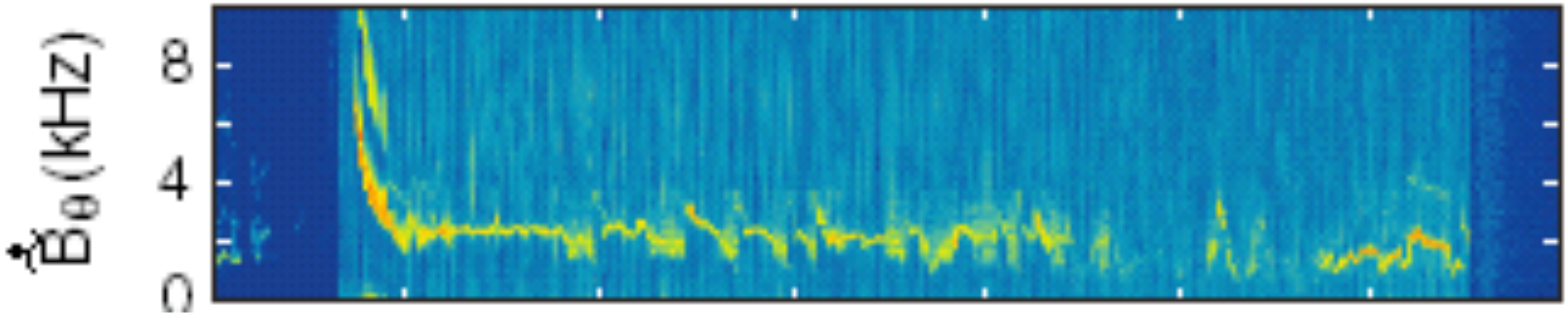
Introduction

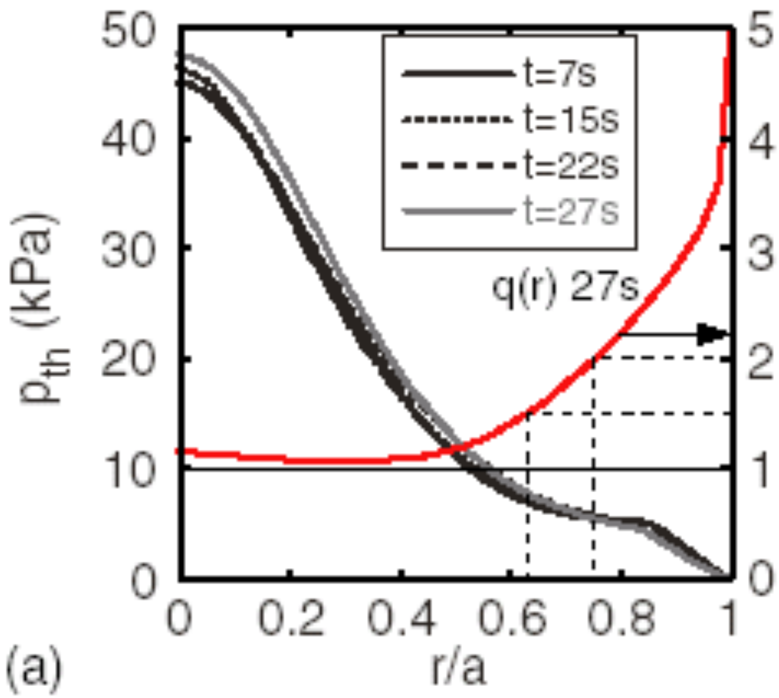
- "Infernal" modes are pressure-driven MHD instabilities that can occur in tokamaks well below high-*n* ballooning stability limit
- They occur for equilibria in which

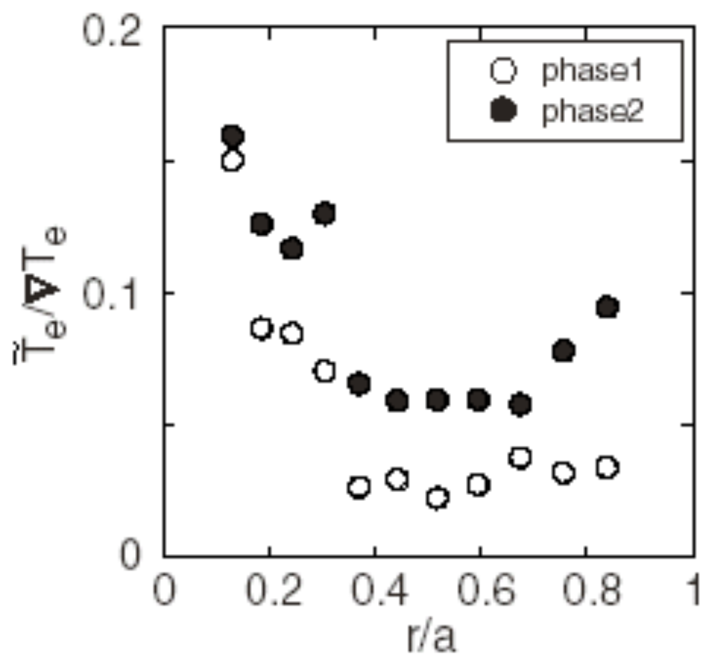
$$q(r) \approx m/n + \mathrm{O}(\varepsilon), \quad 0 \le r < r_1$$

- First predicted by Zakharov (1978) and investigated numerically by Manickam *et al.* (1987)
- For *m/n=1*, this mode is related to the quasi-interchange described by Wesson (1986)
- Theory: Hastie & Hender (1988); <u>Waelbroeck & Hazeltine (1988)</u>
- Hypothesis: below "infernal" stability limit there are low-n Infernal Alfvén Eigenmodes (IAE), which can be excited by energetic ions

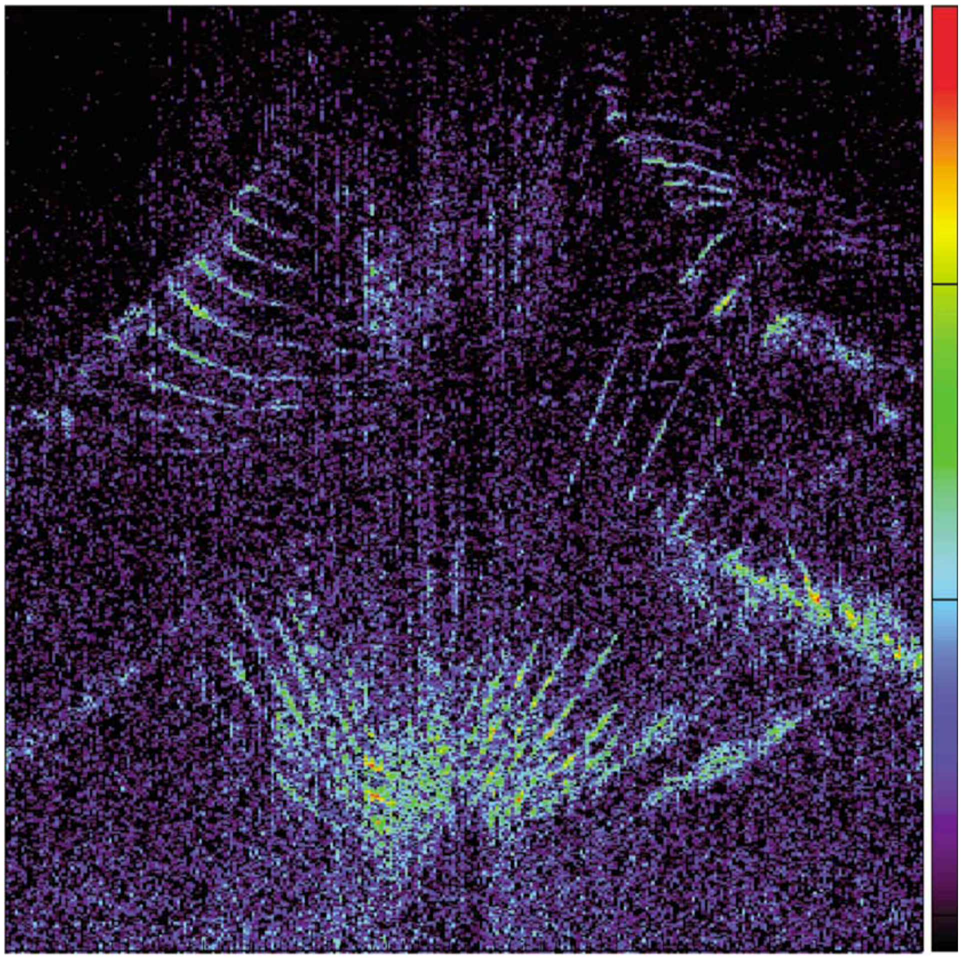
Motivation
Low-frequency modes in hybrid shots on JT-60U
[N. Oyama *et al.*, Nucl. Fusion **49**, 065026 (2009)]

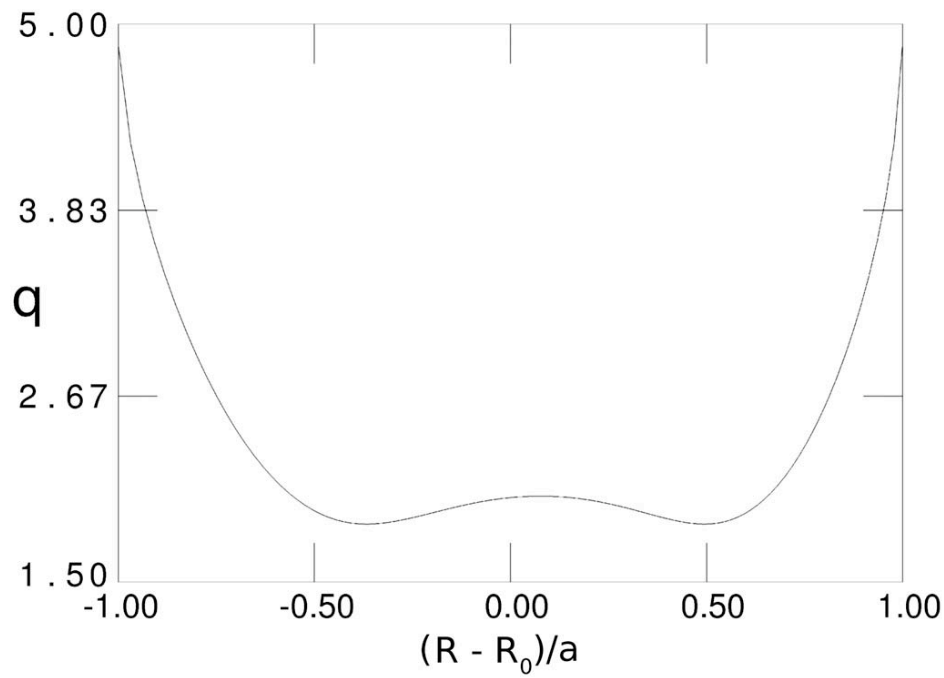






Motivation (cont'd)
Alfvén cascades with downward sweeping on JET
[I.G. Abel *et al.*, Phys. Plasmas **16**, 102506 (2009)]





$m = n = 1$ IAE in hybrids

- Under assumptions $(\omega_0/\omega_A)^2 << \gamma_s\beta$, $q_0-1\sim\varepsilon$, $\beta\sim\varepsilon^2$, minimization of the energy functional yields

  (Crew & Ramos, 1983)

$$\frac{d}{dr}\left\{\varepsilon^{-2}\left[(\iota-1)^2-3\left(\frac{\omega_0}{\omega_A}\right)^2\right]r^3\frac{d\xi_1}{dr}\right\}-4\left(\frac{r}{4}\frac{d\beta_p}{dr}+\beta_p\right)^2 r^3\xi_1=$$

$$=\left(\frac{r}{4}\frac{d\beta_p}{dr}+\beta_p\right)\frac{d}{dr}\left(r^3\hat{\xi}_2\right)$$

$$\frac{d}{dr}\left(r^3\frac{d\hat{\xi}_2}{dr}\right)-3r\hat{\xi}_2=-4r^3\frac{d}{dr}\left[\left(\frac{r}{4}\frac{d\beta_p}{dr}+\beta_p\right)\xi_1\right]$$

$$\beta_p=\frac{8\pi}{B_p^2}\left(\langle p\rangle-p\right)\qquad \xi_2\equiv\varepsilon\hat{\xi}_2$$

m = n = 1 IAE (cont'd)

- Solution in the inner region

$$\hat{\xi}_2 = r^{-3}\int_0^r \hat{r}^4 \beta_p(\hat{r}) \frac{d\xi_1}{d\hat{r}} d\hat{r} + \left[C - \beta_p(r)\xi_1(r)\right]r$$

$$\frac{d\xi_1}{dr} = \frac{\varepsilon^2 C r \beta_p}{(\iota - 1)^2 - 3(\omega_0/\omega_A)^2}$$

- In the outer region

$$\frac{d}{dr}\left[\left(\iota - \frac{1}{2}\right)^2 r^3 \frac{d\hat{\xi}_2}{dr}\right] - 3\left(\iota - \frac{1}{2}\right)^2 r\hat{\xi}_2 = 0$$

$$\Downarrow$$

$$\hat{\xi}_2 \propto \frac{r}{r_2} + \sigma\left(\frac{r}{r_2}\right)^{-3}$$

m = n = 1 IAE dispersion relation

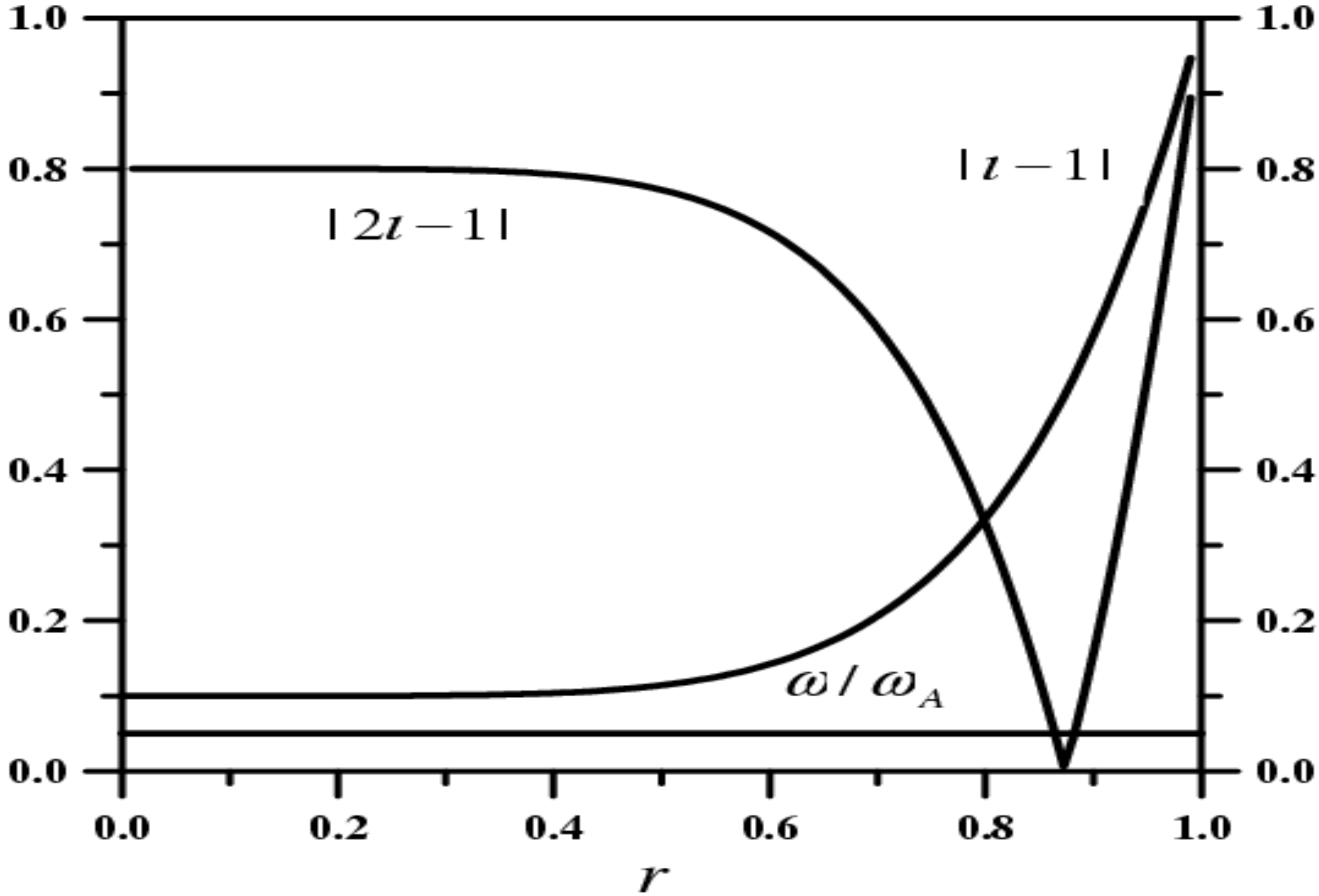
$$\sigma = \left(\frac{r_2}{a}\right)^2 \int_0^a \frac{\left[\varepsilon\beta_p(r)\right]^2}{(\iota - 1)^2 - 3(\omega_0/\omega_A)^2} \left(\frac{r}{r_2}\right)^5 d\left(\frac{r}{r_2}\right)$$

$$p(r) = p_0\left[1 - (r/a)^{2\nu}\right], \iota = 0.5 + (\iota_0 - 0.5)\left[1 - (r/r_2)^{2\lambda}\right], \lambda >> 1$$

$$\Downarrow$$

$$\omega_0 \cong \omega_A \sqrt{A[(\iota_0 - 1)^2 - B(\varepsilon\beta_{p1})^2]}, r_1 = r_2\left(\frac{1 - \iota_0}{\iota_0 - 0.5}\right)^{\frac{1}{2\lambda}}, \beta_{p1} = \beta_p(r_1)$$

$$\beta_0 = 0.1, \iota_0 = 0.9, \nu = 1, \lambda = 3 \Rightarrow \omega_0/\omega_A \approx 3\times 10^{-2}$$



1.0
0.8
0.6
0.4
0.2
0.0
|2ι − 1|
|ι − 1|
ω / ω_A
0.0
0.2
0.4
0.6
0.8
1.0
r

IAE excitation

- Sideband equation in the vicinity of the Alfvén resonance

$$\frac{d}{dr}\left\{\left[\left(\iota-\frac{1}{2}\right)^2-\left(\frac{3}{2}\frac{\omega_0}{\omega_A}\right)^2\right]r^3\frac{d\xi_2}{dr}\right\}=\frac{R}{2}\frac{d}{dr}\left(\frac{d\beta}{dr}r^2\xi_1\right)-\frac{R}{2}\xi_1\frac{d}{dr}\left(\frac{d\beta}{dr}r^2\right)$$

- Threshold

$$-\frac{I(\kappa_0^2)}{3}\frac{R^{3/2}}{r_1}i\pi\frac{\omega_0}{\overline{\omega}_{dm}}\int_0^a r^{3/2}\frac{d\langle\beta_\alpha\rangle}{dr}\xi_1^2 dr=\frac{R}{2}\left(\frac{d\beta}{dr}r^2\xi_1\right)_{r=r_2}\sum_{i=1,2}\int_{r_{Ai}-0}^{r_{Ai}+0}\frac{d\xi_2}{dr}dr$$

$$\overline{\omega}_{dm}=\omega_0\ ,\ \kappa_0^2=1,\ \langle\beta_\alpha\rangle=\beta_{\alpha 0}\left[1-(r/a)^2\right]^2\Rightarrow\beta_{\alpha 0}^{th}\approx 3\times 10^{-3}$$

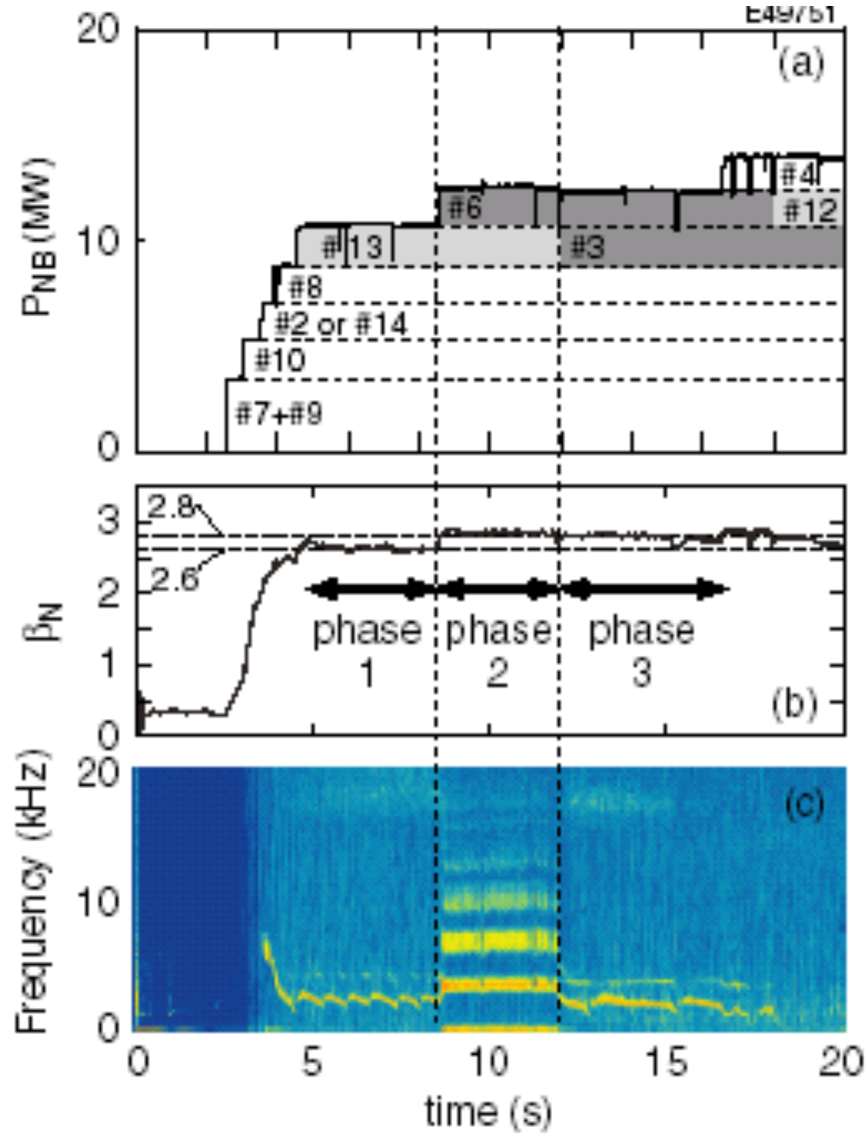
Eigenmodes with m = n > 1

$$\omega_n = \frac{\omega_A}{\sqrt{3}}\sqrt{(\iota_0 - 1)^2 - K_m[(\varepsilon / n)\beta_{p1}]^2}$$

$$K_m = \frac{1}{4\sigma_m}\frac{(\nu+1)^2(1+m)}{(\nu+m)^2(2\nu+m)}\left(\frac{r_1}{r_2}\right)^{2m+2}\left(\frac{r_1}{a}\right)^2$$

- These eigenmodes suffer from higher continuum damping because Alfvén resonances of the sideband harmonic are located closer to the low-shear core
- This is consistent with experiment on JT-60U, where multi-mode excitation has been observed only at highest NBI power

Multi-harmonic excitation in JT-60U
[N. Oyama *et al.*, NF **49**, 065026 (2009)]



Alfvén cascades with downward frequency sweeping

Ware & Haas, 1966

$$(L_m + T_m)\xi_m - \left(\frac{\varepsilon}{mn}\right)^2 \left[\frac{1}{2}\left(r\beta_p' + 4\beta_p\right)^2 + \left(1 - \frac{n^2}{m^2}\right)\left(r\beta_p' + 4\beta_p\right)\right] r^3 \xi_m =$$

$$= \sum \frac{\varepsilon^2}{2nm^2(1 \pm m)} r^{1\mp m}\left(r\beta_p' + 4\beta_p\right)\frac{d}{dr}\left(r^{2\pm m}\xi_{m\pm1}\right)$$

$$\frac{d}{dr}\left(r^3 \frac{d\xi_{m\pm1}}{dr}\right) - \left[(m\pm1)^2 - 1\right] r\xi_{m\pm1} = -\frac{1\pm m}{2n} r^{2\pm m}\frac{d}{dr}\left[\left(r\beta_p' + 4\beta_p\right) r^{1\mp m}\xi_m\right]$$

$$L_m \xi_m = \frac{d}{dr}\left[\left(\frac{1}{nq_0} - \frac{1}{m}\right)^2 r^3 \frac{d\xi_m}{dr}\right] - \left(m^2 - 1\right)\left(\frac{1}{nq_0} - \frac{1}{m}\right)^2 r\xi_m,$$

$$2q_0^2 >> 1 \Rightarrow T_m \xi_m = \frac{d}{dr}\left(\frac{\omega_G^2 - \omega_m^2}{m^2\omega_A^2} r^3 \frac{d\xi_m}{dr}\right) - \left(m^2 - 1\right)\frac{\omega_G^2 - \omega_m^2}{m^2\omega_A^2} r\xi_m.$$

$$\xi_m = \frac{Cr_1}{Br}\left[\frac{I_m\left(\sqrt{\frac{B}{A}}\frac{r}{r_1}\right)}{I_m\left(\sqrt{\frac{B}{A}}\right)} - \left(\frac{r}{r_1}\right)^m\right]$$

$$n\xi_{m\pm1} = -2\beta_p(1\pm m)r^{-(2\pm m)}\int_0^r \hat{r}^{2\pm m}\xi_m d\hat{r} + e_\pm r^{m\pm1-1},$$

$$A = \left(\frac{1}{nq_0} - \frac{1}{m}\right)^2 + \frac{\omega_G^2 - \omega_m^2}{m^2\omega_A^2}, B = \left(\frac{2\varepsilon}{mn}\right)^2\left(\frac{r_1}{a}\right)^2\left(1 - \frac{n^2}{m^2}\right)\beta_p, C = \left(\frac{r_1}{a}\right)^{m+1}\left(\frac{2\varepsilon}{mn}\right)^2 e_+\beta_p$$

external region:

$$\frac{d}{dr}\left[\left(\frac{1}{nq} - \frac{1}{m+1}\right)^2 r^3\frac{d\xi_{m+1}}{dr}\right] - \left[(m+1)^2 - 1\right]\left(\frac{1}{nq} - \frac{1}{m+1}\right)^2 r\xi_{m+1} = 0$$

$$\Downarrow$$

$$\xi_{m+1} \propto \left(\frac{r}{r_{m+1}}\right)^m + \sigma_m\left(\frac{r}{r_{m+1}}\right)^{-(2+m)}$$

Dispersion relation

$$\sigma_m = \beta_p \left(\frac{r_1}{r_0}\right)^{2(m+1)} \frac{m^2}{m^2 - n^2}\left[1 - 2(m+1)\sqrt{\frac{A}{B}}\,\frac{I_{m+1}\left(\sqrt{\frac{B}{A}}\right)}{I_m\left(\sqrt{\frac{B}{A}}\right)}\right]$$

$B << A$:

$$\omega_m^2 = \left(\frac{m}{nq_0} - 1\right)^2 \omega_A^2 + \omega_G^2 - \left(\frac{\varepsilon\beta_p}{n}\right)^2 \frac{\omega_A^2}{(m+1)(m+2)\sigma_m}\left(\frac{r_1}{r_{m+1}}\right)^{2(m+1)}\left(\frac{r_1}{a}\right)^2$$

$$\iota \equiv \frac{1}{q} = \frac{n_0}{m_0+1} + \left(\iota_0 - \frac{n_0}{m_0+1}\right)\left[1-\left(\frac{r}{r_0}\right)^{2\lambda}\right]$$

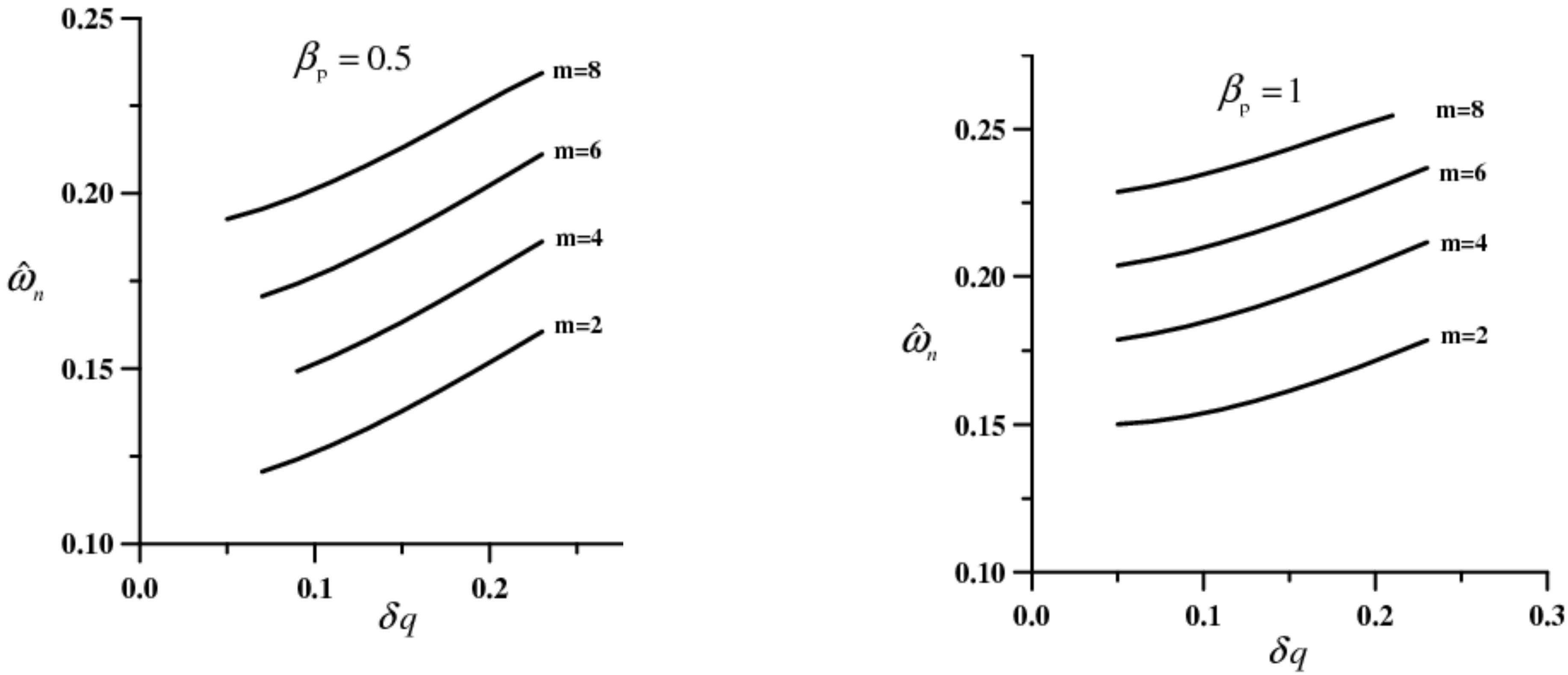
$$\Downarrow$$

$$\xi_{m+1} = C_1\left(\frac{r}{r_0}\right)^{m} F\left[a_1, b_1; c_1; \frac{\iota_0 - \frac{n_0}{m_0+1}}{\iota_0 - \frac{n}{m+1}}\left(\frac{r}{r_0}\right)^{2\lambda}\right] + C_2\left(\frac{r}{r_0}\right)^{-(2+m)} F\left[a_2, b_2; c_2; \frac{\iota_0 - \frac{n_0}{m_0+1}}{\iota_0 - \frac{n}{m+1}}\left(\frac{r}{r_0}\right)^{2\lambda}\right]$$

$$\Downarrow$$

$$\sigma_m = \frac{C_2}{C_1} = -\left(\frac{\iota_0 - \frac{n}{m+1}}{\iota_0 - \frac{n_0}{m_0+1}}\right)^{\frac{m+1}{\lambda}} \frac{\Gamma(c_1)\Gamma(a_2)\Gamma(b_2)}{\Gamma(c_2)\Gamma(a_1)\Gamma(b_1)}$$

$$a_{1,2}(b_{1,2}) = 1 \pm \frac{m+1 \pm (\mp)\sqrt{(2\lambda+1)^2 + m(m+2)}}{2\lambda}, \; c_{1,2} = 1 \pm \frac{m+1}{\lambda}$$



Spectrum of IAEs with $n = 1 - 4$ in the lab frame for $m/n = 2$, $q_0 = 2 + \delta q$, $\varepsilon = 1/3$, $(\omega_G / \omega_A)^2 = 0.016\beta_p$, $\omega_{rot} / \omega_A = 0.025$, and $\lambda = 10$.

Comparison with “quasi-modes”

- Continuum damping of the IAE

$$\frac{\gamma_{cd}}{\omega_A} \sim \varepsilon_{m+1}^2 \left[\frac{\xi_m(r_{m+1})}{\xi_m(0)}\right]^2 \left(\frac{\omega_A}{\omega_0}\right)^2 \left| r_{m+1}\iota'_{m+1}\right|^{-1} < \varepsilon_{m+1}^2 (\delta\, q)^4 \left(\frac{\omega_A}{\omega_G}\right)^2$$

$$\varepsilon_{m+1} = \delta\, q = 0.2, \left(\omega_G / \omega_A\right)^2 = 0.016 \Rightarrow \gamma_{cd} / \omega_A < 4\times 10^{-3}$$

- Radiative damping of the “quasi-mode” [I.G. Abel *et al.,* PoP **18**, 040701 (2009)]

$$\frac{\gamma_{rd}}{\omega_A} \approx \sqrt{\frac{1}{8m}\frac{\omega_{c\alpha}}{\omega_A}\frac{n_\alpha}{n_i}\frac{r_{\min}^3 q''_{\min}}{L_\alpha q_{\min}(nq_{\min}-m)}} \left[\left(\frac{m-nq_{\min}}{q_{\min}}\right)^2 + \left(\frac{\omega_G}{\omega_A}\right)^2\right]^{1/4}$$

$$\omega_{c\alpha}/\omega_A = 10^2, n_\alpha / n_i = 10^{-3}, r_{\min}/L_\alpha = 3, r_{\min}^2 q''_{\min}/q_{\min} = 0.4$$

$$\Downarrow$$

$$\gamma_{rd} / \omega_A \approx 5\times 10^{-2}$$

Summary: *m = n = 1* IAE

- In contrast to the cylindrical GAE, the novel IAE has eigenfrequency well below the minimum of the Alfvén continuum, and can exist in plasmas with $V_A(r)$ = const.
- The properties of this mode are consistent with observations in hybrid discharges with high plasma pressure on the JT-60U tokamak
- With FLR taken into account, eigenfrequency is slightly up-shifted

$$\omega_{pi}(r) \cong const \Rightarrow \omega_0 = \frac{\omega_{pi}}{2} + \sqrt{\frac{\omega_{pi}^2}{4} + \omega_A^2 A\left[(\iota_0 - 1)^2 - B(\varepsilon\beta_{p1})^2\right]}$$

- Ion Landau damping is weak

$$(\omega_0 / \omega_A)^2 << \beta \Leftrightarrow \omega_0 << \omega_{ti}$$

- The analysis is restricted to plasmas slightly below the ideal MHD stability limit. The general case requires several complications, such as geodesic compression. Although study of such BAE would be interesting, for q_0~1 these modes should be suppressed by strong ion Landau damping (consistent with experiment)

Summary: downward cascades

- IAE can resolve the problem with interpretation of the AC with downward frequency sweeping, observed in JET exclusively in shots with almost flat *q*-profile in the wide central region
- IAE frequency lies below the Alfvén continuum
- Consistent with experiments, the obtained frequency spectra are almost degenerate in the plasma frame
- Continuum damping of the IAE is at least an order of magnitude lower than radiative damping of the “quasi-modes”
- In contrast to “quasi-modes”, IAE occupy the whole low-shear core