Energetic particle modes: from bump on tail to tokamak plasmas

M. K. Lilley¹

B. N. Breizman², S. E. Sharapov³, S. D. Pinches³

¹ Physics Department, Imperial College London, London, SW7 2AZ, UK

² IFS, University of Texas at Austin, Austin, Texas, 78712, USA

³ EURATOM/UKAEA Fusion Association, Culham Science Centre, Abingdon, OX14 3DB, UK
Outline

• Experimental observations
• Marginal stability and the role of collisions
• Bump-on-tail model including drag
• Generalisation to toroidal systems
Experimental observations
Question: Why disparity between NBI and ICRH?

ICRH drive (JET)

NBI drive (MAST)

Heeter et.al PRL 85, 3177 (2000)

Pinches et.al PPCF, 46, S47 (2004)
Marginal stability and the role of collisions
Marginal stability

- System evolves through a threshold

\[ |\gamma_L - \gamma_d| \ll \gamma_L, \gamma_d \]
Collisionality - previous analysis

• Marginal stability allows collisions to compete with mode growth

\[ \left| \gamma_L - \gamma_d \right| \sim \nu_{\text{eff}} \]

• Krook and diffusion were studied in bump on tail

\[ \left. \frac{dF}{dt} \right|_{\text{coll}} = \beta (F - F_0) \quad \left. \frac{dF}{dt} \right|_{\text{coll}} = \frac{\nu^3}{k^2} \left( \frac{\partial^2 F}{\partial \nu^2} - \frac{\partial^2 F_0}{\partial \nu^2} \right) \]

\[ \nu_{\text{eff}} = \max \{ \beta, \nu \} \]

Berk, Breizman et.al PRL, 76, 1256 (1996)
Breizman, Berk et.al PoP, 4 1559 (1997)
Bump on tail - Basic ingredients

- Particle injection and effective collisions, \( v_{\text{eff}} \), create an inverted distribution of energetic particles \( F_0(v) \)
- Discrete spectrum of unstable electrostatic modes
- Instability drive, \( \gamma_L \sim dF_0/dv \), due to wave-particle resonance \((\omega-kv=0)\)
- Background dissipation rate, \( \gamma_d \), determines the critical gradient for the instability

![Diagram showing critical slope and separatrix](image)
Collisionality affects mode saturation

$$\hat{\nu} = \frac{\nu}{\gamma_L - \gamma_d}$$

Heeter et. al PRL, 85, 3177 (2000)
Low collisionality – Frequency chirping

\[ \delta \omega \sim \sqrt{t} \]
Frequency chirping – Holes and Clumps

- Slowly accelerating trapped particles release energy that balances dissipation $\rightarrow \delta \omega \sim \sqrt{t}$
Collisionality not low enough to explain MAST
Yet another collisional effect to analyse

- NBI-produced fast ions slow down due to electron drag
- Dynamical friction (drag) should be included

$$\left. \frac{dF}{dt} \right|_{\text{coll}} = \frac{\alpha^2}{k} \left( \frac{\partial F}{\partial v} - \frac{\partial F_0}{\partial v} \right)$$

- Could this explain the bursting behaviour?
Bump-on-tail model including drag
Bump on tail - formalism

• Linear cold background with sinusoidal field
  \[ E = \frac{1}{2} \left[ \hat{E}(t) e^{i\zeta} + c.c. \right] \]
  \[ \zeta \equiv kx - \omega t \]
  \[ u \equiv kv - \omega \]

\[ \frac{\partial V}{\partial t} = - \frac{|e|}{m} \frac{E}{m} - \nu_c V \]

• Kinetic fast particle population
  \[ F = F_0 + f_0 + \sum_{n=1}^{\infty} \left[ f_n \exp(in\zeta) + c.c. \right] \]

\[ \frac{\partial F}{\partial t} + u \frac{\partial F}{\partial \zeta} - \frac{|e|}{2m} \left[ \hat{E}(t) e^{i\zeta} + c.c. \right] \frac{\partial F}{\partial u} = \frac{dF}{dt} \]_{coll}

• Current from cold background obtained perturbatively using smallness of wave growth and dissipation

\[ \frac{\partial \hat{E}}{\partial t} = - \frac{\omega}{\varepsilon_0 k^2} e \int f_1 du - \gamma_d \hat{E} \]
\[ \gamma_d = \nu_c / 2 \]
Near threshold ordering

• Perturbative approach applied $\rightarrow$ time scales $\tau$ shorter than non-linear bounce period of the wave $\omega_B^{-1}$

$$\omega_B^2 = e k \hat{E} / m$$

• Can be maintained indefinitely if collision frequency is much larger than bounce frequency

• Marginal stability allows truncation of expansion at cubic order in wave amplitude

$$\frac{\partial \hat{E}}{\partial t} \sim \gamma_L \hat{E} \left(1 + c_3 \hat{E}^2 + \ldots \right) - \gamma_d \hat{E}$$
Mode evolution equation

Normalised wave equation. First term gives exponential growth, second term is cubic nonlinearity

\[
\frac{dA}{d\tau} = A(\tau) - \frac{1}{2} \int_0^{\tau/2} dz z^2 A(\tau - z) \int_0^{\tau - 2z} dx e^{-\hat{\nu}^2(2z/3+x) - \hat{\beta}(2z+x) + i\hat{\alpha}^2 z(z+x)}
\]

\[
\hat{\nu} - \text{Diffusion coefficient}
\]

\[
\hat{\beta} - \text{Krook coefficient}
\]

\[
\hat{\alpha} - \text{Drag coefficient}
\]

Drag gives oscillatory behaviour, in contrast to the Krook and diffusive cases.
Sign of cubic nonlinearity

• First term leads to exponential growth, we must have a negative second term to have saturation.

\[
\frac{dA}{d\tau} = A(\tau) - \frac{1}{2} \int\limits_0^{\tau/2} d\tau \frac{z^2}{2} A(\tau - z) \int\limits_0^{\tau - 2z} dx e^{\nu^3 z^2 (2z/3 + x) - \beta(2z + x) + i\alpha^2 z(z + x)} \times
\]

\[
A(\tau - z - x) A^* (\tau - 2z - x)
\]

• For Krook and diffusion – Sign can only flip for low collisionality
• For drag – The oscillatory nature allows the sign to flip often
Diffusion + drag

- For diffusion drag steady state solutions do exist
- For an appreciable amount of drag these solutions become unstable (pitch fork splitting etc.)
- Explosive solutions again when drag dominates
Collisionality affects mode saturation

• This was only a perturbative analysis (cubic order in $E$)

• Fully non-linear treatment requires numerical techniques

• Techniques should take advantage of separation of times scales ($\gamma \ll \omega$) i.e. Use BOT code: Fourier space code that runs in a couple of minutes on a laptop

• What happens in the explosive regime?
Pure drag
Pure drag – No steady state

Holes grow
Clumps die

Pure drag – Growing holes

• Drag collision operator has a slowing down force and a sink

\[ \frac{dF}{dt}_{\text{coll}} = \frac{\alpha^2}{k} \left( \frac{\partial F}{\partial v} - \frac{\partial F_0}{\partial v} \right) \]

• Slowing down + sink returns distribution to equilibrium

• E field however can hold the hole in place working against slowing down force

• Sink still acts to lower F → deeper hole over time

• Deeper hole → larger density perturbation → larger E
Now add some diffusion
Drag + diffusion – Steady state hole
Add a bit more diffusion
Drag + diffusion – Undulating frequency

Lilley et.al PoP, 17, 092305 (2010)
Keep adding diffusion
Drag + diffusion – Hooked frequency chirp

- Hooked frequency chirp seen in BOT
- Also seen in MAST (NBI) and JET (ICRH)
Drag + diffusion – Hooked frequency chirp

Hooks for the holes, clumps die sooner

Lilley et.al PoP, 17, 092305 (2010)
Drag Diffusion competition

Poisson Equation

\[ \delta \omega \omega_B^2 = \gamma_L \omega_B g \]

Diffusion fills, chirping and drag deepen

\[
\frac{\partial g}{\partial t} + \frac{v^3}{\omega_B^2} g = \frac{\partial \delta \omega}{\partial t} + \alpha^2
\]

Power balance

\[
\gamma_d \omega_B^4 = \gamma_L g \omega_B \left( \frac{\partial \delta \omega}{\partial t} + \alpha^2 \right)
\]
0-D Equations

\[ x^2 = y \left( \frac{\partial y}{\partial \tau} + 1 \right) \]

\[ a \frac{\partial (xy)}{\partial \tau} + \frac{y}{x} = \left( 1 + \frac{\partial y}{\partial \tau} \right) \]

- \( x=y=1 \) is steady state
- Unstable for \( a<1 \)
- Stable for \( a>1 \)

Lilley et.al PoP, 17, 092305 (2010)
Generalisation to toroidal systems
Toroidal systems – A first glance (low freq.)

- Phase space resonance is more sophisticated
  \[ u \equiv k v - \omega = 0 \rightarrow \Omega \equiv n \omega_{\phi} (P_{\phi}, E) - p \omega_{\theta} (P_{\phi}, E) - \omega = 0 \]
- Location of resonance varies

\[ E - \frac{\omega}{n} p_{\phi} = \text{const.} \]

Breizman et.al PoP, 4, 1559 (1997)
Toroidal systems – Reduction to 1-D model (low freq.)

- Particle motion along resonance does not lead to large gradients in $F$, so neglect them
- Need projection of motion and collisions across resonance

\[ E - \frac{\omega}{n} p_\phi = \text{const.} \]

Toroidal systems – Reduction to 1-D model (low freq.)

- Transform to coordinates that straighten resonance
- Motion across the resonance 1-D for given \( E \) and \( \mu \)
- Must integrate over all \( E \) and \( \mu \) to get the result

\[
\Omega \equiv n\omega_\phi \left( P_\phi, E \right) - p\omega_\theta \left( P_\phi, E \right) - \omega = 0
\]

Breizman et.al PoP, 4, 1559 (1997)
Chirikov *Phys. Rep.* 52, 263 (1979)
Experimental estimate for MAST and ITER

\[
\frac{\nu_{\text{TAE}}}{\alpha_{\text{TAE}}} \sim \frac{T_e^{3/4} n_e^{1/6}}{B_0^{5/6}}
\]

Drag vs diffusion depends on plasma parameters

\[
\frac{\nu_{\text{TAE}}}{\alpha_{\text{TAE}}} \approx 0.2 - 1.6
\]

MAST - beams - Drag can dominate \(\rightarrow\) explosive

\[
\frac{\nu_{\text{TAE}}}{\alpha_{\text{TAE}}} \approx 1.4
\]

ITER – alphas – drag and diffusion comparable

Lilley et.al PRL, 102, 195003 (2009)
Drag in toroidal systems – HAGIS

HAGIS shows symmetric chirp without collisions

HAGIS shows hooking tendency with drag and Krook collisions
Conclusions and future work

• Drag is destabilising and acts as a seed for energetic particle modes

• Drag gives asymmetry as a lowest order effect, giving steady state holes and hooks

• Possible explanation of NBI vs. ICRH TAE observations

• Experiments underway (ITPA) to get drag/diffusion database

• HAGIS now includes drag
BOT Code

• Email bumpontail@gmail.com from your work email
• Please give your name, institution and your position
• It is free for you to use, modify and also distribute, but I encourage others to contact me for the code so that I can send updates as they become available