Measurement and theoretical modelling of the damping rate of medium-n TAEs in JET

D.Testa\textsuperscript{1}, T.Panis\textsuperscript{1}, A.Fasoli\textsuperscript{1}, P.Blanchard\textsuperscript{1,2}, H.Carfantan\textsuperscript{3}, A.Goodyear\textsuperscript{4}, N.Mellet\textsuperscript{1,5}, S.E.Sharapov\textsuperscript{4}, D.Spong\textsuperscript{6}, JET-EFDA contributors

JET-EFDA, Culham Science Centre, OX14 3DB, Abingdon, UK

[2] JET-EFDA Close Support Unit, Culham Science Centre, Abingdon, UK
[4] Culham Center for Fusion Energy, Culham Science Centre, Abingdon, UK
[5] Association Euratom CEA, Cadarache, Saint-Paul-lez-Durance, France
[6] Oak Ridge National Laboratory, Fusion Energy Theory Group, Oak Ridge, USA
Alfvén Eigenmode Active Diagnostics

Aim: address physics of mode damping, identify modes most prone to instability in different burning plasma scenarios, and parameters to control stability.

 ITER-relevance for size and shape scaling, scenarios

high field & density, $T_e \sim T_i$

New antennas

Old saddle coils

紧l aspect ratio, broad range of $\beta$
JET AE antennas

Unique capability: real-time tracking to follow mode evolution as plasma parameters change
Ex. of $\gamma/\omega$ measurements for n=7 TAE

Note: in the absence of fast ion drive, $\gamma/\omega$ is the mode damping rate
Outline

• Real-time mode decomposition and tracking

• Measurements of TAE damping rate
  – Database and trends
    • Parameter ranges in which measurements are possible
    • Parametric dependencies for dominant damping mechanism(s)
  – Individual discharge analysis
    • Essential for detailed comparisons with theory
    • Ex.: modeling of n=3 TAEs using LEMan, CASTOR, TAEFL

• Outlook: Active diagnostic upgrade
Real-time detection and tracking of individual n-components

- Antenna spectrum contains several frequency-degenerate modes
- Discrimination of n’s done in real-time using sparse representation method (SparSpec)
- Computation within 850µs

Example
- Antenna configuration to drive odd modes (3<|n|<11)
- n=3 mode dominates
- Weaker n=5, 7 modes

Post-pulse analysis (no time limitations for calculation) reveals entire set of modes
For modes also measured in real-time the difference in f, γ/ω and n are within 20%
### AE damping database

**Range of plasma parameters**

<table>
<thead>
<tr>
<th>Plasma parameter</th>
<th>min</th>
<th>max</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_p$ (MA)</td>
<td>0.93</td>
<td>2.58</td>
</tr>
<tr>
<td>$B_0$ (T)</td>
<td>1.87</td>
<td>3.37</td>
</tr>
<tr>
<td>$q_0$</td>
<td>0.72</td>
<td>2.70</td>
</tr>
<tr>
<td>$q_{95}$</td>
<td>2.63</td>
<td>6.56</td>
</tr>
<tr>
<td>$s_{95}$</td>
<td>2.21</td>
<td>4.73</td>
</tr>
<tr>
<td>$\kappa_{95}$</td>
<td>1.25</td>
<td>1.71</td>
</tr>
<tr>
<td>$\delta_u$</td>
<td>-0.02</td>
<td>0.34</td>
</tr>
<tr>
<td>$\delta_l$</td>
<td>-0.01</td>
<td>0.33</td>
</tr>
<tr>
<td>ROG (m)</td>
<td>0.01</td>
<td>0.12</td>
</tr>
<tr>
<td>$\beta_N$</td>
<td>0.15</td>
<td>0.54</td>
</tr>
<tr>
<td>$\beta_t$ (%)</td>
<td>0.06</td>
<td>0.46</td>
</tr>
<tr>
<td>$\beta_p$</td>
<td>0.13</td>
<td>0.25</td>
</tr>
<tr>
<td>$n_e0$ (10^{19} \text{ m}^{-3})</td>
<td>\sim 1</td>
<td>\sim 3</td>
</tr>
<tr>
<td>$T_e0$ (keV)</td>
<td>\sim 1</td>
<td>\sim 4</td>
</tr>
</tbody>
</table>

~10000 TAE damping rate measurements in ohmic plasmas
### AE damping database
Looking for correlations

| plasma quantity | $|n|$ |
|----------------|-----|
|                | 1   | 2   | 3   | 4   | 5   | 6   | 7   |
| $q_0$          | -0.05 | -0.00 | -0.30 | -0.17 | -0.05 | 0.09 | -0.42 |
| $q_{95}$       | -0.05 | -0.03 | -0.18 | -0.06 | 0.11 | 0.04 | -0.57 |
| $q_{95} - q_0$ | -0.05 | -0.05 | -0.09 | 0.04 | 0.20 | 0.02 | -0.52 |
| $q_{95}/q_0$   | 0.14 | -0.04 | 0.24 | 0.20 | 0.16 | -0.21 | 0.11 |
| $s_{95}$       | 0.60 | 0.22 | 0.31 | 0.29 | 0.09 | -0.28 | 0.02 |
| $\kappa_{95}$ | 0.49 | 0.19 | 0.22 | 0.25 | -0.13 | -0.26 | -0.18 |
| $\delta_u$    | 0.36 | 0.12 | 0.10 | 0.21 | 0.07 | -0.39 | -0.05 |
| $\delta_l$    | 0.30 | 0.22 | 0.04 | 0.29 | 0.20 | -0.29 | -0.13 |
| $I_p$          | -0.40 | -0.04 | -0.00 | 0.12 | -0.24 | -0.14 | -0.24 |
| $\beta_N$     | 0.22 | 0.04 | 0.32 | 0.13 | 0.01 | 0.04 | 0.38 |
| $\beta_t$     | 0.28 | 0.07 | 0.38 | 0.17 | -0.09 | 0.02 | 0.50 |
| $\beta_p$     | 0.09 | -0.07 | 0.04 | 0.09 | 0.21 | -0.03 | -0.09 |
| $B_0$          | -0.69 | -0.09 | -0.34 | -0.07 | -0.24 | -0.06 | -0.51 |
| ROG            | -0.01 | -0.07 | 0.03 | 0.21 | -0.05 | 0.05 | 0.12 |
| $T_{e0}$       | -0.20 | -0.01 | -0.13 | 0.09 | -0.38 | 0.01 | -0.10 |
| $n_{e0}$       | -0.16 | -0.09 | -0.02 | -0.07 | -0.23 | -0.27 | -0.29 |
| $\lambda$     | 0.75 | 0.18 | 0.23 | 0.29 | -0.05 | 0.05 | 0.05 |

**Correlation coefficient**

$$r = \frac{\sum_{i=1}^{N}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{N}(x_i - \bar{x})^2 \sum_{i=1}^{N}(y_i - \bar{y})^2}}$$
## AE damping database

Looking for correlations

<table>
<thead>
<tr>
<th>plasma quantity</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_0$</td>
<td>-0.05</td>
<td>-0.00</td>
<td>-0.30</td>
<td>-0.17</td>
<td>-0.05</td>
<td>0.09</td>
<td>-0.42</td>
</tr>
<tr>
<td>$q_{95}$</td>
<td>-0.05</td>
<td>-0.03</td>
<td>-0.18</td>
<td>-0.06</td>
<td>0.11</td>
<td>0.04</td>
<td>-0.57</td>
</tr>
<tr>
<td>$q_{95} - q_0$</td>
<td>-0.05</td>
<td>-0.05</td>
<td>-0.09</td>
<td>0.04</td>
<td>0.20</td>
<td>0.02</td>
<td>-0.52</td>
</tr>
<tr>
<td>$q_{95}/q_0$</td>
<td>0.14</td>
<td>-0.04</td>
<td>0.24</td>
<td>0.20</td>
<td>0.16</td>
<td>-0.21</td>
<td>0.11</td>
</tr>
<tr>
<td>$s_{95}$</td>
<td>0.60</td>
<td>0.22</td>
<td>0.31</td>
<td>0.29</td>
<td>0.09</td>
<td>-0.28</td>
<td>0.02</td>
</tr>
<tr>
<td>$\kappa_{95}$</td>
<td>0.49</td>
<td>0.19</td>
<td>0.22</td>
<td>0.25</td>
<td>-0.13</td>
<td>-0.26</td>
<td>-0.18</td>
</tr>
<tr>
<td>$\delta_u$</td>
<td>0.36</td>
<td>0.12</td>
<td>0.10</td>
<td>0.21</td>
<td>0.07</td>
<td>-0.39</td>
<td>-0.05</td>
</tr>
<tr>
<td>$\delta_l$</td>
<td>0.30</td>
<td>0.22</td>
<td>0.04</td>
<td>0.29</td>
<td>0.20</td>
<td>-0.29</td>
<td>-0.13</td>
</tr>
<tr>
<td>$I_p$</td>
<td>-0.40</td>
<td>-0.04</td>
<td>-0.00</td>
<td>0.12</td>
<td>-0.24</td>
<td>-0.14</td>
<td>-0.24</td>
</tr>
<tr>
<td>$\beta_N$</td>
<td>0.22</td>
<td>0.04</td>
<td>0.32</td>
<td>0.13</td>
<td>0.01</td>
<td>0.04</td>
<td>0.38</td>
</tr>
<tr>
<td>$\beta_t$</td>
<td>0.28</td>
<td>0.07</td>
<td>0.38</td>
<td>0.17</td>
<td>-0.09</td>
<td>0.02</td>
<td>0.50</td>
</tr>
<tr>
<td>$\beta_p$</td>
<td>0.09</td>
<td>-0.07</td>
<td>0.04</td>
<td>0.09</td>
<td>0.21</td>
<td>-0.03</td>
<td>-0.09</td>
</tr>
<tr>
<td>$B_0$</td>
<td>-0.69</td>
<td>-0.09</td>
<td>-0.34</td>
<td>-0.07</td>
<td>-0.24</td>
<td>-0.06</td>
<td>-0.51</td>
</tr>
<tr>
<td>ROG</td>
<td>-0.01</td>
<td>-0.07</td>
<td>0.03</td>
<td>0.21</td>
<td>-0.05</td>
<td>0.05</td>
<td>0.12</td>
</tr>
<tr>
<td>$T_{e0}$</td>
<td>-0.20</td>
<td>-0.01</td>
<td>-0.13</td>
<td>0.09</td>
<td>-0.38</td>
<td>0.01</td>
<td>-0.10</td>
</tr>
<tr>
<td>$n_{e0}$</td>
<td>-0.16</td>
<td>-0.09</td>
<td>-0.02</td>
<td>-0.07</td>
<td>-0.23</td>
<td>-0.27</td>
<td>-0.29</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.75</td>
<td>0.18</td>
<td>0.23</td>
<td>0.29</td>
<td>-0.05</td>
<td>0.05</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Correlation coefficient

$$r = \frac{\sum_{i=1}^{N}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{N}(x_i - \bar{x})^2 \sum_{i=1}^{N}(y_i - \bar{y})^2}}$$
$\gamma/\omega$ for $1<|n|<7$ TAEs vs. $q_0$

- No clear general trend in $\gamma/\omega=f(q_0)$
- Try selection of shots with similar $q_{95}$ and varying $q_0$ during $q$-profile relaxation
\(\gamma/\omega\) for 1<\(|n|<7\) TAEs vs. \(q_0\)

Selection of shots with fixed \(q_{95}\)

- For same \(q_{95}\), Alfvén continuum gaps get less and less aligned as q-profile relaxes (\(q_0\) decreases)
- This effect is quantified by \(1/[q\sqrt{n_e(r)}]\)
- Estimate of continuum damping
\(\gamma/\omega\) for \(1<|n|<7\) TAEs vs. \(q_0\)

Selection of shots with fixed \(q_{95}\)

- For same \(q_{95}\), Alfvén continuum gaps get less and less aligned as q-profile relaxes (\(q_0\) decreases)
- This effect is quantified by \(1/[q\sqrt{n_e}(r)]\)
- Estimate of continuum damping

- Evidence for difference between \(n\)-ranges: continuum and radiative damping less effective as \(n\) increases

D. Testa, CRPP-EPFL

IAEA-TCM-EP, Austin, 07-10 September 2011
\( \gamma / \omega \) for \( 1 < |n| < 7 \) TAEs vs. \( \lambda \)

- Non-ideal parameter \( \lambda \propto q_{95} s_{95} \sqrt{T_{e0}/B_0} \) provides estimate of radiative damping
- Clear trend for \( |n|=1 \): \( \gamma / \omega |n|=1 \propto \lambda \)
- Similar trend for \( 2 \leq |n| \leq 4 \), only for large \( \lambda \)
- No clear trend for \( |n| \geq 5 \)
Edge shape effect on TAE damping

Old, saddle coil driven $n=1$ measurements indicated a clear trend

$n=1$ TAE
\( \gamma/\omega \) for \( 1<|n|<7 \) TAEs vs. edge shape

- No data points for which \( \gamma/\omega<7\% \) for large values of \( \kappa_{95} \) and \( s_{95} \)
- Clear trend only visible in general for low-\( n \)
- To assess effect for medium-\( n \) a single shot approach is necessary
\( \gamma/\omega \) for \(|n|=3, 7\) TAEs vs. edge shape

Single shot approach

- In single shot, single-\( n \) measurements, edge elongation leads to increase in damping
- Why is this trend not visible in database?
- *Disentangle two dependences, e.g. q-profile and edge shape in specific experiment*
Disentangling influence of edge shape and q-profile on n=3 TAE damping

• Differences in $\gamma/\omega$ for same $\kappa_{95}$ in different discharges seem related to ‘span’ of q-profile, quantified by $q_{95}/q_0$ and $q_{95}-q_0$ (proportional to number of poloidal harmonics)

• Damping increases with $q_{95}/q_0$ and $q_{95}-q_0$

• Trend consistent with electron Landau damping scaling $(\gamma/\omega)_{ELD} \propto n^2(q_{95}-q_0)^2$
n=3 TAE damping: modeling shape effect

- **CASTOR**: fluid
  - Large discrepancies with data

- **TAEFL**: gyro-fluid
  - $\gamma/\omega$ extrapolated back from marginal stability threshold for fast-ion driven modes
  - Good agreement with data, important to test validity of modeling approach

- **LEMan**: gyro-kinetic
  - Good agreement with data
  - Electron Landau damping of mode converted kinetic AW
  - *Note: good agreement was also found in comparisons with gyro-kinetic code LIGKA*

IAEA-TCM-EP, Austin, 07-10 September 2011
Summary

• JET AE exciter drives and detects real-time selected spectrum of medium-n Aes

• Damping measurements database suggests some global trends for damping rates

• Individual discharge and n analysis is needed to disentangle complex dependences of $\gamma/\omega$ on various parameters and profiles

• Agreement with numerical codes based on gyro-kinetic or gyro-fluid models has reached quantitative level
Limitations of present AE system

• Coupling
  – Even with optimal matching and coils from both antennas, with one amplifier ($I_{\text{max}} \sim 5\text{A}$), core amplitudes are very small
    • $\sim 0.05\text{mG}/1\text{A}$ in the plasma core for modes with $n\sim 7-15$
  – Tracking is difficult in the presence of noise (e.g. during strong additional heating) and if LCFS is distant from antennas
  – Mode identification requires sophisticated $n$-detection algorithms

• Mode selection
  – Wide spectrum; plasma preferentially selects low-$n$'s

![n distribution of damping rate measurements](image)
Upgrade to 8 independent amplifiers

• Maximize antenna currents within feed-through limits (25A)
• Better definition of antenna spectrum
  – Gain in single mode excitation by ~ factor of 4
  – More balanced distribution of currents without transformer coupling
• Arbitrary phasing
  – Definition of sign of n (traveling wave): identification of fast ion contribution
  – Simultaneous excitation of selected modes (different n’s or frequencies)
    • Stronger constraint on theory simulations for ITER extrapolation
    • Multi-point diagnostic applications
      – \(q(r_1, r_2, ...)\), \(A_{\text{EFF}}(r_1, r_2, ...)\), \(\omega_{\text{TOR}}(r_1, r_2, ... )\)
• Test of burn control ideas
  – Real time control of fast ion stability by mode tracking
the Sparse Representation Method

SparSpec minimizes the L1-norm penalized criterion:

\[
J(x) = \frac{1}{2} \| y - W x \|^2 + \lambda \sum_{k=-K}^{K} |x_k|_{L1}
\]

\(y\): vector of data taken at time \(t_k\) \([\equiv \text{position } \phi_k]\)
\(W\): spectral window \(\exp(i2\pi t_k f_n)\) \([\equiv \exp(i2\pi \phi_k n)]\)
\(x\): vector of \((I,Q)\) signals for frequencies \(f_n\)
\(\lambda\): parameter fixed to obtain a satisfactory sparse solution \(\Rightarrow\) penalty criterion for invoking more modes to find adequate solution
\(\lambda\) can be fixed a-priori from known noise variance

\(SparSpec\) minimizes the L1-norm penalized criterion:

\[
J(x) = \frac{1}{2} \| y - W x \|^2 + \lambda \sum_{k=-K}^{K} |x_k|_{L1}
\]

- the sparse signal representation method is ideally suited for mode number analysis in fusion plasmas:
  - specifically designed for un-evenly distribution of sensors
  - allowable mode numbers are discretized: \(|n| = 0, \pm1, \pm2, \pm3\ldots\)
  - large \((n,m)\)-range, number of modes not assumed a priori
  - amplitude and phase equally important for fitting algorithm
  - no need for a-posteriori tresholding to discriminate between solutions as \(\lambda\)-penalty determined a-priori from knowledge of noise variance
  - very efficient, very fast convergence \(\Rightarrow\) ideal for real-time applications
  - now implemented and fully validated in JET real-time and post-pulse mode tracking algorithm for stable Alfvén Eigenmodes
  - accuracy \(\Rightarrow\) need correct interpretation of the spectral window
\( \gamma/\omega \) measurements for medium-n AEs: data available for theory comparisons

- database compiled of \( \gamma/\omega(n) \) as function of plasma parameters and configurations for individual mode numbers
  - in excess of 10'000 individual \( \gamma/\omega(n) \) measurements already analyzed
  - in excess of 60 individual discharges already analyzed
  - various dedicated scans in plasma parameters have been run:
    - elongation scan during ohmic phase, \( 1.25 < \kappa_95 < 1.65 \) without ICRF
      - add ICRF with PRF=2MW and PRF=3MW, different phasing (dipole and +/-90)
      - add PRF modulations 2MW +1MW/300ms, different phasing (dipole and +/-90)
    - ohmic Bfield/ne scan, change RF deposition profile and edge continuum
      - add PRF with power ramp-up to 4.5MW, different phasing (dipole and +/-90)
- damping rate as function of plasma isotope composition and ion Larmor radius
- damping rate for medium-n (n=3-7) TAEs at ICRF power switch off with constant plasma parameters
  - direct measurement of MeV-ions drive to the modes?
- effect of ripple in the magnetic field medium-n (n=3-7) TAEs with/out fast ions:
  - fast ion losses (resonant NBI ions with \( V_{||} \sim V_A/3 \)), affecting drive for the modes?
  - change density scale length at plasma edge, affecting the continuum damping?