Imperial College London

CHALMERS

# Energetic particle modes: from bump on tail to tokamak plasmas

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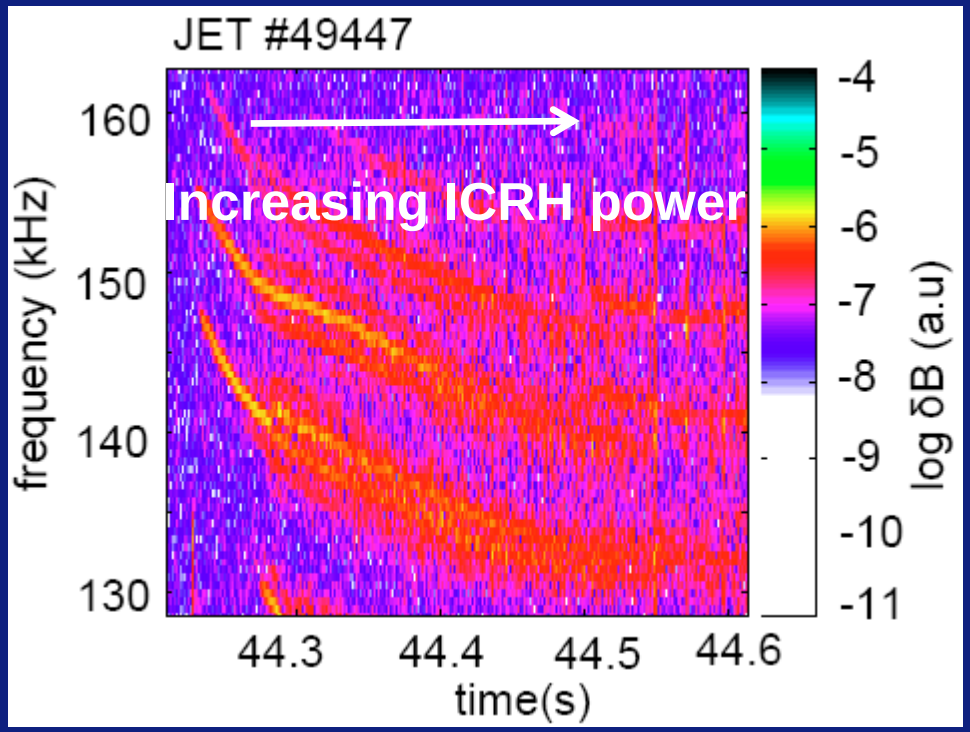
# Outline

- Experimental observations
- Marginal stability and the role of collisions
- Bump-on-tail model including drag
- Generalisation to toroidal systems

# Experimental observations

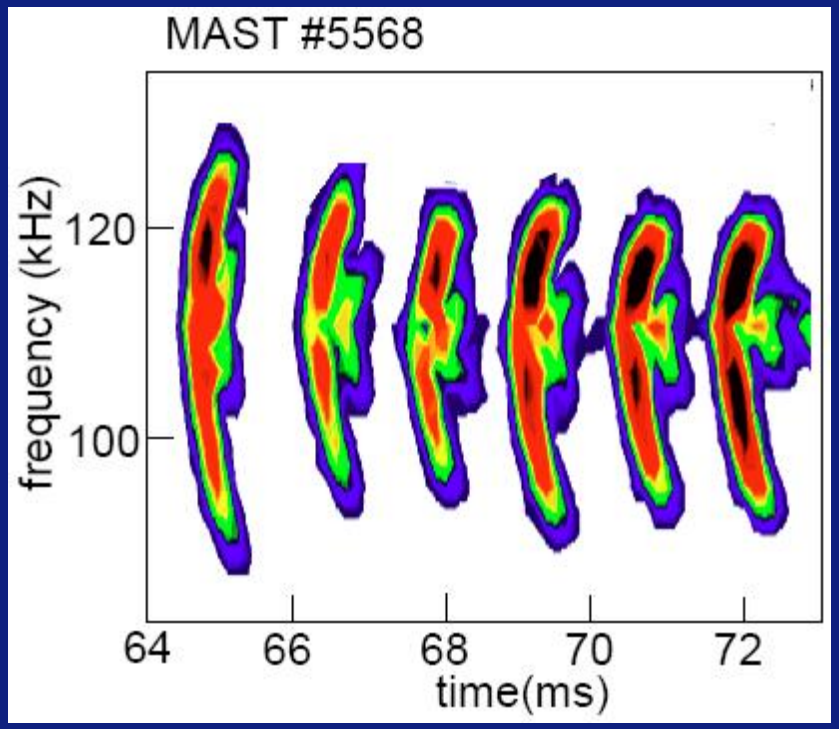
# Question: Why disparity between NBI and ICRH?

## ICRH drive (JET)




Heeter et.al PRL 85, 3177 (2000)

## NBI drive (MAST)



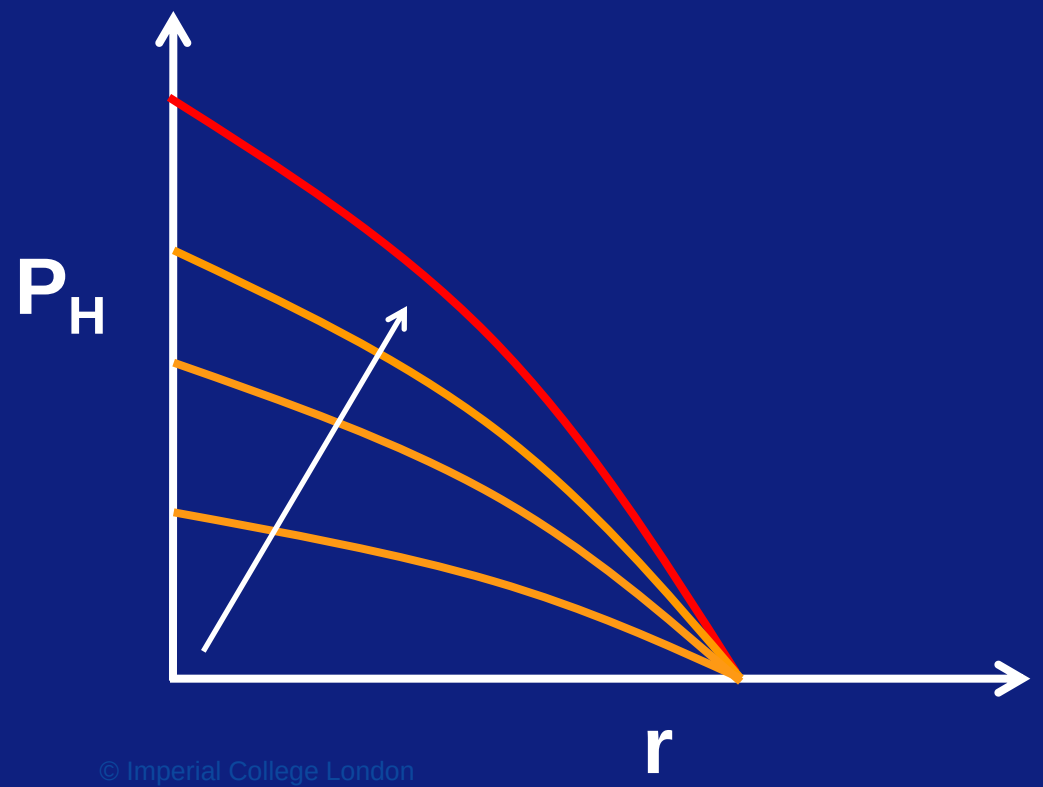

Pinches et.al PPCF, 46, S47 (2004)

# Marginal stability and the role of collisions

# Marginal stability

- System evolves through a threshold

$$\left|\gamma_L - \gamma_d\right| \ll \gamma_L, \gamma_d$$



# Collisionality - previous analysis

- Marginal stability allows collisions to compete with mode growth

$$\left|\gamma_L - \gamma_d\right| \sim \nu_{\text{eff}}$$

- Krook and diffusion were studied in bump on tail

$$\left.\frac{dF}{dt}\right|_{\text{coll}} = \beta\left(F - F_0\right) \qquad \left.\frac{dF}{dt}\right|_{\text{coll}} = \frac{\nu^3}{k^2}\left(\frac{\partial^2 F}{\partial \mathrm{v}^2} - \frac{\partial^2 F_0}{\partial \mathrm{v}^2}\right)$$

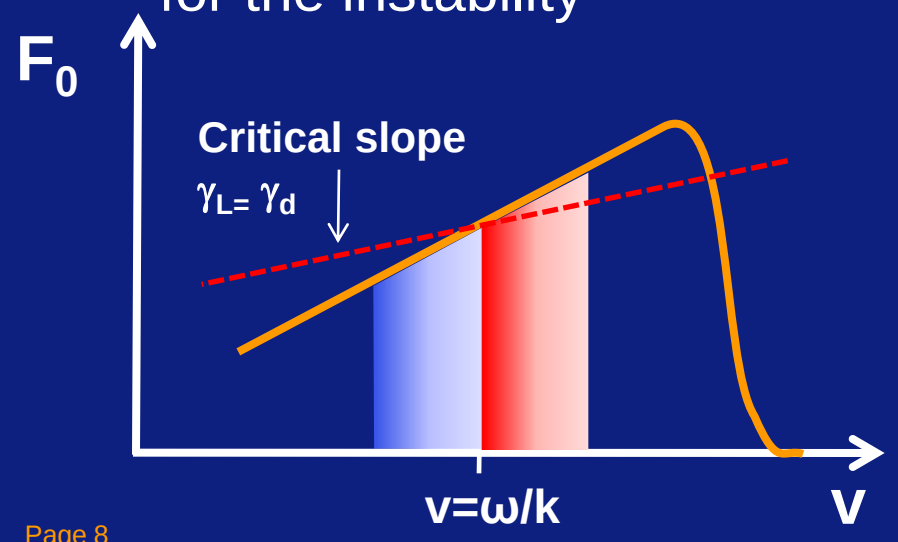
$$\nu_{\text{eff}} = \max\left\{\beta, \nu\right\}$$

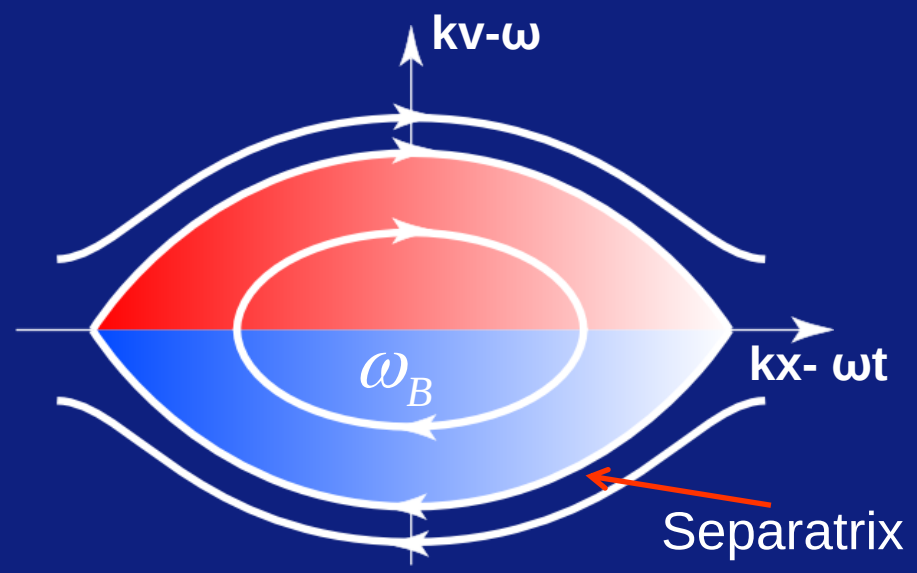
Berk, Breizman et.al PRL, 76, 1256 (1996)

Breizman, Berk et.al PoP, 4 1559 (1997)

# Bump on tail - Basic ingredients

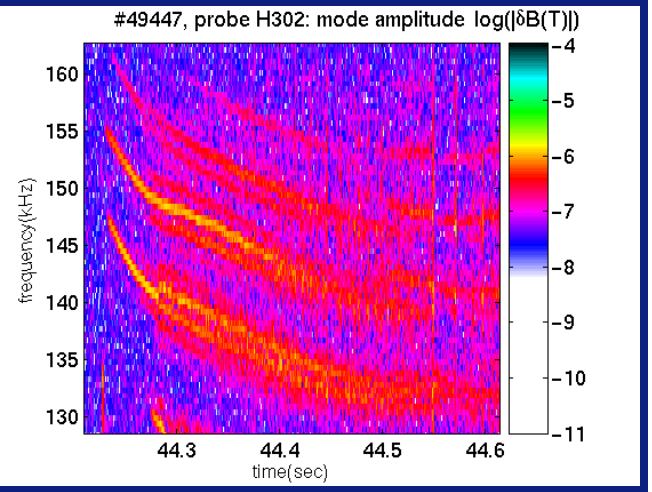
- Particle injection and effective collisions, $\nu_{eff}$, create an inverted distribution of energetic particles $F_0(v)$
- Discrete spectrum of unstable electrostatic modes
- Instability drive, $\gamma_L \sim dF_0/dv$, due to wave-particle resonance ($\omega$-kv=0)
- Background dissipation rate, $\gamma_d$, determines the critical gradient for the instability

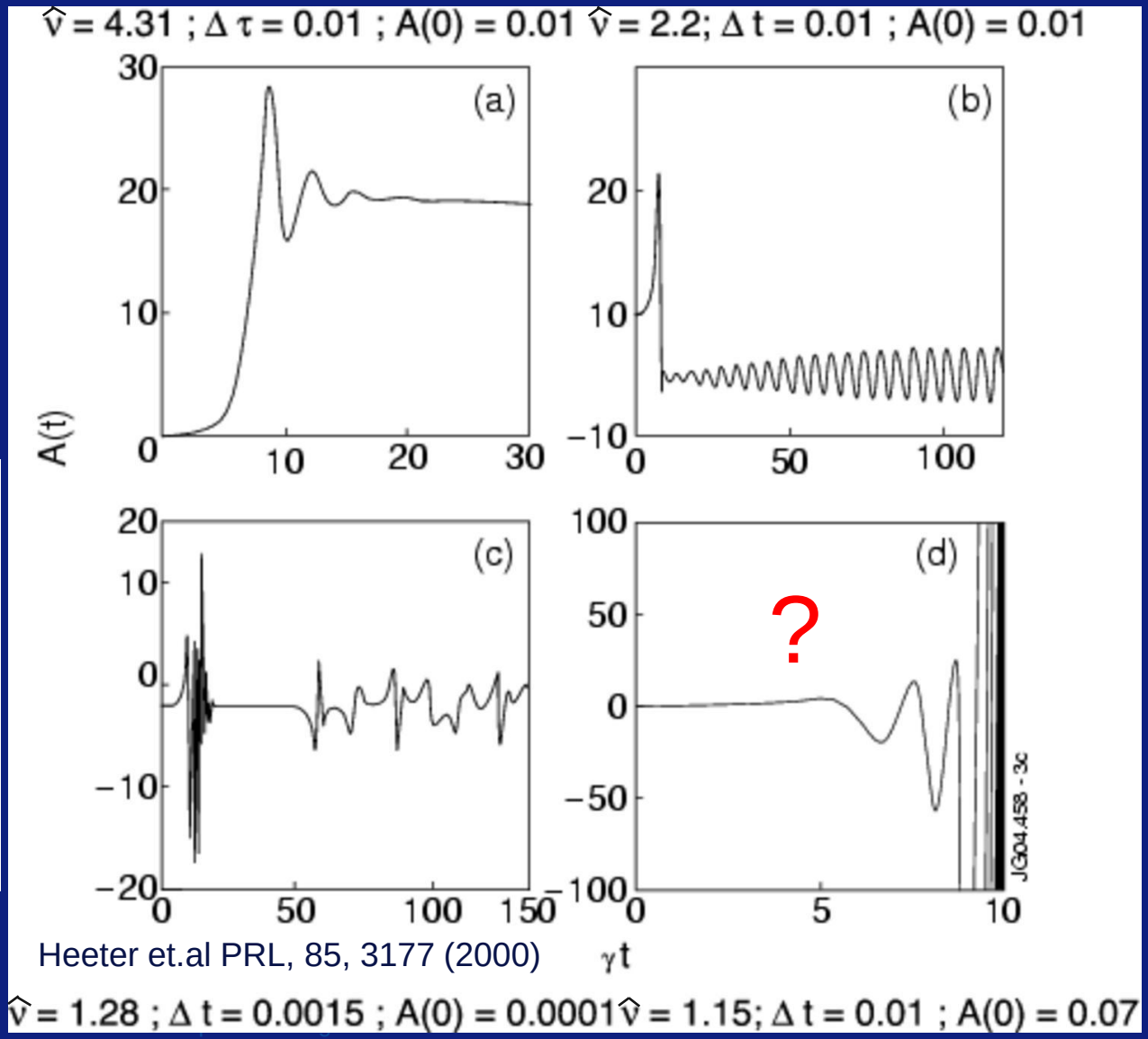


# Collisionality affects mode saturation

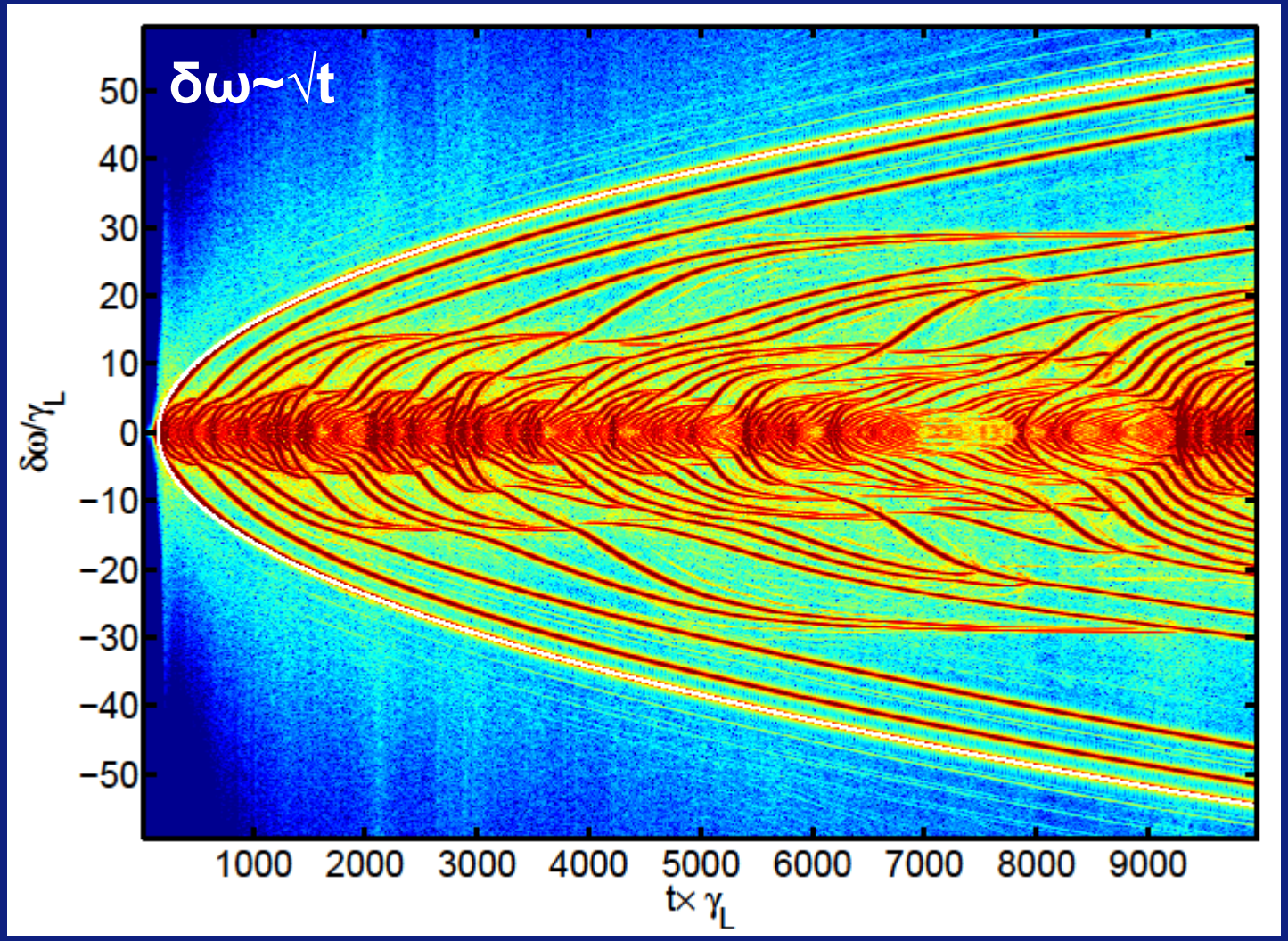
$$\hat{\nu} = \frac{\nu}{\gamma_L - \gamma_d}$$



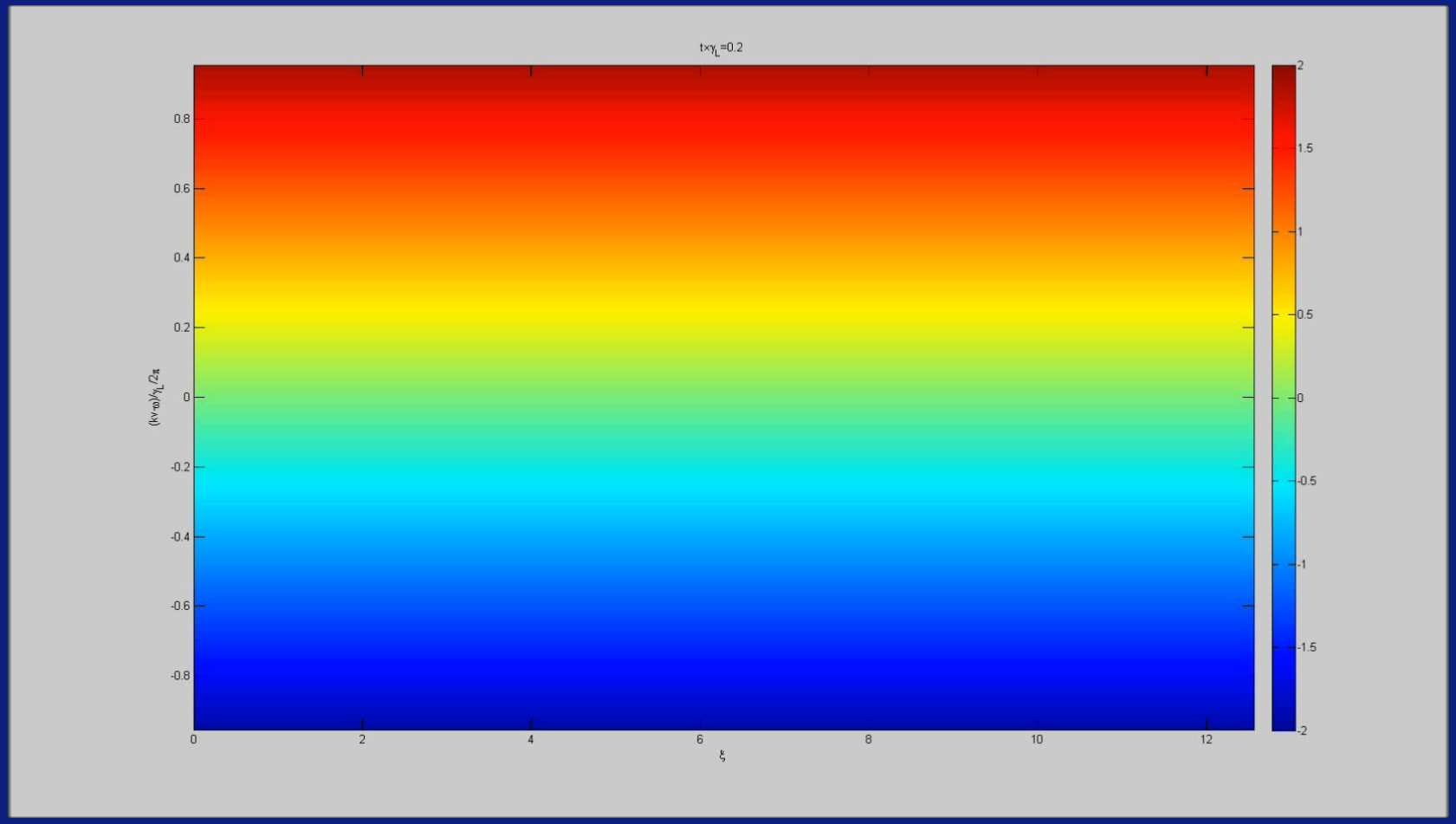

Heeter et.al PRL, 85, 3177 (2000)

# Low collisionality – Frequency chirping



# Frequency chirping – Holes and Clumps

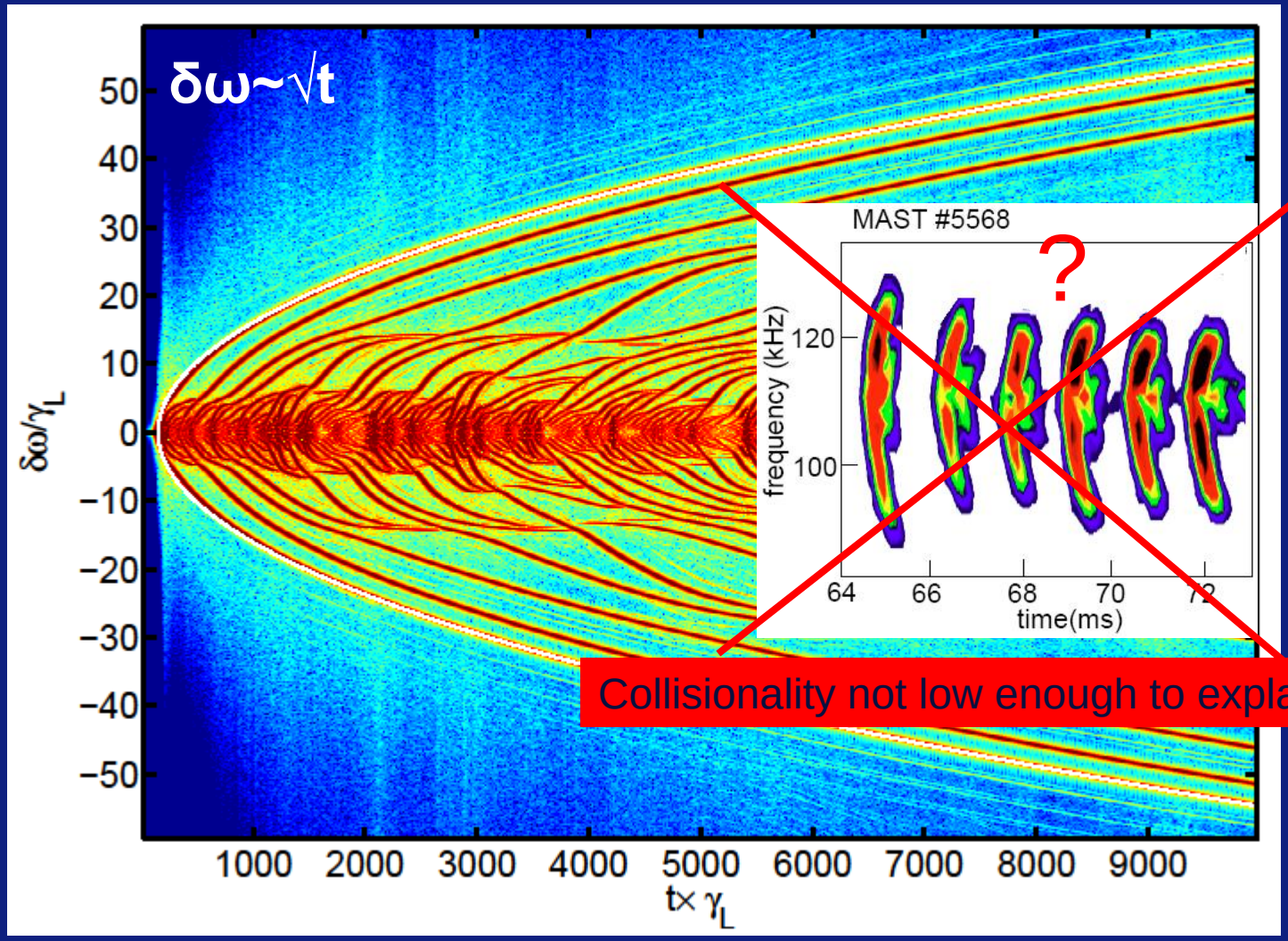
kv-ω




kx-ωt

- Slowly accelerating trapped particles release energy that balances dissipation → **δω~√t**

# Low collisionality – Frequency chirping




Collisionality not low enough to explain MAST

# Yet another collisional effect to analyse

- NBI-produced fast ions slow down due to electron drag
- Dynamical friction (drag) should be included

$$\left.\frac{dF}{dt}\right|_{\mathrm{coll}} = \frac{\alpha^2}{k}\left(\frac{\partial F}{\partial \mathrm{v}} - \frac{\partial F_0}{\partial \mathrm{v}}\right)$$

- Could this explain the bursting behaviour?

# Bump-on-tail model including drag

# Bump on tail - formalism

- Linear cold background with sinusoidal field $E = \frac{1}{2}\left[\hat{E}(t)e^{i\zeta} + \text{c.c.}\right]$

$$\boxed{\frac{\partial V}{\partial t} = -\frac{|e|E}{m} - \nu_c V}$$

$$\zeta \equiv kx - \omega t$$
$$u \equiv kv - \omega$$

- Kinetic fast particle population $F = F_0 + f_0 + \sum_{n=1}^{\infty}\left[f_n \exp(in\zeta) + c.c.\right]$

$$\boxed{\frac{\partial F}{\partial t} + u\frac{\partial F}{\partial \zeta} - \frac{|e|k}{2m}\left[\hat{E}(t)e^{i\zeta} + \text{c.c.}\right]\frac{\partial F}{\partial u} = \left.\frac{dF}{dt}\right|_{\text{coll}}}$$

- Current from cold background obtained perturbatively using smallness of wave growth and dissipation

$$\boxed{\frac{\partial \hat{E}}{\partial t} = -\frac{\omega}{\varepsilon_0 k^2} e\int f_1 du - \gamma_d \hat{E}}$$

$$\gamma_d = \nu_c / 2$$

# Near threshold ordering

- Perturbative approach applied → time scales $\tau$ shorter than non-linear bounce period of the wave $\omega_B^{-1}$

$$\omega_B^2 = ek\hat{E} / m$$

- Can be maintained indefinitely if collision frequency is much larger than bounce frequency

- Marginal stability allows truncation of expansion at cubic order in wave amplitude

$$\frac{\partial \hat{E}}{\partial t} \sim \gamma_L \hat{E}\left(1 + c_3 \hat{E}^2 + \ldots\right) - \gamma_d \hat{E}$$

# Mode evolution equation

Normalised wave equation. First term gives exponential growth, second term is cubic nonlinearity

$$\frac{dA}{d\tau} = A(\tau) - \frac{1}{2}\int_0^{\tau/2} dz\, z^2 A(\tau - z) \int_0^{\tau-2z} dx\, e^{-\hat{\nu}^3 z^2 (2z/3+x) - \hat{\beta}(2z+x) + i\hat{\alpha}^2 z(z+x)} \times$$

$$A(\tau - z - x)\, A^*(\tau - 2z - x)$$

$\hat{\nu}$ - Diffusion coefficient

$\hat{\beta}$ - Krook coefficient

$\hat{\alpha}$ - Drag coefficient

Drag gives oscillatory behaviour, in contrast to the Krook and diffusive cases.

# Sign of cubic nonlinearity

- First term leads to exponential growth, we must have a negative second term to have saturation.

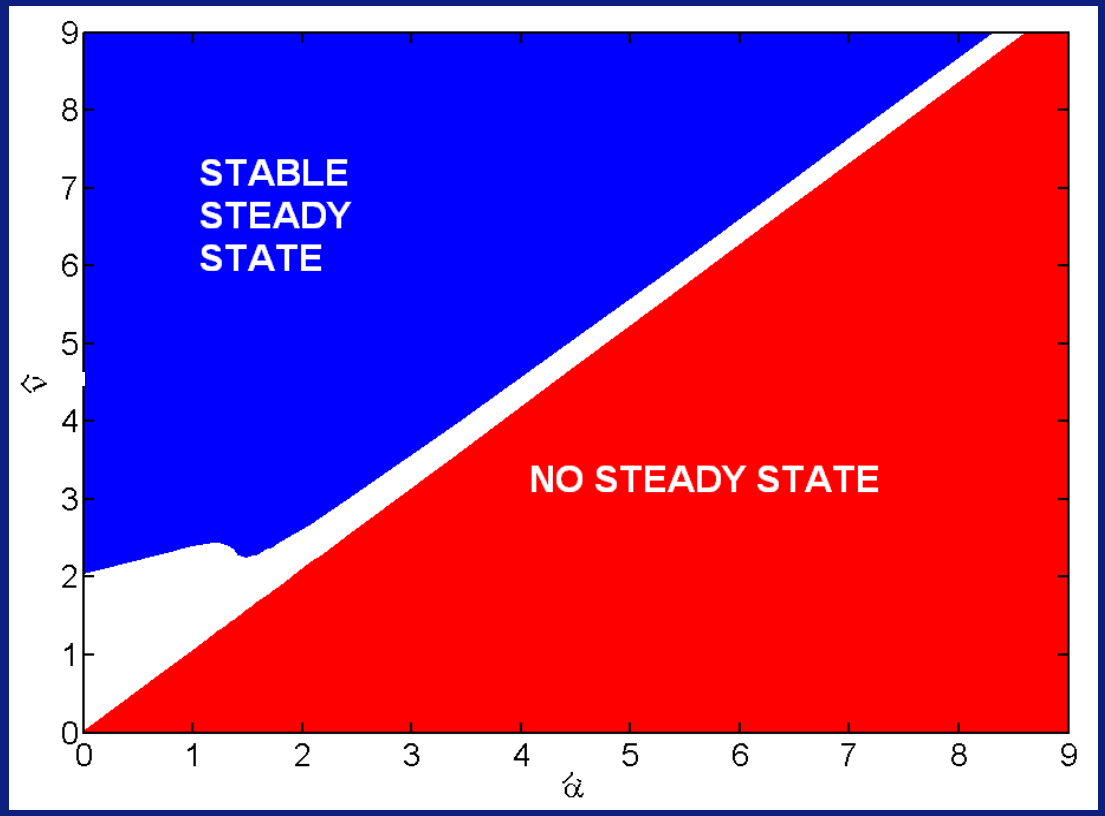
$$\frac{dA}{d\tau} = A(\tau) - \frac{1}{2}\int_0^{\tau/2} dz\, z^2 A(\tau - z) \int_0^{\tau-2z} dx\, e^{-\hat{\nu}^3 z^2 (2z/3+x) - \hat{\beta}(2z+x) + i\hat{\alpha}^2 z(z+x)} \times$$

$$A(\tau - z - x) A^*(\tau - 2z - x)$$

- For Krook and diffusion – Sign can only flip for low collisionality
- For drag – The oscillatory nature allows the sign to flip often

# Diffusion + drag

- For diffusion drag steady state solutions do exist
- For an appreciable amount of drag these solutions become unstable (pitch fork splitting etc.)
- Explosive solutions again when drag dominates

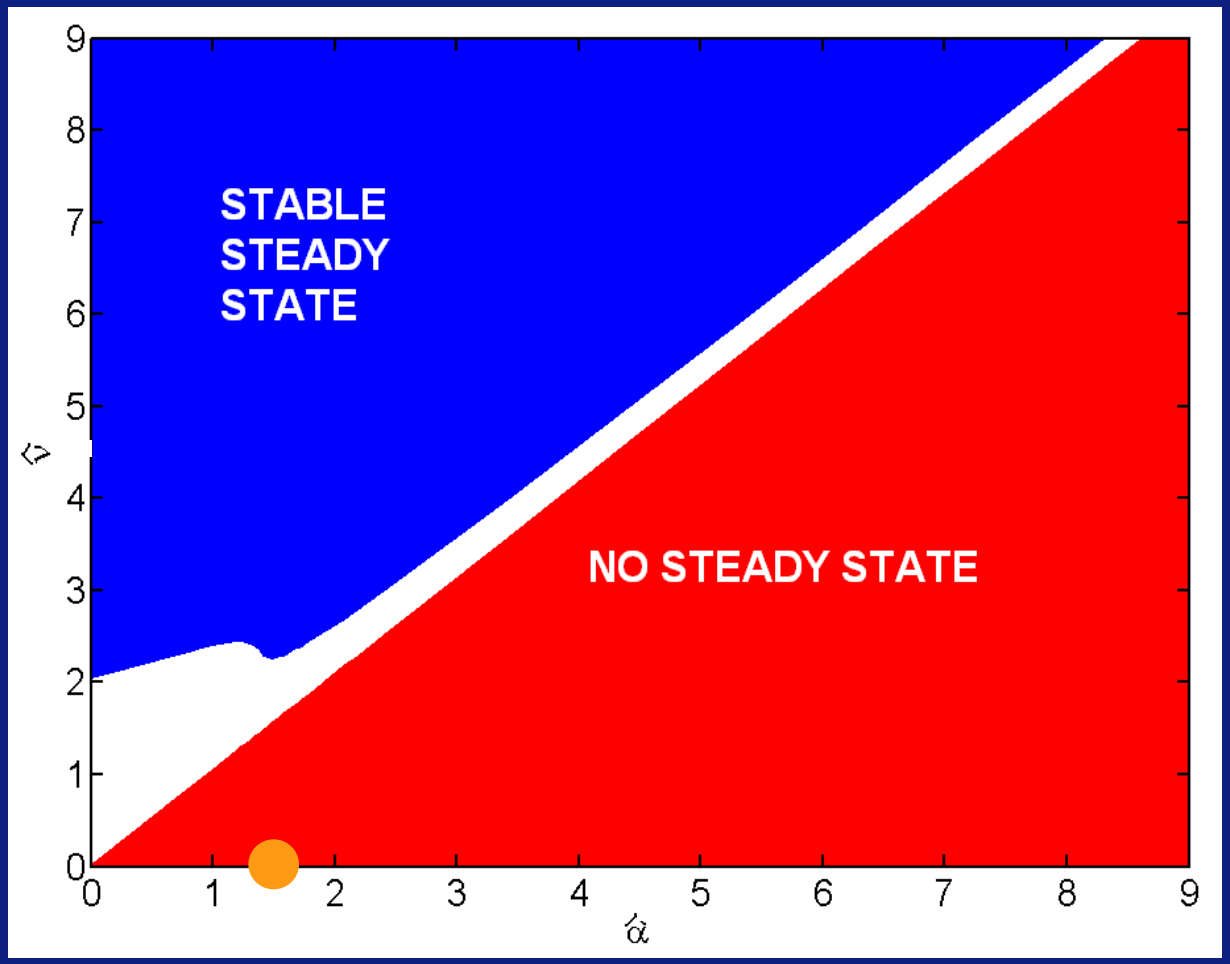



Lilley et.al PRL, 102, 195003 (2009)

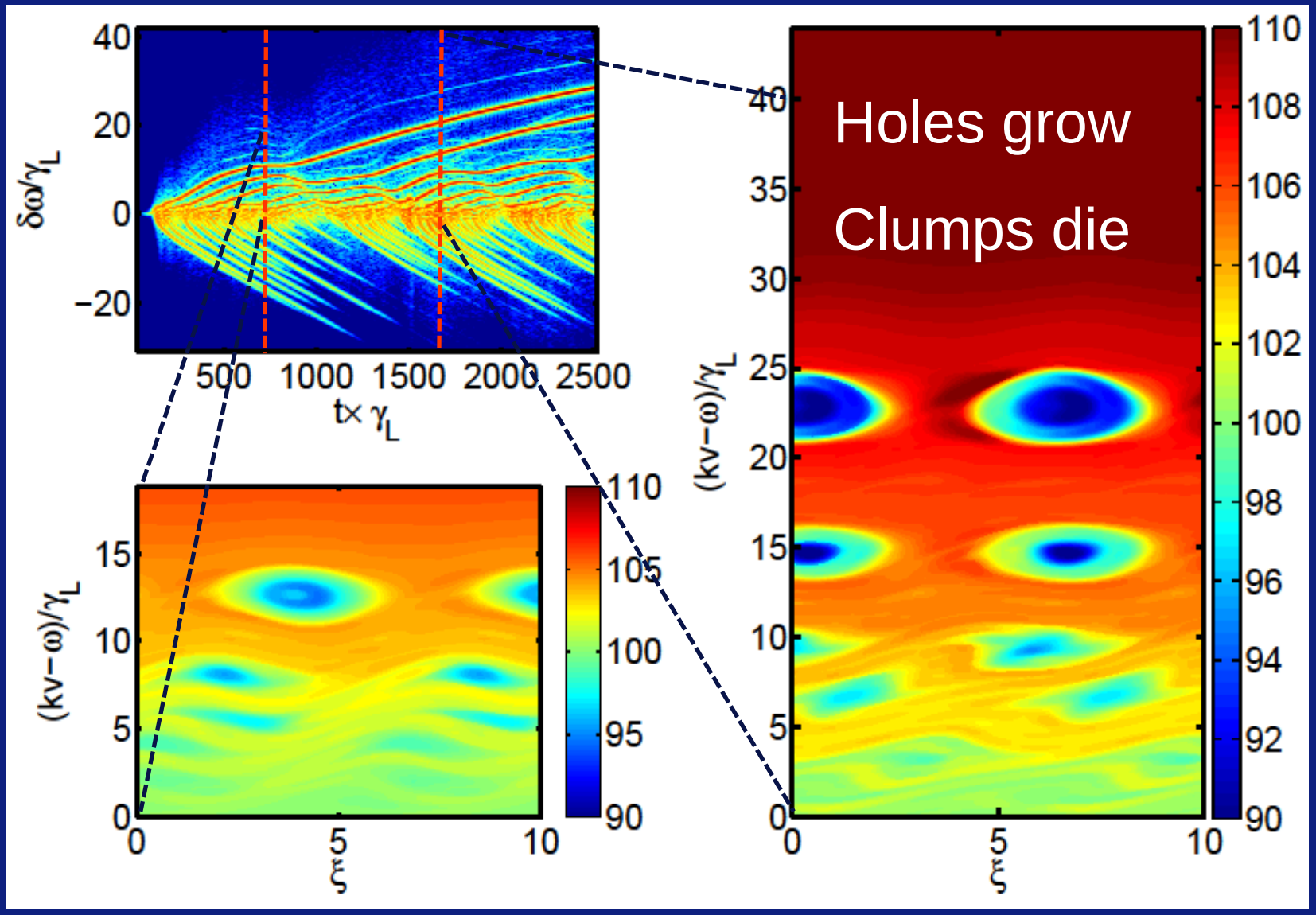
# Collisionality affects mode saturation

- This was only a perturbative analysis (cubic order in E)
- Fully non-linear treatment requires numerical techniques
- Techniques should take advantage of separation of times scales $(\gamma \ll \omega)$ i.e. Use BOT code: Fourier space code that runs in a couple of minutes on a laptop
- What happens in the explosive regime?

# Pure drag


STABLE
STEADY
STATE
NO STEADY STATE
0
1
2
3
4
5
6
7
8
9
0
1
2
3
4
5
6
7
8
9

# Pure drag – No steady state






Lilley et.al PoP, 17, 092305 (2010)

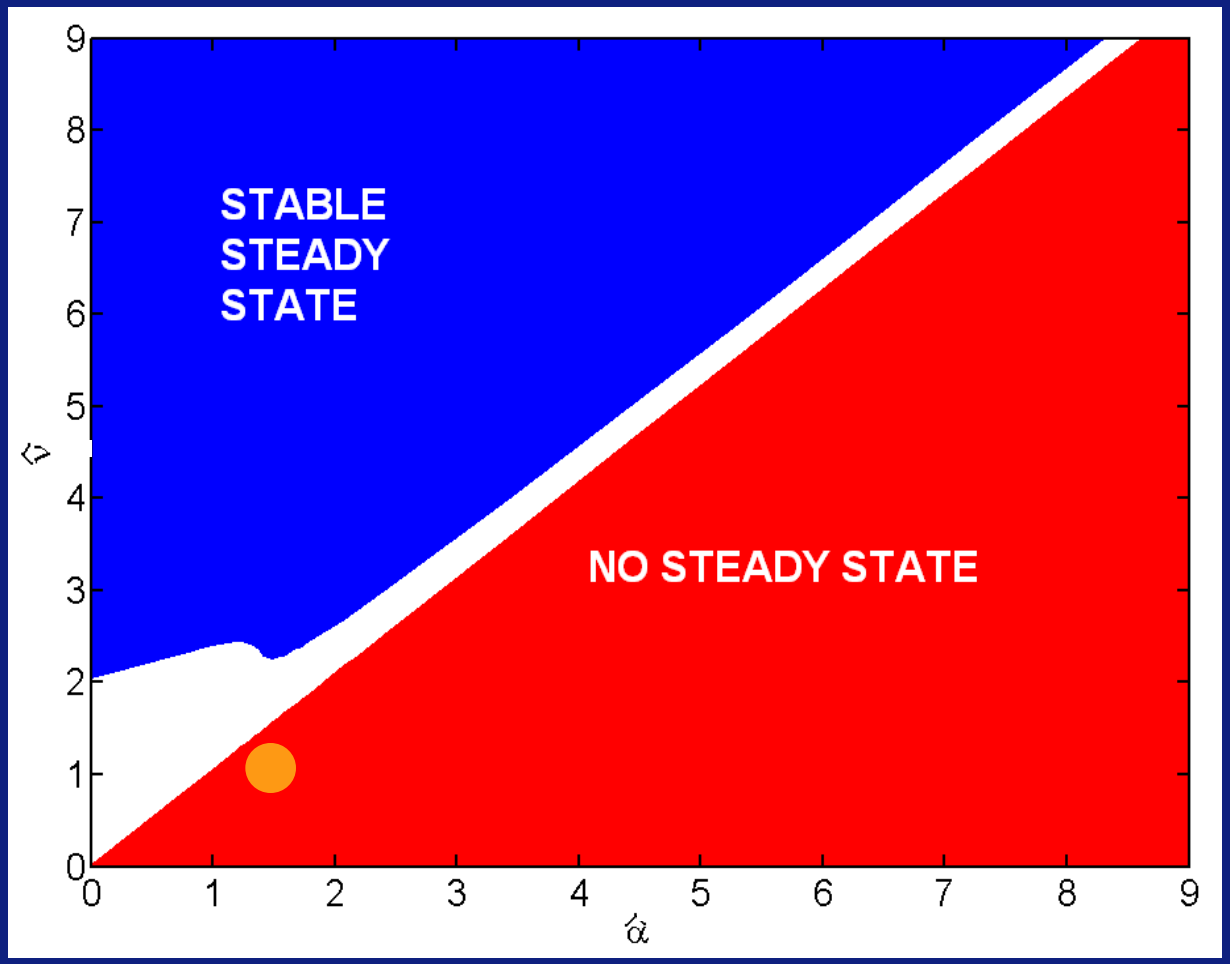
# Pure drag – Growing holes

- Drag collision operator has a slowing down force and a sink

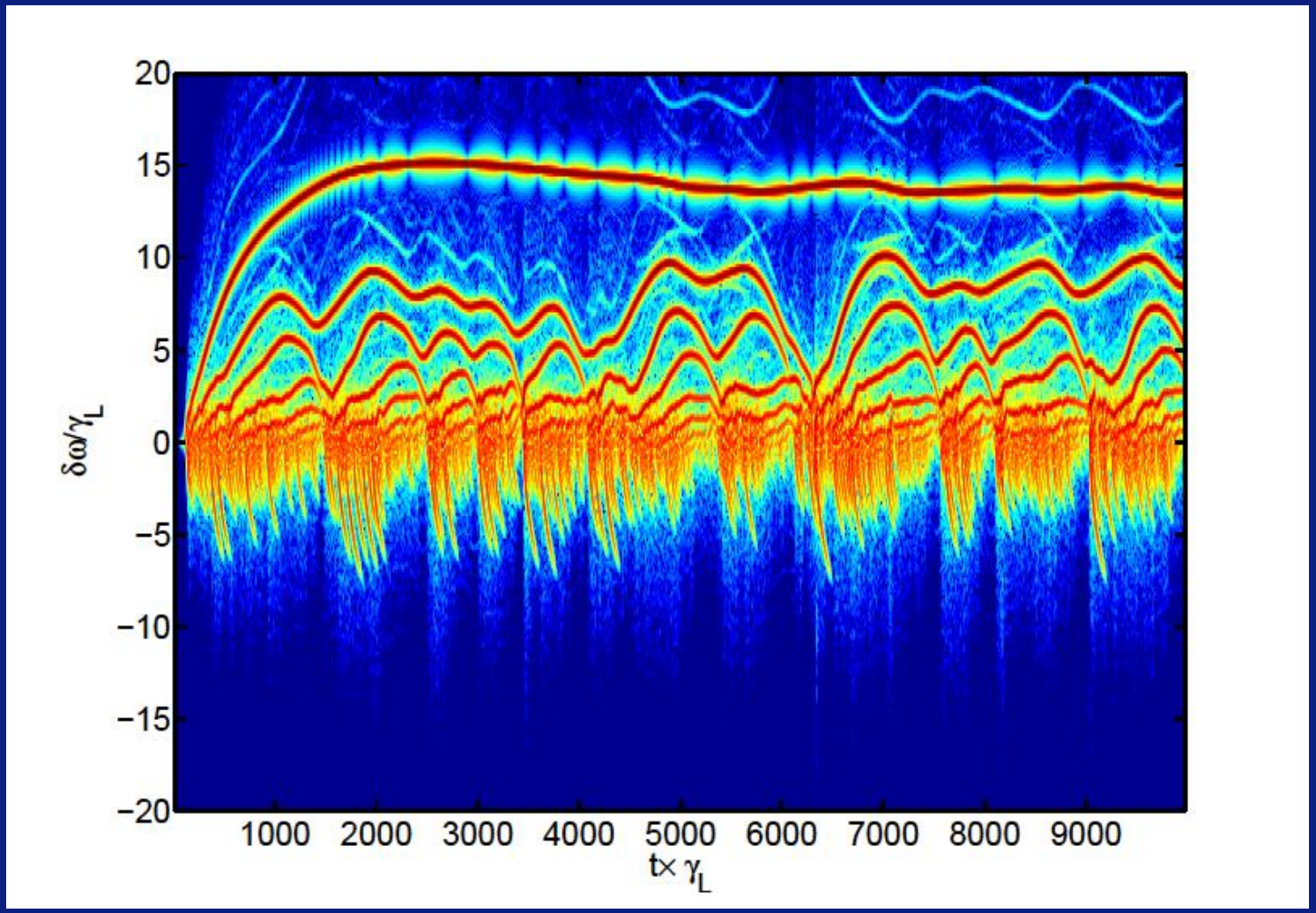
$$\left.\frac{dF}{dt}\right|_{\mathrm{coll}} = \frac{\alpha^2}{k}\left(\frac{\partial F}{\partial \mathbf{v}} - \frac{\partial F_0}{\partial \mathbf{v}}\right)$$

- Slowing down + sink returns distribution to equilibrium
- E field however can hold the hole in place working against slowing down force
- Sink still acts to lower F → deeper hole over time
- Deeper hole → larger density perturbation → larger E

# Now add some diffusion

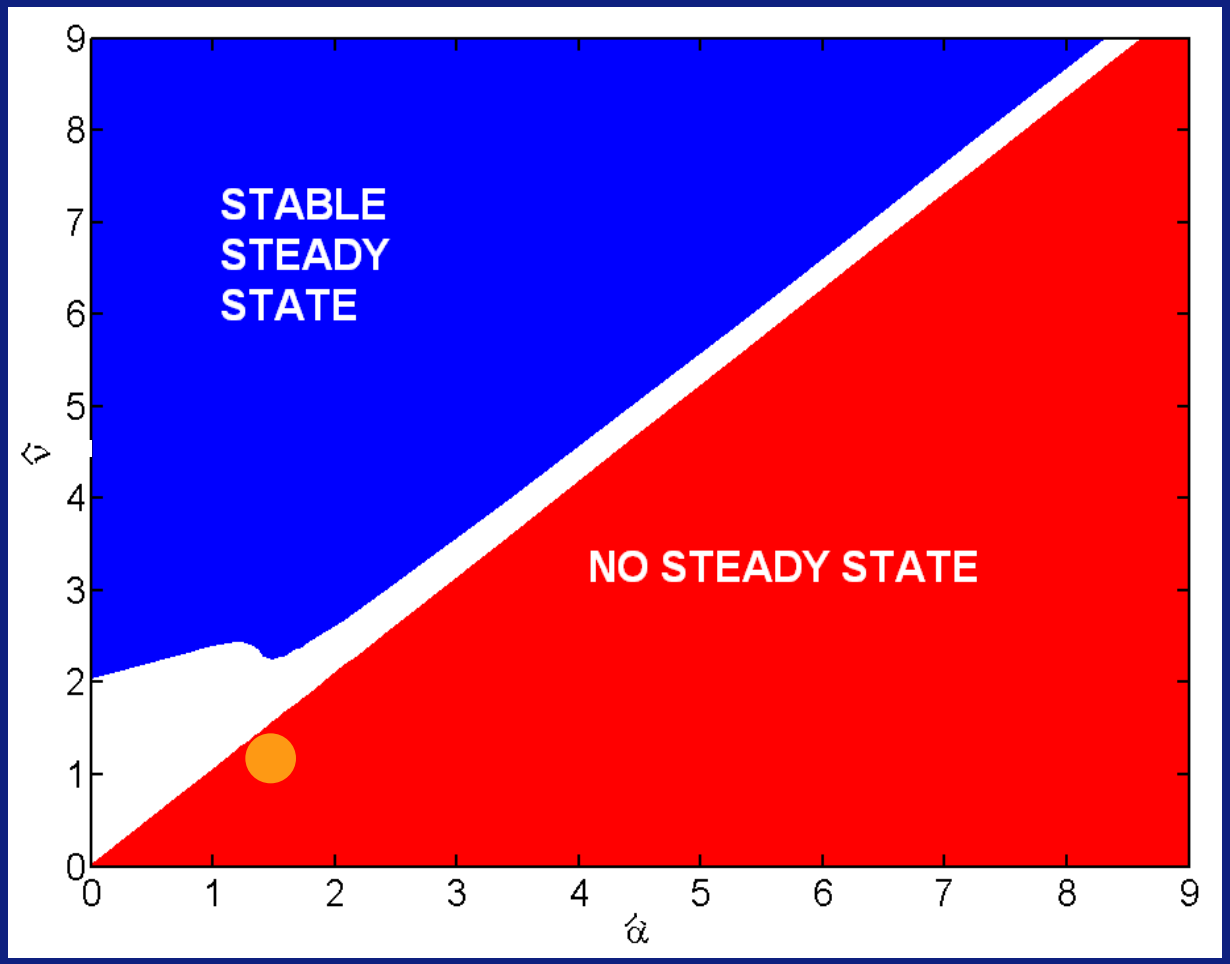


# Drag + diffusion – Steady state hole

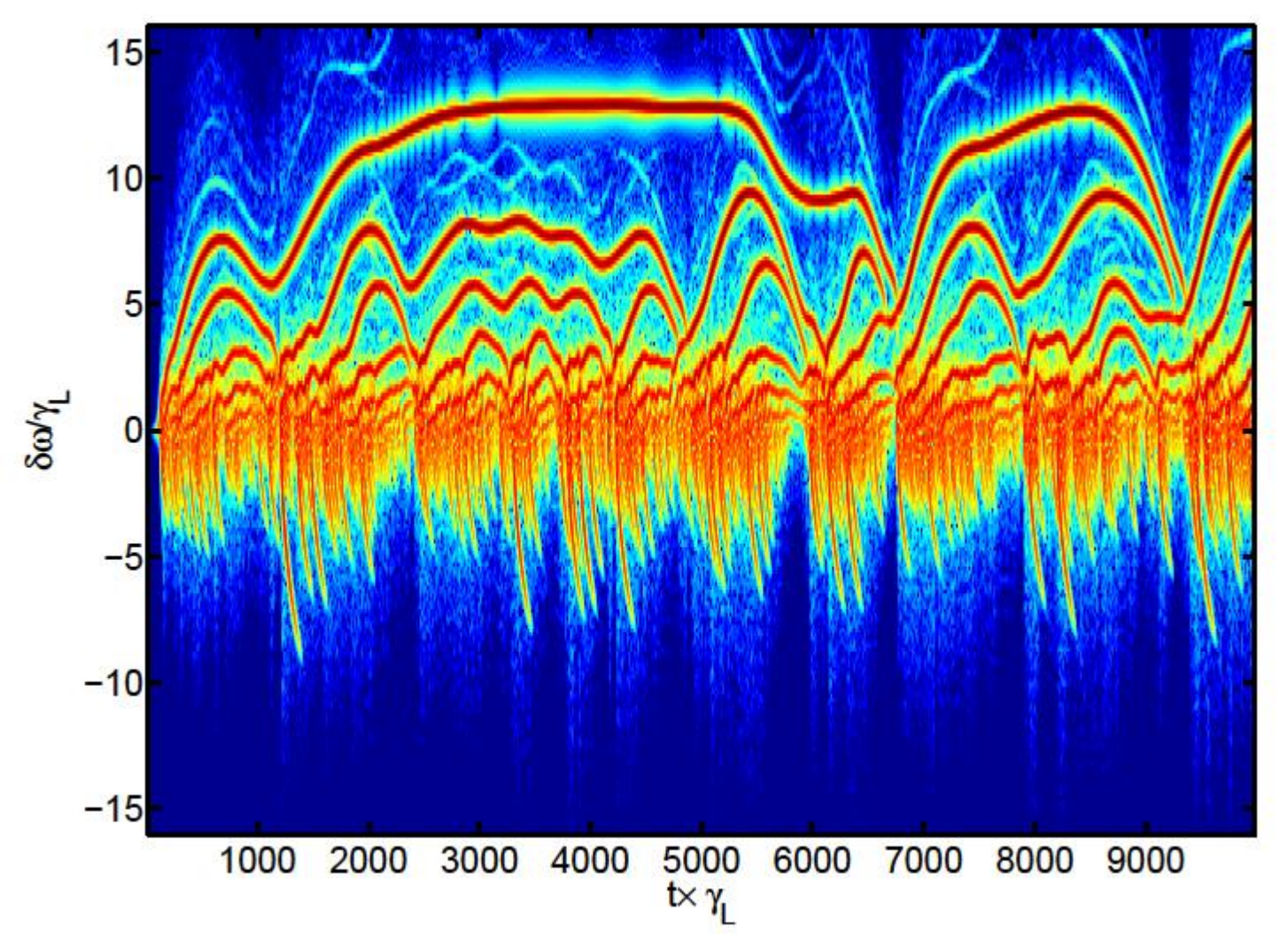



Lilley et.al PoP, 17, 092305 (2010)

# Add a bit more diffusion

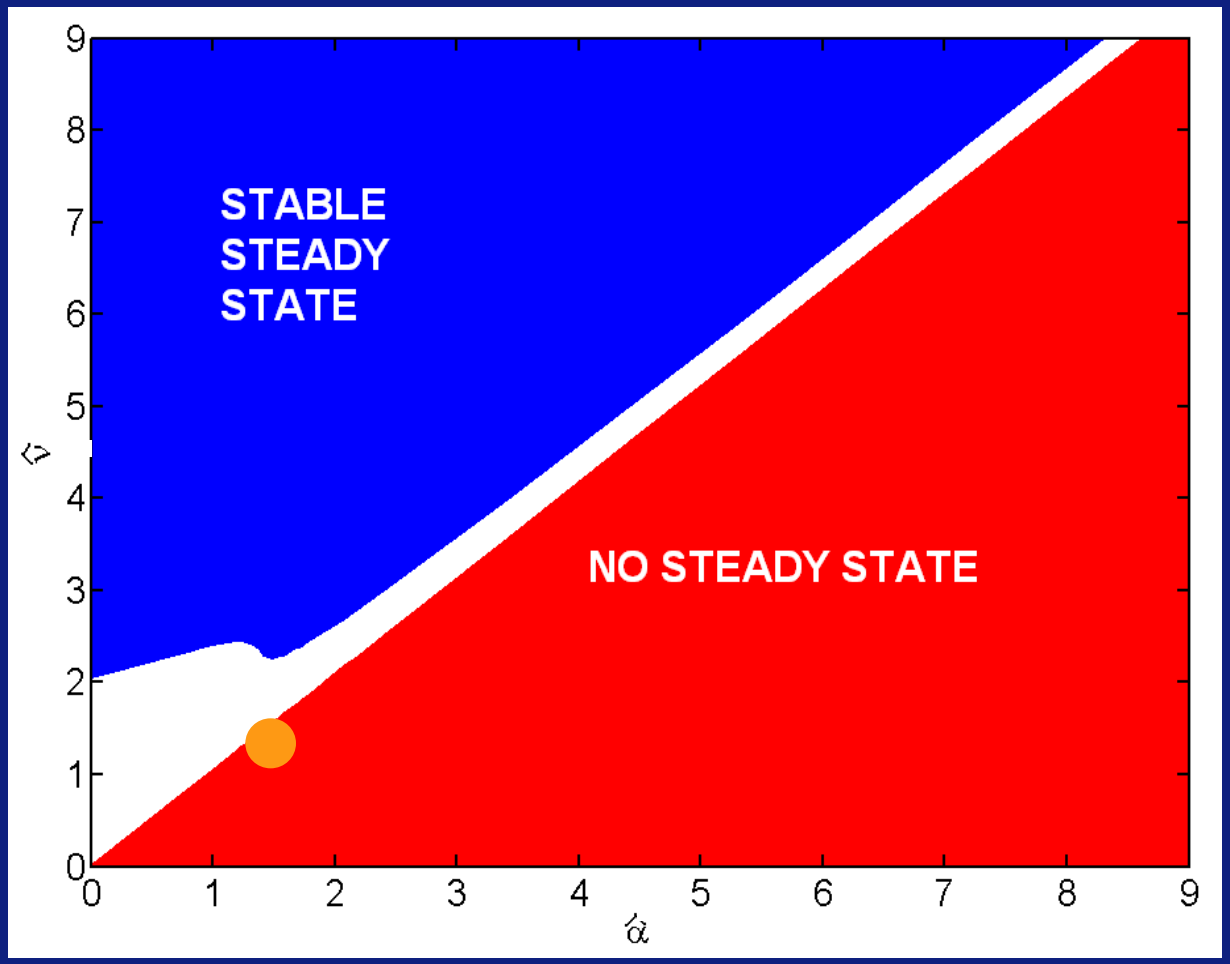


# Drag + diffusion – Undulating frequency

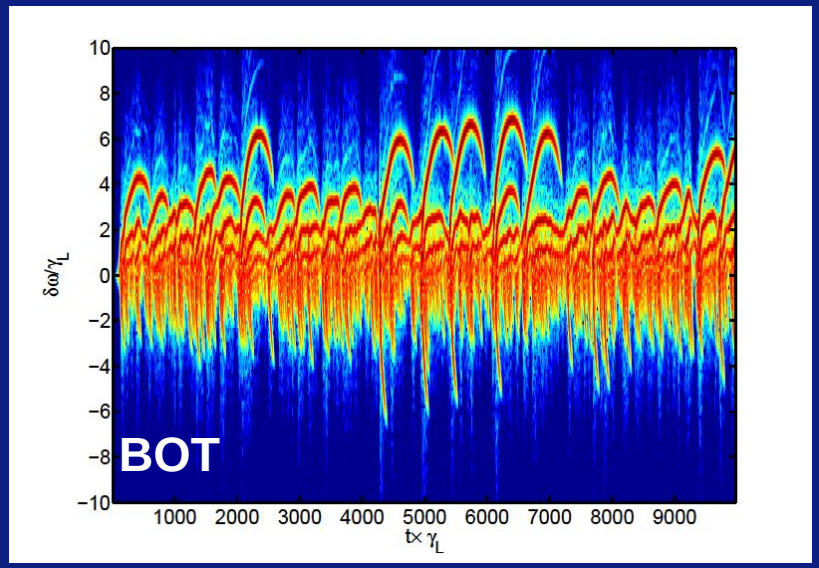



Lilley et.al PoP, 17, 092305 (2010)

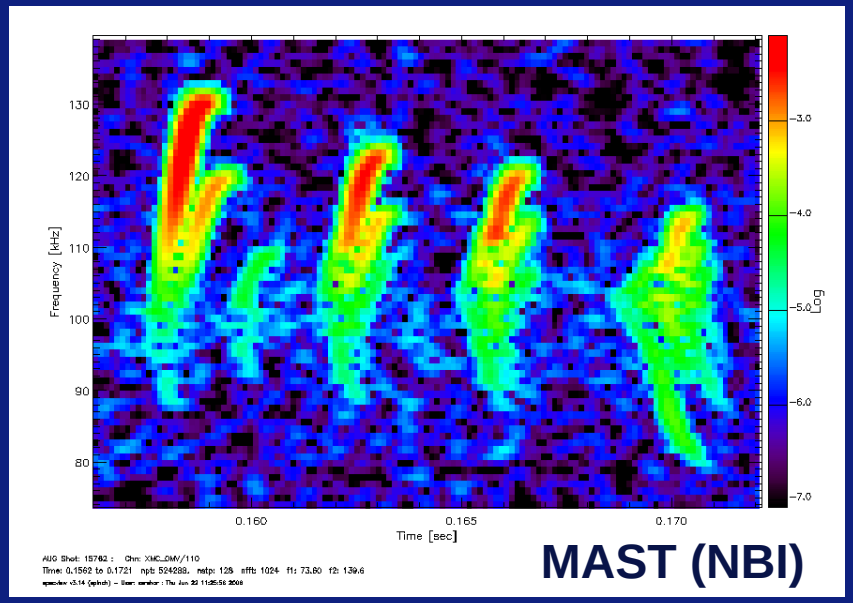
# Keep adding diffusion

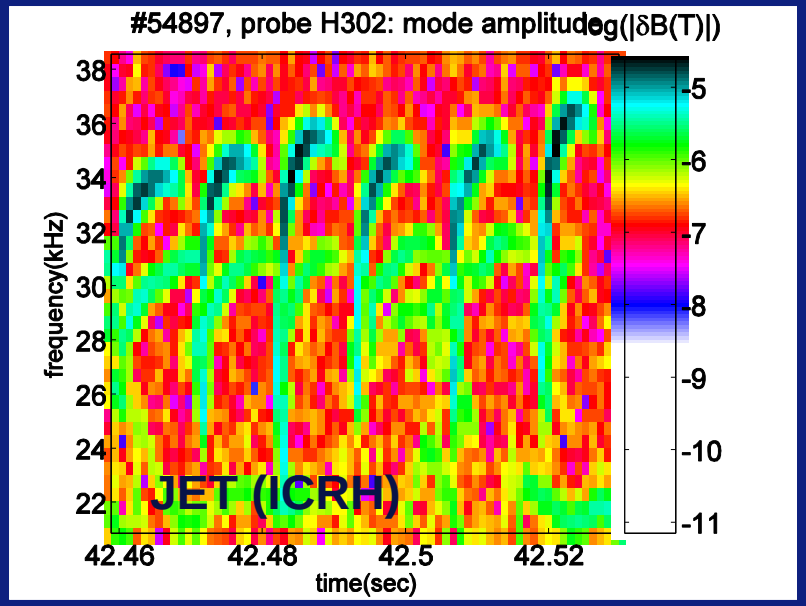


# Drag + diffusion – Hooked frequency chirp

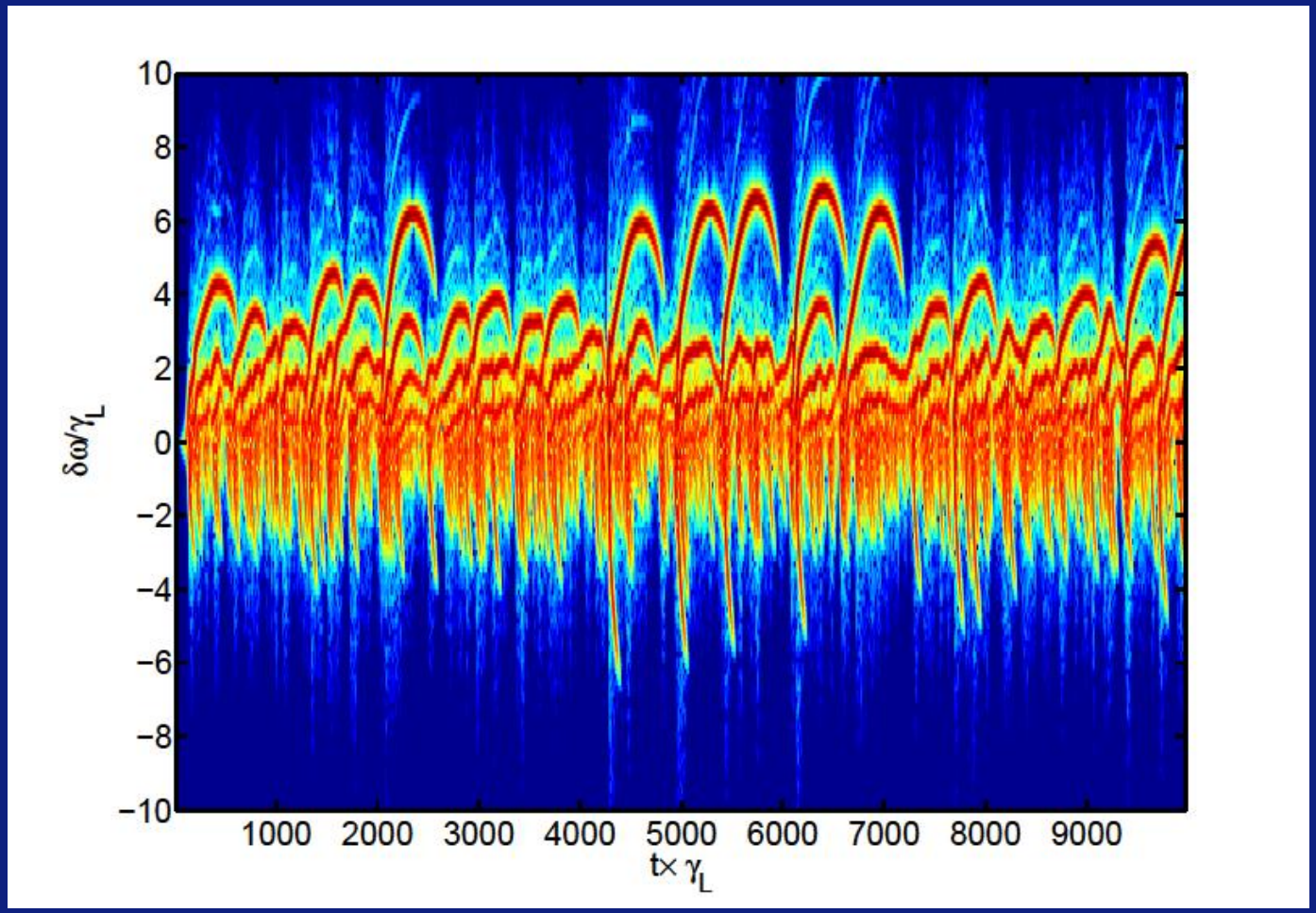



- Hooked frequency chirp seen in BOT
- Also seen in MAST (NBI) and JET (ICRH)



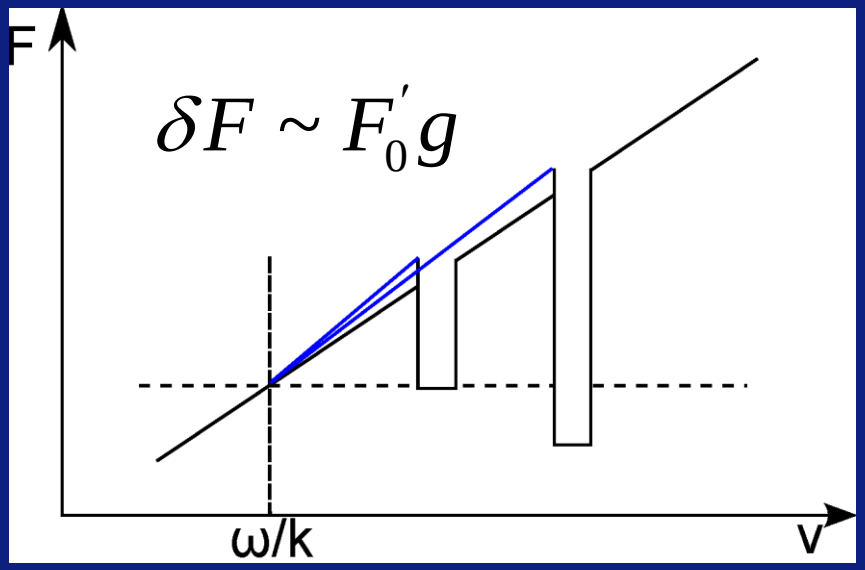
# Drag + diffusion – Hooked frequency chirp




Hooks for the holes, clumps die sooner

Lilley et.al PoP, 17, 092305 (2010)

# Drag Diffusion competition




Poisson Equation

$$\delta\omega\omega_B^2 = \gamma_L \omega_B g$$

Diffusion fills, chirping and drag deepen

$$\frac{\partial g}{\partial t} + \frac{\nu^3}{\omega_B^2} g = \frac{\partial \delta\omega}{\partial t} + \alpha^2$$

Power balance

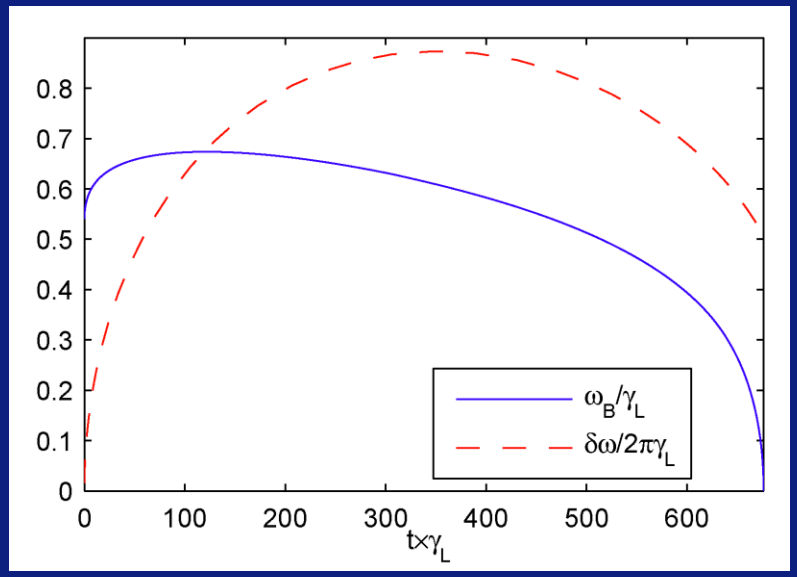
$$\gamma_d \omega_B^4 = \gamma_L g \omega_B \left( \frac{\partial \delta\omega}{\partial t} + \alpha^2 \right)$$

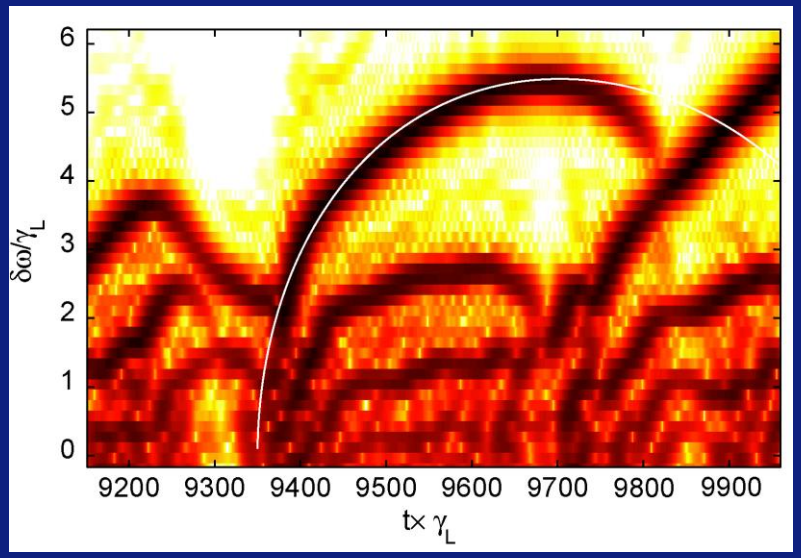
# 0-D Equations

$$x^2 = y\left(\frac{\partial y}{\partial \tau} + 1\right)$$

$$a\frac{\partial(xy)}{\partial \tau} + \frac{y}{x} = \left(1 + \frac{\partial y}{\partial \tau}\right)$$

- x=y=1 is steady state
- Unstable for a<1
- Stable for a>1




Lilley et.al PoP, 17, 092305 (2010)

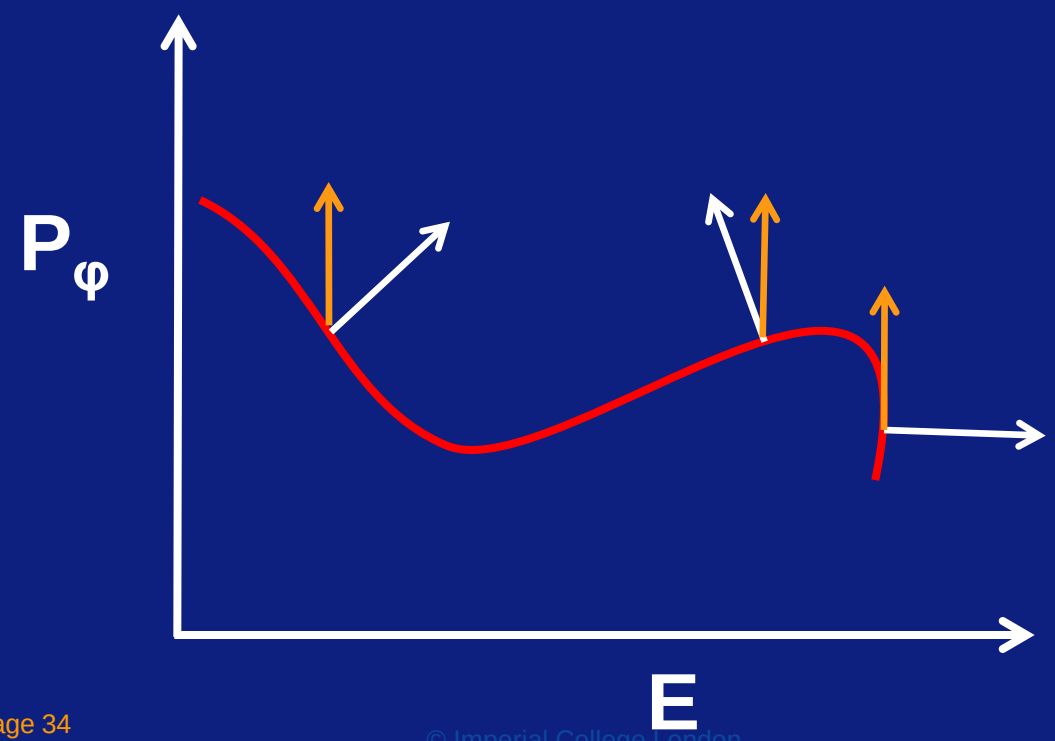
# Generalisation to toroidal systems

# Toroidal systems – A first glance (low freq.)

- Phase space resonance is more sophisticated

$$u \equiv k\mathrm{v} - \omega = 0 \;\rightarrow\; \Omega \equiv n\omega_{\varphi}\left(P_{\varphi}, E\right) - p\omega_{\theta}\left(P_{\varphi}, E\right) - \omega = 0$$

- Location of resonance varies




**Resonance**

**Motion across resonance**

**Motion due to wave**

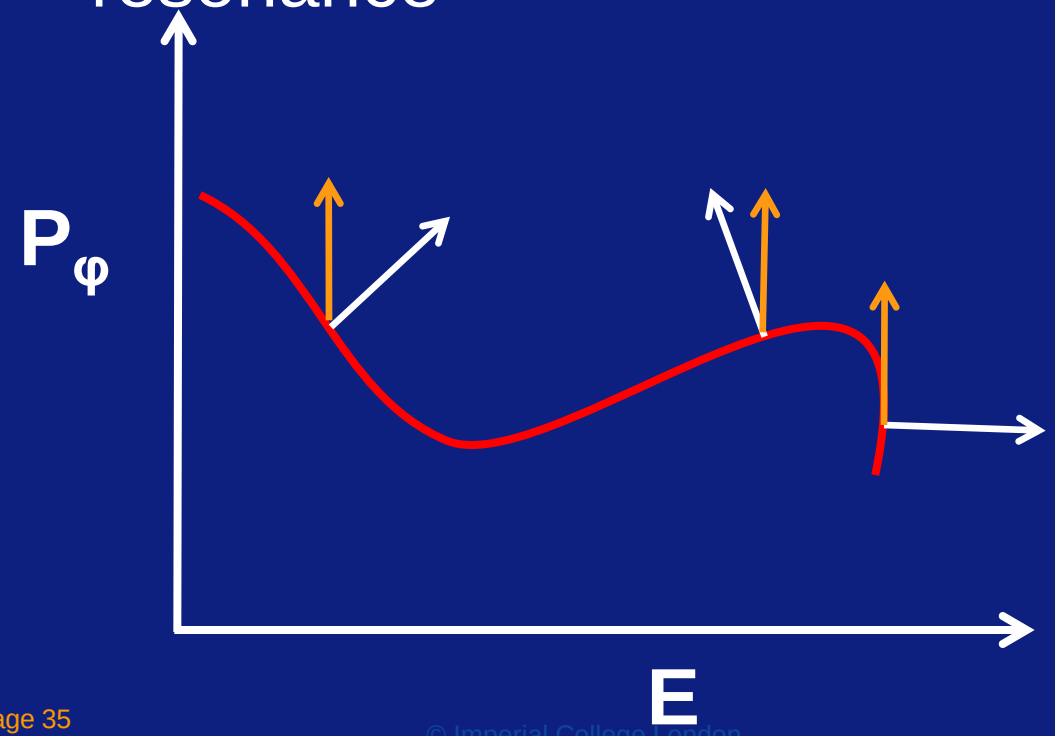
$$E - \frac{\omega}{n} p_{\varphi} = \mathrm{const.}$$

Breizman et.al PoP, 4, 1559 (1997)

Chirikov *Phys. Rep*. 52 263 (1979)

# Toroidal systems – Reduction to 1-D model (low freq.)

- Particle motion along resonance does not lead to large gradients in F, so neglect them
- Need projection of motion and collisions across resonance




**Resonance**

**Motion across resonance**

**Motion due to wave**

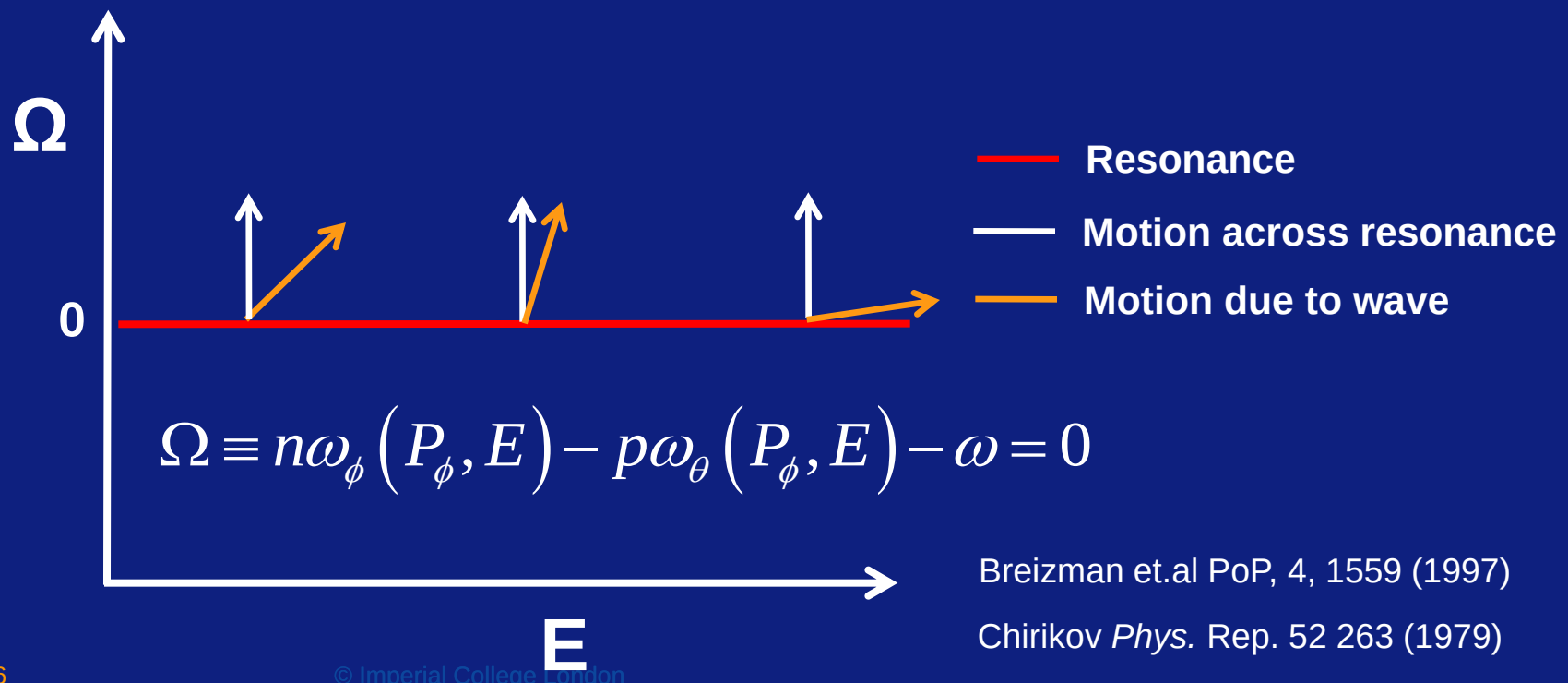
$$E - \frac{\omega}{n} p_\varphi = \text{const.}$$

Breizman et.al PoP, 4, 1559 (1997)

Chirikov *Phys. Rep.* 52 263 (1979)

# Toroidal systems – Reduction to 1-D model (low freq.)

- Transform to coordinates that straighten resonance
- Motion across the resonance 1-D for given E and μ
- Must integrate over all E and μ to get the result




$$\Omega \equiv n\omega_\phi\left(P_\phi, E\right) - p\omega_\theta\left(P_\phi, E\right) - \omega = 0$$

Breizman et.al PoP, 4, 1559 (1997)

Chirikov *Phys.* Rep. 52 263 (1979)

# Experimental estimate for MAST and ITER

$$\frac{\nu_{\mathrm{TAE}}}{\alpha_{\mathrm{TAE}}} \sim \frac{T_e^{3/4} n_e^{1/6}}{B_0^{5/6}}$$

Drag vs diffusion depends on plasma parameters

$$\frac{\nu_{\mathrm{TAE}}}{\alpha_{\mathrm{TAE}}} \approx 0.2 - 1.6$$

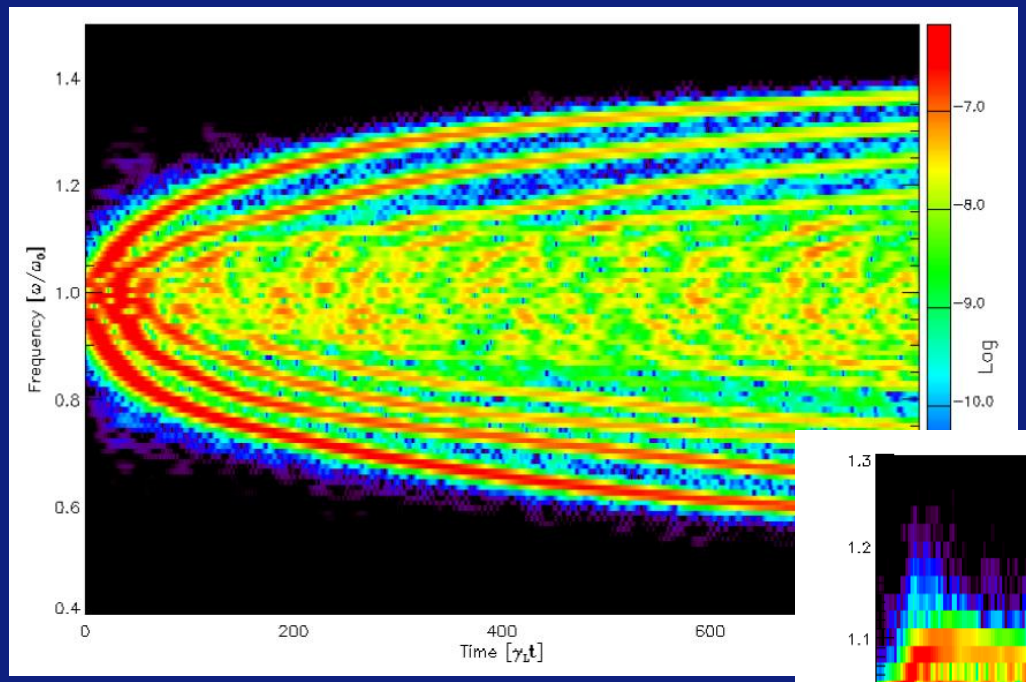
MAST - beams - Drag can dominate → explosive

$$\frac{\nu_{\mathrm{TAE}}}{\alpha_{\mathrm{TAE}}} \approx 1.4$$

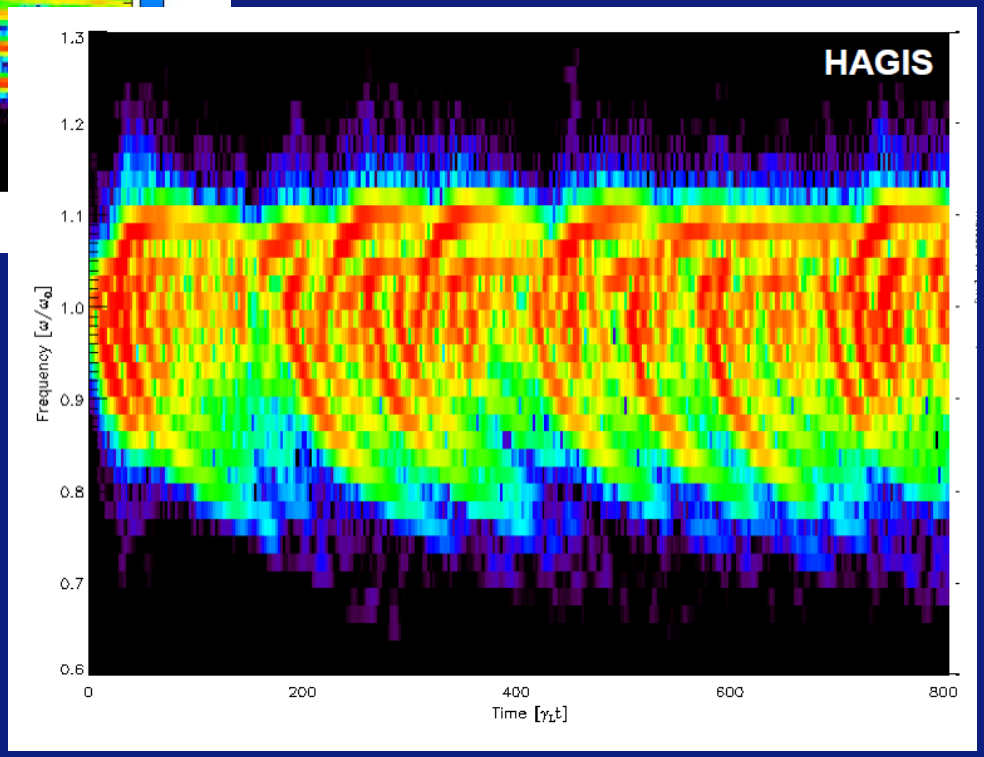
ITER – alphas – drag and diffusion comparable

Lilley et.al PRL, 102, 195003 (2009)

# Drag in toroidal systems – HAGIS




HAGIS shows symmetric chirp without collisions




HAGIS shows hooking tendency with drag and Krook collisions

# Conclusions and future work

- Drag is destabilising and acts as a seed for energetic particle modes
- Drag gives asymmetry as a lowest order effect, giving steady state holes and hooks
- Possible explanation of NBI vs. ICRH TAE observations
- Experiments underway (ITPA) to get drag/difusion database
- HAGIS now includes drag

# BOT Code

- Email bumpontail@gmail.com from your work email
- Please give your name, institution and your position
- It is free for you to use, modify and also distribute, but I encourage others to contact me for the code so that I can send updates as they become available