Study of chirping Toroidicity-induced Alfvén Eigenmodes in the National Spherical Torus Experiment

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Bursting toroidicity-induced Alfvén eigenmodes (TAEs) can lead to enhanced fast-ion transport

• Multiple TAEs can be simultaneously destabilized
  – Possible overlap of many resonances in phase space
  – Non-linear development into “TAE avalanches”
Bursting toroidicity-induced Alfvén eigenmodes (TAEs) can lead to enhanced fast-ion transport

- Multiple TAEs can be simultaneously destabilized
  - Possible overlap of many resonances in phase space
  - Non-linear development into “TAE avalanches” -> fast ion losses

Spherical tokamaks such as NSTX provide excellent test-bed

- Need to understand the physics of bursting TAEs, improve predictive capability for future devices (ITER)
Outline

• Experimental scenario, diagnostics
• General features of TAEs on NSTX
  • Frequency, amplitude dynamics
  • Mode structure
  • Role of fast ion drive
• Non-linear dynamics, mode-mode coupling
• Summary
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NSTX parameters

- Major radius: 0.85 m
- Aspect ratio: 1.3
- Elongation: 2.7
- Triangularity: 0.8
- Plasma current: ~1 MA
- Toroidal field: <0.55 T
- Pulse length: <2 s

3 Neutral Beam sources
- $P_{\text{NBI}} \leq 6 \text{ MW, } E_{\text{injection}} \leq 95 \text{ keV}$
- $1 < \frac{v_{\text{fast}}}{v_{\text{Alfvén}}} < 5$

This work:
- Focus on TAEs in L-mode plasma
- Center-stack limited
- Deuterium plasma
- $B_{\text{tor}} = 0.55 \text{ T, } I_p = 0.7\text{-}0.9 \text{ MA}$
Mode activity and fast ion diagnostics on NSTX

- Mirnov coils
  - Magnetic fluctuations up to 2.5 MHz
- Multi-channel reflectometer
  - Mode structure (monotonic profiles)
    FFT analysis complemented by analysis in time domain to study mode dynamics over short time scale

- Fast Ion diagnostics
  - Fast Ion D-Alpha systems (fast ion radial profile and spectrum)
  - Neutral Particle Analyzers, Fast Ion Loss probe, neutron rate

shot#135404, t=320 ms
Experimental scenario:
\[ P_{NB} < 3 \text{MW}, \quad n_e \sim 3-4 \times 10^{19} \text{m}^{-3}, \quad T_i \sim T_e = 1-1.5 \text{keV} \]

- NB-heated, L-mode plasmas
- Plasma limited on center-stack
- NB power and timing varied to affect mode stability
- Plasma profiles evolving in time
  - Monotonic, centrally peaked: OK for reflectometer measurements
  - Central plasma rotation up to 40kHz
  - Large Doppler shift of mode frequency
Safety factor profile evolves from strongly to slightly-reversed shear

- NB-heated, L-mode plasmas
- Plasma limited on center-stack
- NB power and timing varied to affect mode stability
- Plasma profiles evolving in time
- Reversed-shear $q$ profile
  - $q_{\text{min}} \sim 1$ toward end of discharge
- Safety factor evolution reconstructed through LRDFIT code constrained by MSE data
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TAEs with low-intermediate toroidal mode number ($n=2→7$) are observed, with dominant $n=2$-$4$ modes

- Burst separation 0.5 – 2 ms
  - No systematic variation with $n_e$, $T_e$, $P_{NB}$, …
  - Frequency evolution does not follow unique pattern (e.g. $t^{-1/2}$, linear, exponential)
- Usually, each mode chirps independently of the others...
- ... but, eventually, avalanches occur:
  - Drop in neutron rate, FIDA
Up to ~30% of fast ions can be lost during a single TAE avalanche

- Fast ion density (FIDA) drops over most of minor radius
- Loss results in a relaxation of the radial gradient → drive for TAEs is reduced
- Comparable losses estimated from FIDA and neutron rate
  - Large portion of phase space affected
- Losses increase with (total) mode amplitude
  - Linear+threshold? Quadratic?
Frequency vs. amplitude are correlated; reminiscent of driven, non-linear system

- E.g., non-linear Van der Pol oscillator:
  \[ \ddot{x} - \varepsilon (1 - \beta x^2) \omega_0 \dot{x} + (\omega_0^2 + \eta x^2) x = M \omega_D^2 \sin(\omega_D t) \]
  - damping
  - ‘non-linearity’ factor
  - ‘restoring’ term
  - driving force

- Can get info on damping, drive (resonant) frequency and their temporal variations?
- Comparison with chirping TAE data under way

\[ Koepeke \text{ et al., PRA 44 (1991) 6877} \]
On average, TAE frequencies are consistent with a common frequency in the plasma frame

Measured frequency consistent with:

\[ f_{\text{lab},n} = f_0^{\text{TAE}} + n f_{\text{Doppler}} \]

- Valid for time scales >1 ms
- In general, each mode shows a different sub-millisecond dynamic

FFT window 1.3ms

Shot #135414

Light symbols: time-domain
Dark symbols: FFT
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- Valid for time scales >1 ms
- In general, each mode shows a different sub-millisecond dynamic...
- ...except during large bursts:
  - Doppler shift only slightly changed here
  - Chirp mainly due to decrease in \( f_0^{\text{TAE}} \)

![Graph showing measured frequency vs. time](image)

- Light symbols: time-domain
- Dark symbols: FFT

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Multi-channel reflectometer is main tool to unravel details of mode structure

\begin{itemize}
  \item Makes it possible comparison with codes – e.g. NOVA – in terms of $dn/n$
\end{itemize}
NOVA eigenmodes selected from matching with measured frequency, mode number, structure

- NOVA provides *ideal* eigenmodes in a given frequency range
- Match mode structure \( (dn/n) \) from reflectometer to NOVA solutions
- Select ‘observed’ mode; re-scale NOVA solution according to measured \( dn/n \)
- [Use as input for ORBIT, calculate fast ion transport]

Fredrickson et al., PoP 2009

Fredrickson, P1.7

Darrow, O1
Mode structure maintains its shape even during strong, multi-step avalanches

- Re-scaled $dn/n$ shown (compare radial structure, not amplitude)
- Outward propagation of *unstable front* during burst not observed
  - Broad mode structure, $\sim$ minor radius
  - Incomplete transition TAE $\rightarrow$ EPM?

Zonca et al., NF 2005

Magnetic axis: $R\sim105$ cm
Edge: $R\sim155$ cm
Modes have broad mode structure, extending over good fraction of minor radius; peak at mid-radius

- sh#141711
- t=470ms
- Magnetic axis: R~105 cm
- Edge: R~155 cm
Comparison with NOVA (ideal MHD) is satisfactory; however, no good match for $n=1$ ... if ‘TAE’

- NOVA results assume ideal MHD, no fast ion effects
- Need to carefully check profile consistency between experimental data (MPTS, CHERS) and TRANSP output / NOVA input

- Frequency from NOVA ~ consistent with experiment
- Doppler shift is included
- But: radial shift of 2-5 cm between measured and simulated dn/n

<table>
<thead>
<tr>
<th>$n_{tor}$</th>
<th>$f_{meas}$ [kHz]</th>
<th>$f_{NOVA}$ [kHz]</th>
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<tr>
<td>2</td>
<td>60-80</td>
<td>77.8</td>
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<tr>
<td>3</td>
<td>80-100</td>
<td>91.4</td>
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<td>4</td>
<td>110-125</td>
<td>103</td>
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<tr>
<td>5</td>
<td>130-140</td>
<td>111.9</td>
</tr>
</tbody>
</table>
Similar features observed in L- and H-mode plasmas and during combined NB+RF: robust dynamics

- Profiles different from L-mode
- Higher safety factor than for L-mode discharges
- Reversed shear in both L- and H-mode
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Onset of bursting/chirping regime strongly dependent on injected NB (and RF) power

- Take fractional frequency variation $\Delta f/f_{0,\text{pl}}$ as metric for severity of bursts
  - $f_{0,\text{pl}}$: mode frequency in the plasma frame
- Each symbol represents values for single $n$ mode, 5 ms average

![Graph showing $\Delta f/f_{0,\text{pl}}$ vs $P_{\text{NB}}$]
High ratio of fast ion to thermal $\beta$ leads to bursts/chirps – but other factors determine mode dynamics

- Modes show more bursting character as discharge evolves
  - NB power increases, fast ion population and $\beta_{f,i}$ builds up
  - Typical bursts have edge $\delta B/B \sim 10^{-4}$ (from Mirnov coils)
    > Can increase more than x10 during avalanches
Destabilization of TAEs correlates with variations of heating scheme, e.g. NB vs. NB+RF

- Example: H-mode discharges with NB and NB+RF heating
  - Different profiles with respect to L-mode (e.g. higher safety factor)
  - Reversed shear in both L- and H-mode

- Here NB alone does not drive TAEs
- NB+RF does: increase in fast ions
Strong dependence on NB drive leads to entangled evolution of TAEs, fast ions and thermal plasma

- **Mode location**, \( R^{TAE} \), obtained by matching \( f^{TAE}_{Doppler} \) with measured rotation profile:
  \[
  f_{rot}(R^{TAE}) = f^{TAE}_{Doppler}
  \]

- Correlation between
  - Mode location \( R @ f^{TAE}_{Doppler} \)
  - Max fast ion gradient \( max \; grad(n_f) \)
  - Max rotation shear

---

**Coupling through common “source term”, i.e. NB injection**

**Modes cluster at similar radius: may enhance coupling**

*Podestà et al., PoP 2010*
Once set up, bursting/chirping TAE regime is rather insensitive to variations in plasma parameters

- Burst separation 0.5 – 2 ms
  - No systematic variation with $n_e$, $T_e$, $P_{NB}$, …
  - Frequency evolution does not follow unique patter (e.g. $t^{-1/2}$, linear, exponential)
- Usually, each mode chirps independently of the others...
- ... but, eventually, **avalanches** occur:
  - Multiple modes lock on similar dynamic, mode-mode coupling
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Coupling between multiple TAEs with $\Delta n_{tor}=1$, enhanced losses observed during explosive modes’ growth

- Multiple modes follow similar dynamic during the burst
  - Transition from single- to multi-mode regime

- Coupling generates higher/lower frequency modes

Podestà et al., NF 2011
‘New’ modes appear in the spectrum above/below TAE range during large bursts

- Modes can be classified into three groups
  - Discriminant: frequency, temporal evolution

- Picture consistent with primary TAEs
  - coupling to each other
  - generating/pumping secondary modes through sum/difference with $\Delta n=1$

- High-frequency band: appears during bursts
- “Primary” TAEs: observed during most of the time interval of interest
- Low-frequency band: appears during large bursts only? Masked by other MHD modes?

- Poloidal plot: Toroidal mode structure vs. time and mode number from 11 Mirnov coils
  - Note regions of constructive/destructive overlap between “primary” TAEs
  - Shown are $n=1$, $n=3$, $n=4$
Simple model based on quadratic interactions can be used to investigate coupling between TAEs

\[ \dot{s}_n = c_{(n_1, n_2)} s_{n_1} s_{n_2} \]

\[ s_{n_2} \rightarrow s_{n_2}^\ast \] (complex conjugate) for difference interaction

Right-hand side filtered around frequency \( f_{n_3} \)

Modes must satisfy matching conditions

\[ \begin{align*}
  n_3 & = n_1 \pm n_2 \\
  f_{n_3} & = f_{n_1} \pm f_{n_2}
\end{align*} \]

\( c(n_1, n_2) \) is the coupling coefficient, here used as ‘free scaling parameter’

In practice:

- Real signals \( s_{n_1}, s_{n_2}, s_{n_3} \) measured for each possible triplet, e.g. from Mirnov coils
- “Reconstruct” \( \dot{s}_{n_3} \rightarrow \dot{s}_{n_3,\text{rec}} \) from measured \( s_{n_1}, s_{n_2} \)
- Compare measured and reconstructed \( \dot{s}_{n_3} \)
- **Frequency match must be verified in the plasma frame:**
  - Rotation profile and location of each mode must be accurately known
Good agreement with quadratic interactions’ model: amplitude evolution and frequency matching

- $n=1$ mode fades away when either amplitude of pump modes or frequency matching vanishes
Mode number matching condition verified

- "Reconstructed" toroidal structure of $n=1$ mode also agrees with measured one
  - Phase shift of ~180 degrees during strongest mode activity

-Symbols: rms mode amplitude data from 11 Mirnov coils
- Solid lines: fit for a given $n$ ($n=1$ here)
- Dashed line: unit circle (zero-amplitude reference)
Phase matching condition is *transiently* verified for tens of (primary) mode cycles during large bursts

- Stationary phase during quadratic interaction is important!
  - $n=1$ mode fades away $\Leftrightarrow$ phase changes rapidly in time
  - “Single mode” dynamic, with each mode following its own chirp/burst cycle, is effective in reducing efficiency of quadratic interactions
  - Result: small bursts (single mode), and occasional multi-mode avalanches

- $n=1$ mode mediates coupling: what is it?
$f_{n=1}$ consistent with central plasma rotation: avalanches drive $f_0^{TAE} \rightarrow 0$ (plasma frame), cause coupling to ‘kinks’?

- Kink-like modes destabilized during avalanche
- Sustained fast ion losses
- TAE mode structure ~ maintained
- Mode location $R^{TAE}$ sweeps from plasma center out in ~2 ms

![Graph showing frequency vs. time](image)

Bortolon, O-12

Axis: R~105 cm
Edge: R~155 cm
Meaning of $f_0^{TAE}$: mode frequency at the mode location $R^{TAE}$, i.e. where drive is maximum

TAE continuum from NOVA-K

$t=410\text{ms}$

Weak bursts/chirp

$R^{TAE}$

$f_{TAE}$

$n_f^{TAE}$

$\tilde{f}_{\text{Doppler}}$

$n=3$

$f_{\text{meas}}$

$\text{shot}=141711$, $n_{10x}=3$
Meaning of $f_0^{TAE} \rightarrow 0$: $f^{TAE}$ beats with $n \times$ rotation frequency at the plasma center

- Coupling with kink-like modes favored when $n_{tor} \times f_{rot}$ on axis $\sim f^{TAE}$
- Observed for $q>1$: fishbone branch involved?
$n=1, m=-2, ..., +2$ kink found through ideal MHD code NOVA with no rotation, *free boundaries*

- Large edge perturbation, consistent with reflectometer’s data
Measured perturbation evolves during burst; large edge component, \(dn/n>5\%\)

- Data from inversion of multi-channel reflectometer signals
- Roughly consistent with (ideal) NOVA solution

![Density perturbation](image)

![Graph](image)
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- After onset, bursting/chirping TAE regime is “robust” against small variations of plasma parameters
  - L-mode vs. H-mode, NB only, NB+RF, ... : all show similar features
- Non-linearities at play in both single-mode and multi-mode (avalanching) TAE dynamics
  - But only avalanches seem to cause significant fast ion losses
  - Coupling manifests as intermittent, chaotic process
  - Coupling can encompass multiple ‘scales’: TAE, kink/fishbones (, GAE, ...)
  - Dynamics complicated by link between fast ions, TAEs, thermal plasma through NB injection
- Different physics for weak chirps vs. avalanches?
  - Single-mode; weak chirps regulated by phase-space, fast ion profile effects
  - Multi-mode; avalanches lock on underlying kink/fishbone (‘global’)
- Present experiments allow thorough benchmark of codes
  - Linear MHD satisfactory for first-order estimates of mode structure
  - Non-linear, self consistent codes required to capture full dynamics