Simulation of the interaction between Alfvén waves and fast particles in stellarators

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Outline

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Introduction

Alfvén waves

- Alfvén waves are present in magnetised plasmas
- There are global eigenmodes in the gaps of the Alfvén wave spectrum
- These modes could be driven unstable by energetic particles

CKA-EUTERPE Hybrid Code

- To describe wave particle interaction
- In real 3D geometry of stellarators
- Tool for perturbative stability analysis

The hybrid approach

- The particles are described by kinetic equations
- The Alfvén waves are calculated from MHD
- The particles move in the calculated wavefield
- Nonlinear: the energetic particles modify the wavefield

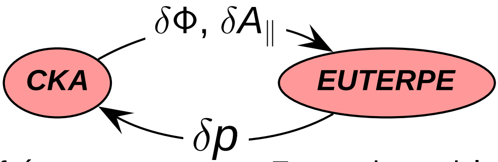
Why do we need a hybrid model?

- Kinetic simulation of Alfvén waves is complicated
- We need a numerical tool to determine stability
- Better physical model than present hybrid codes (CAS3D-K)

Outline

CKA–EUTERPE Hybrid code

code:



	CKA	EUTERPE
describes:	Alfvén waves	Energetic particles
solves:	MHD EQs	Gyrokinetic EQs
method:	Spline decomposition	PIC

- Real 3D magnetic geometry
- No assumptions on particle orbits

MHD description

- Linearised reduced MHD equations
- Transformed to an eigenvalue problem:

$$\omega^2\left[\nabla\cdot\left(\frac{1}{v_A^2}\nabla_\perp\Phi\right)+\frac{3}{4}\nabla\nabla_\perp\left(\rho_i^2\frac{1}{v_A^2}\nabla\cdot\nabla_\perp\Phi\right)\right]=-\nabla\cdot\left[\mathbf{b}\nabla^2(\mathbf{b}\nabla)\Phi\right]$$

- The CKA code is used to solve the MHD equations in 3D real magnetic geometry
- Determines the mode frequency ω and the mode structure $\Phi(r), A_\parallel(r)$

$$E_\parallel = -\nabla\Phi - \frac{\partial A_\parallel}{\partial t} = 0$$

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$$-\nabla\cdot\left[\mathbf{b}\nabla\left(\mu_0\frac{j_\parallel}{B}\mathbf{b}\times\nabla\Phi\right)\right]-\nabla\cdot\left[\frac{\mu_0\delta p_\perp}{B^2}\mathbf{b}\times\nabla B\right]-\nabla\cdot\left[\frac{\mu_0\delta p_\parallel}{B^2}\mathbf{b}\times\kappa\right]$$

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Kinetic description

$$\frac{\partial \delta F}{\partial t} + \dot{\mathbf{R}}^{(0)} \frac{\partial \delta F}{\partial \mathbf{R}} + \dot{v_\parallel}^{(0)} \frac{\partial \delta F}{\partial v_\parallel} = -\dot{\mathbf{R}}^{(1)} \frac{\partial F_0}{\partial \mathbf{R}} - \dot{v_\parallel}^{(1)} \frac{\partial \delta F_0}{\partial v_\parallel} - \dot{v_\perp}^{(1)} \frac{\partial \delta F_0}{\partial v_\perp}$$

$$\frac{\mathrm{d}\mathbf{R}}{\mathrm{d}t} = \frac{\mathbf{B}^*}{B_\parallel^*}\left(v_\parallel - \frac{q}{m}\langle A_\parallel\rangle\right) + \frac{1}{qB_\parallel^*}\mathbf{b} \times \left(\mu\nabla B + q\nabla\langle\Phi - v_\parallel A_\parallel\rangle\right)$$

$$\frac{\mathrm{d}v_\parallel}{\mathrm{d}t} = -\frac{1}{m}\frac{\mathbf{B}^*}{B_\parallel^*}(\mu\nabla B + q\nabla\langle\Phi - v_\parallel A_\parallel\rangle),$$

- The EUTERPE code is used to integrate the equations
- PIC code, δf method
- Full particle orbits
- Currently the linearised version is used
- Possible to use multiple species

Wave particle interaction

- The field energy

$$\mathcal{E}_{field} = \frac{1}{2}\int mn_0 \frac{|\nabla_\perp \Phi|^2}{B^2} dV + \frac{1}{2\mu_0}\int |\nabla_\perp A_\parallel|^2 dV$$

- The growth rate of the mode is given by:

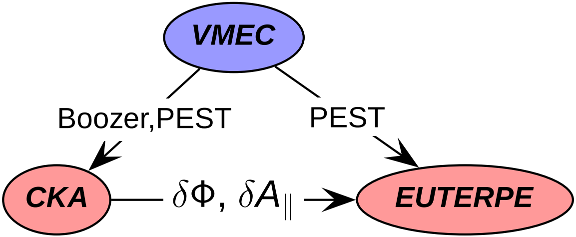
$$\gamma = \frac{1}{2\mathcal{E}_{field}} \frac{\partial \mathcal{E}_{field}}{\partial t} = -\frac{1}{2\mathcal{E}_{field}} \frac{\partial \mathcal{E}_{kin}}{\partial t} = -\frac{1}{2\mathcal{E}_{field}} \int \boldsymbol{j} \cdot \boldsymbol{E} \mathrm{d}^3 r$$

- Later the pressure perturbation will be calculated

$$\delta p_\parallel = \int d\mathbf{v}\, m v_\parallel^2 \delta F, \quad \delta p_\perp = \int d\mathbf{v}\, \mu B \delta F$$

Interface between the codes

- The magnetic equilibrium is calculated by VMEC
- The PEST coordinates are implemented in CKA
- Data transfer routines from CKA to EUTERPE are written
- The feedback from EUTERPE will be implemented later



Outline

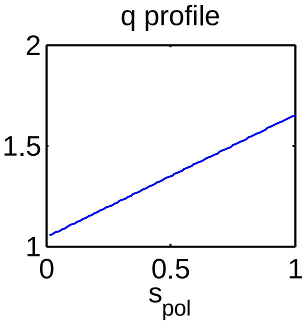
Benchmark 1

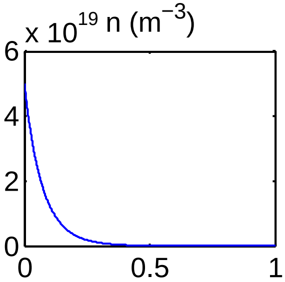
Plasma parameters

- Circular tokamak, $R_0 = 4m$, a=1 m
- B_0 = 5 T
- Deuterium plasma
- $n_e = 5 \cdot 10^{19} m^{-3}$, constant n profile

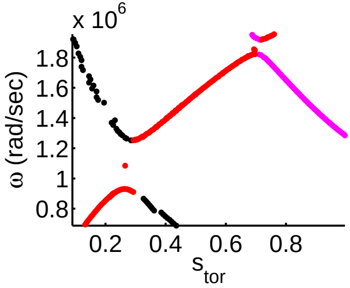
Energetic particles

- Hydrogen
- T = 6keV - 1.4 MeV, constant T profile
- β kept constant
- density gradient
- Maxwellian velocity distribution

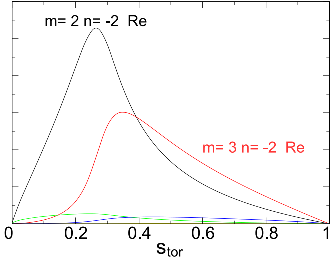


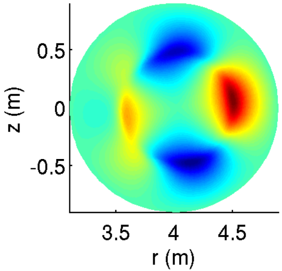


Mode structure



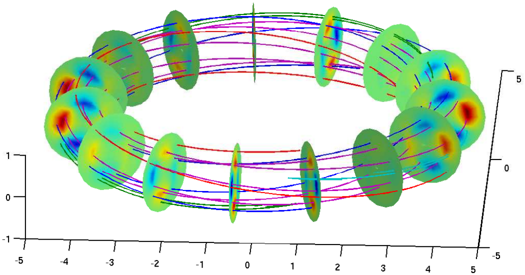
- Alfvén spectrum calculated by CKA
- toroidal mode number n=2
- global TAE at the crossing of (2,2) and (2,3) modes





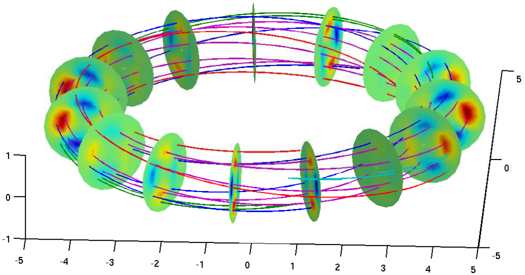
The data is transferred to EUTERPE

Fast particles started



The data is transferred to EUTERPE

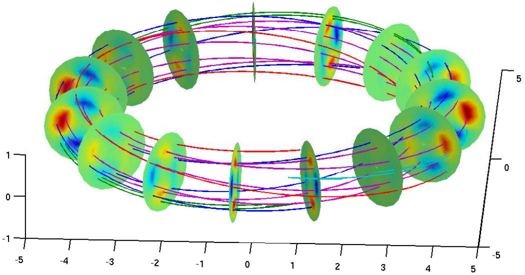
Fast particles started



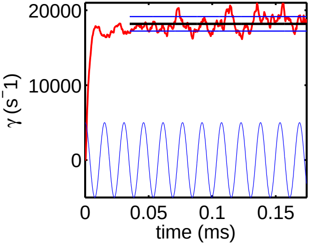
$$\gamma \;=\; -\frac{1}{2\mathcal{E}_{field}}\int \boldsymbol{j}\cdot\boldsymbol{E}\,\mathrm{d}^3r$$

The data is transferred to EUTERPE

Fast particles started



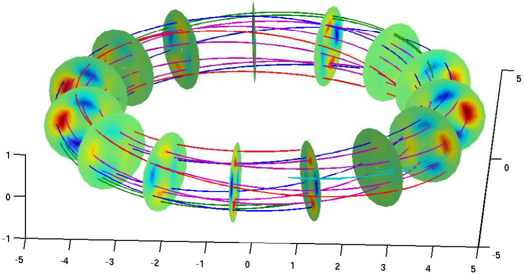
$$\gamma = -\frac{1}{2\mathcal{E}_{field}} \int \boldsymbol{j} \cdot \boldsymbol{E} \, \mathrm{d}^3 r$$



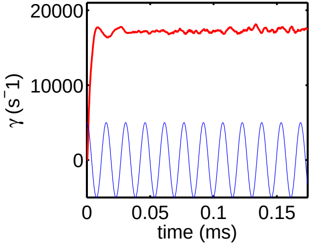
Simulation with EUTERPE

The data is transferred to EUTERPE

Fast particles started

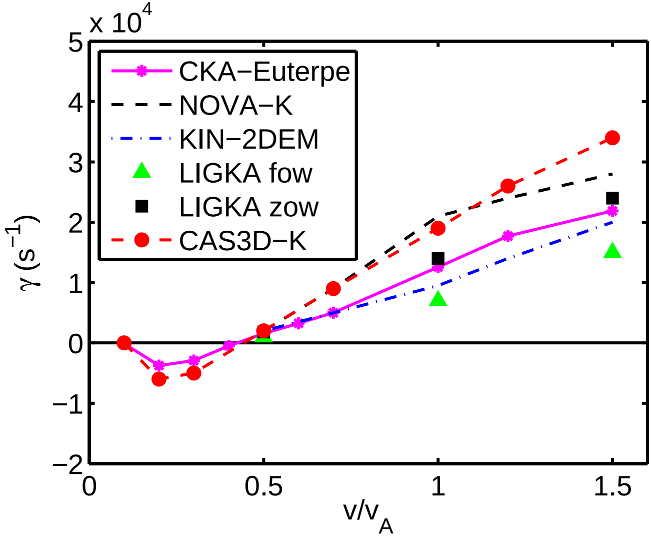


$$\gamma = -\frac{1}{2\mathcal{E}_{field}} \int \boldsymbol{j} \cdot \boldsymbol{E} \, \mathrm{d}^3 r$$



Stability of the mode

The energetic particle velocity was changed

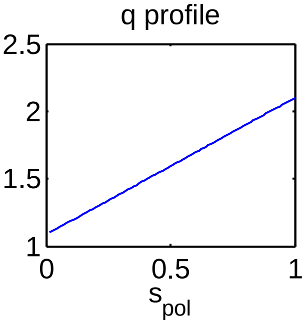


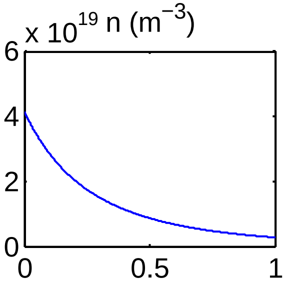
Plasma parameters

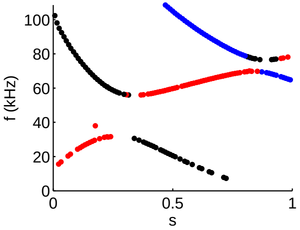
- Circular tokamak, $R_0 = 3m$, a=1 m
- B_0 = 1T, q = 1.1 + Ψ_{pol}
- Deuterium plasma
- $n_e = 4.142 \cdot 10^{19} m^{-3}$, constant n profile
- $T_e = 3.14$ keV

Energetic particles

- Deuterium
- $E_{EP} = 173$ keV, constant T profile
- density profile $\sim exp(-\Psi_{pol}/0.37)$
- Maxwellian velocity distribution
- or slowing down $f(v) \sim \frac{1}{v^3+v_{crit}^3}$

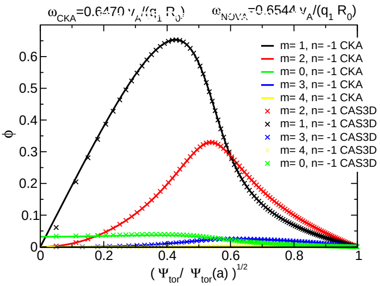


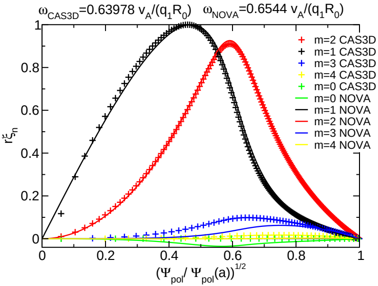




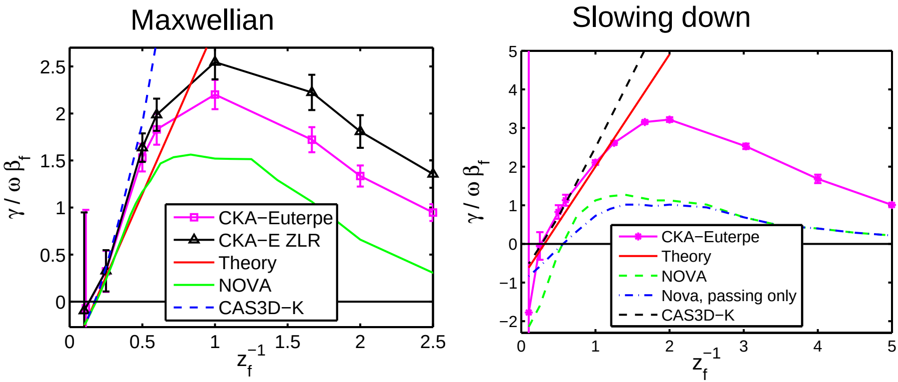
The CKA code is used to calculate the Alfvén mode spectrum

The mode structure from CKA, CAS3D and NOVA:





The charge of the fast particles was changed.

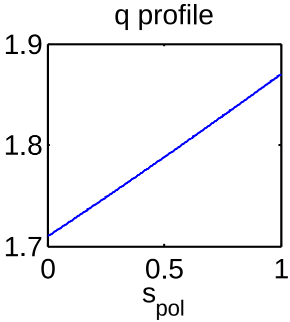


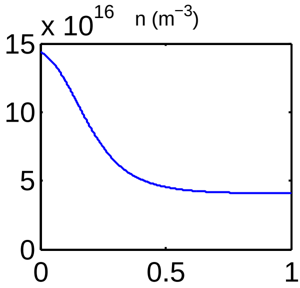
Plasma parameters

- Circular tokamak, $R_0 = 10m$, a=1 m
- B_0 = 3T
- Hydrogen plasma
- $n_e = 2 \cdot 10^{19} m^{-3}$, constant n profile
- $T_e = 1$ keV

Energetic particles

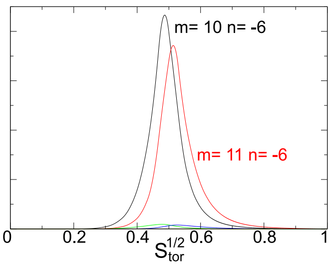
- Deuterium
- T = 100 keV-800 keV constant T profile
- $n_{0hot} = 1.44 \cdot 10^{17}$ m^{-3}, density gradient
- Maxwellian velocity distribution

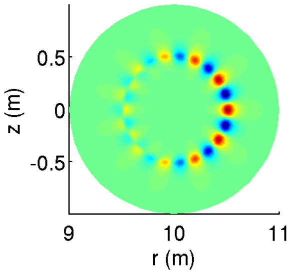




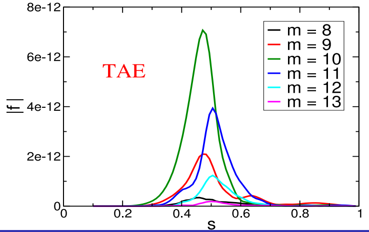
Mode structure, GYGLES benchmark

Mode structure calculated by CKA

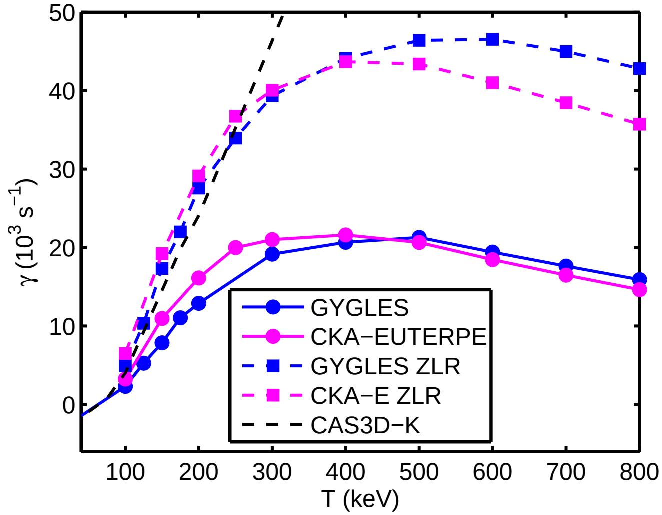




GYGLES mode structure
(T = 400keV):



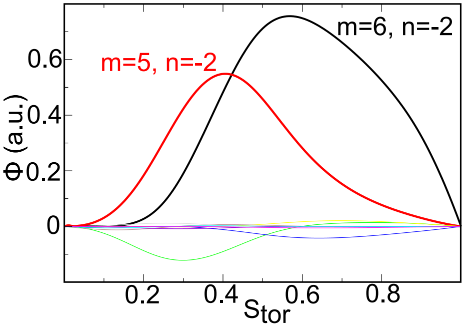
The temperature of the fast particles was changed

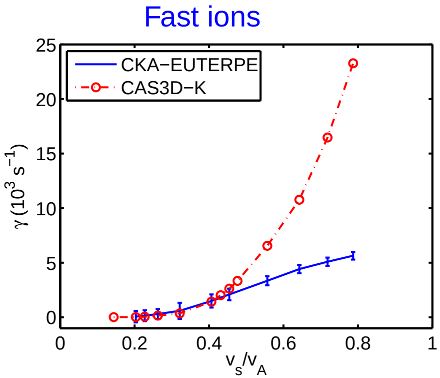


Outline

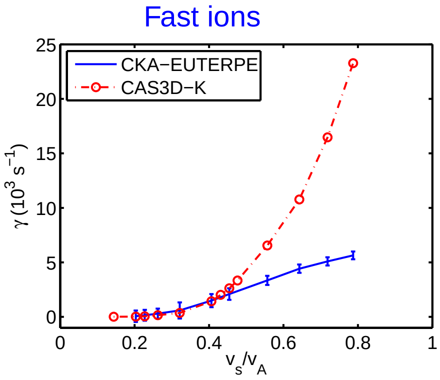
W7-AS #39042

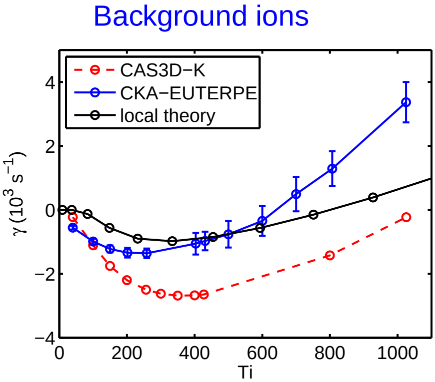
- Alfvén spectrum determined by CKA
- Stability of a global TAE is calculated
- Slowing down distribution function




Fast ions
CKA−EUTERPE
CAS3D−K
γ (10^3 s^−1)
v_s/v_A
0
5
10
15
20
25
0
0.2
0.4
0.6
0.8
1

W7-AS #39024 Energy transfer





Summary

CKA-EUTERPE

- Codes were coupled to create a hybrid code
- Perturbatively study the stability of Alfvén modes

Linear benchmarks

- Benchmark 1: successful
- NOVA benchmark: differences maybe due to the distribution function
- GYGLES benchmark: successful

Stellarator results

- W7-AS case calulated
- FLR and FOW effects are strongly stabilizing