Nonlinear Theory and Simulation of Energetic Particle-induced Geodesic Acoustic Mode

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In collaboration with

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Outline

- Introduction
- Nonlinear Simulation of EGAM in DIII-D
- Nonlinear Theory of EGAM: generation of second harmonic

Energetic Particle-driven GAM (EGAM)

- Energetic particle-driven GAM-like modes have been observed in tokamaks and stellarators (JET, DIII-D, LHD etc).

- The first observation came from JET where an n=0 mode was excited by the fast ICRF tail ions. The mode was interpreted as a MHD GAM eigenmode with frequency above the maximum of GAM continuum.
  (C. J. Boswell et al., Phys. Lett. A 358, 154 (2006)
  H. L. Berk et al., Nucl. Fusion 46, S888 (2006).)

- Recent DIII-D results showed count-injected beam ions can excite a n=0 GAM-like global mode. The mode frequency is well inside the GAM continuum. (R. Nazikian et al., Phys. Rev. Lett. 101,185001 (2008).)

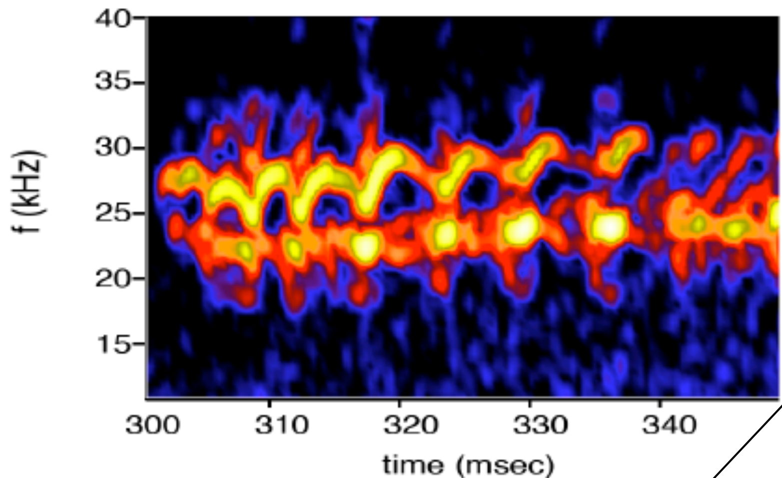
- Analytic theory showed existence of EGAM with global radial mode structure determined by energetic particle's finite orbit width effects.
  (G.Y. Fu, Phys. Rev. Lett. 101,185002 (2008))

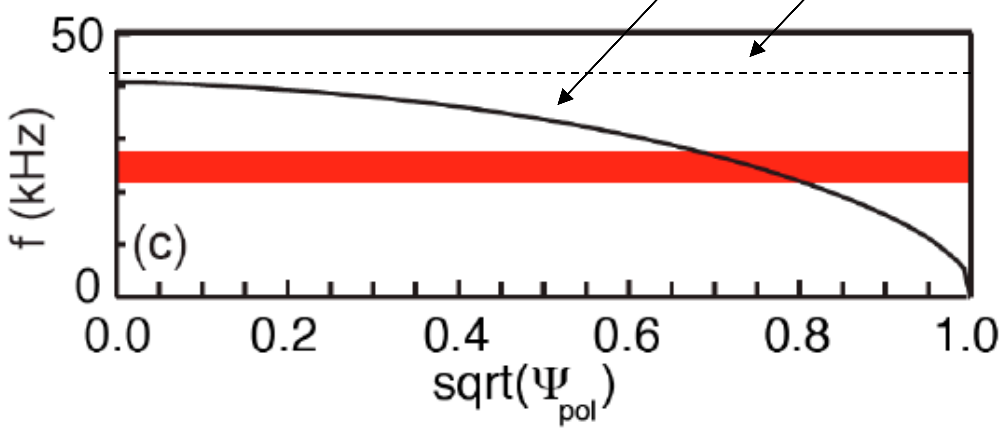
Instability Mechanism of EGAM

- The GAM is usually stable due to n=0 (i.e., no universal drive due to radial gradient). Thus it is typically driven nonlinearly by micro-turbulence.
- However, energetic particles can provide instability drive via velocity space gradient for inverted distribution function (inverse Landau damping for dF/dE >0). In this way, the energetic particle-driven GAM is similar to the bump-on-tail instability.
- dF/dE >0 is possible for NBI beam ions and ICRF tail ions due to bump-on-tail distribution.
- The wave particle resonance ($\omega_{GAM} \sim \omega_{bh}$) is possible for large value of safety factor ($v_h \sim qv_t$)

Mode Frequency Well Below ideal GAM frequency

$\delta B/B \sim 10^{-5}$, n=0 at wall

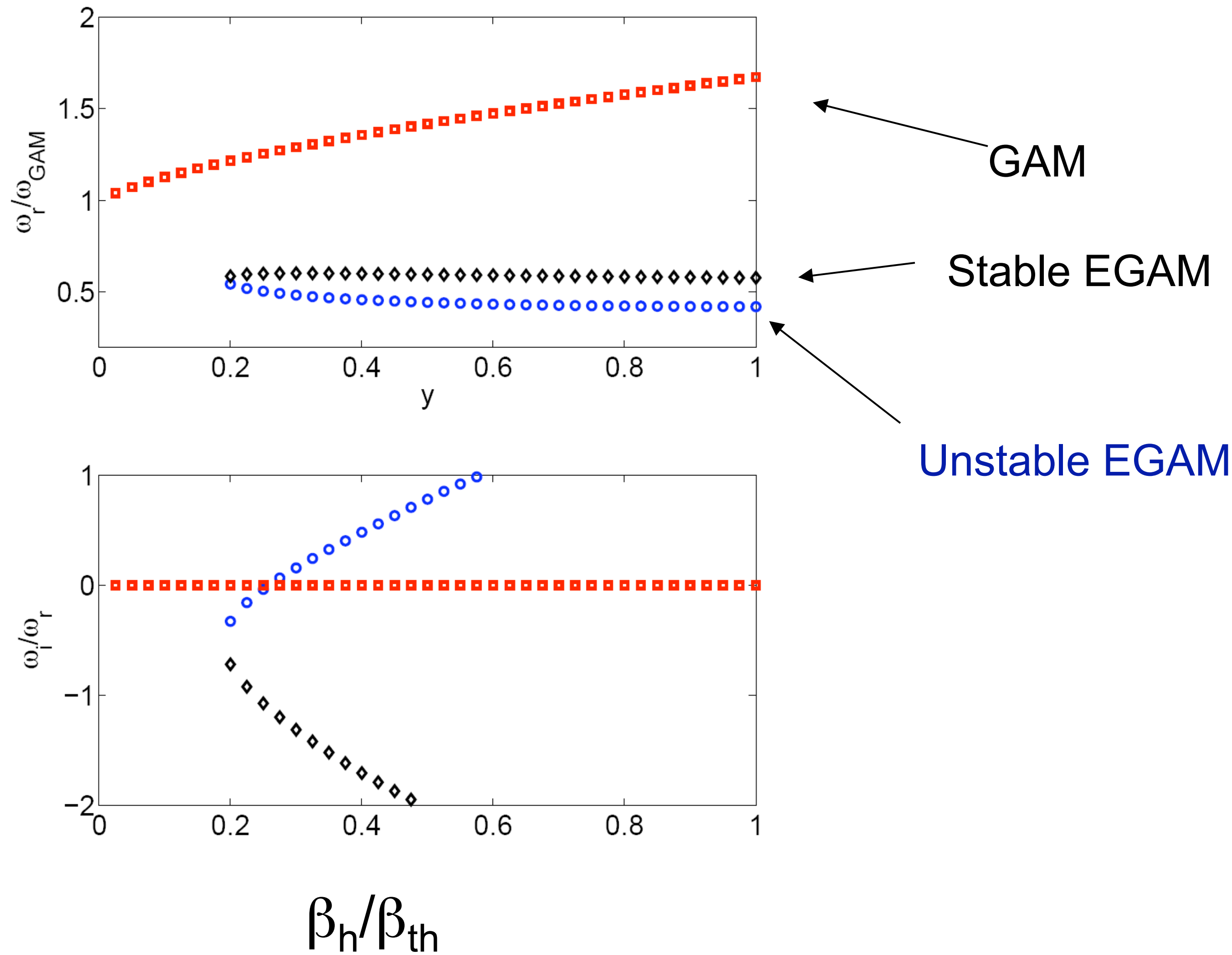




- n=0 GAM continuum
  $\omega \approx 2C_s/R$
- ideal GAM can only exist above the continuum
  - no NOVA solution
- Mode frequencies well below peak in the continuum
  - not the ideal GAM
- Mode structure is global, not the local kinetic GAM

R. Nazikian et al., Phys. Rev. Lett. 101,185001 (2008).)

Energetic particle effects induce two new branches of eigenmode (EGAM)



G.Y. Fu, Phys. Rev. Lett. 101,185002 (2008))

Hybrid Simulation Model for EGAM

- n=0 electrostatic perturbation (Er only)
- fluid model for thermal plasma;
- drift-kinetic model for energetic particles.

Nonlinear Hybrid Equations for EGAM

$$\mathbf{E} = -\nabla\Phi = E_r \nabla r, \qquad \mathbf{v} = \frac{\mathbf{E}\times\mathbf{B}}{B^2}$$

Radial electric field response

$$\frac{\partial}{\partial t} < \frac{\rho|\nabla r|^2}{B^2} > E_r = - < G(r,\theta)(2P_{th} + P_{\|h} + P_{\perp h}) >$$

Geodesic curvature!

$$G(r,\theta) = -\frac{B_\phi R}{JB^3}\frac{\partial B}{\partial \theta}$$

density response

$$\frac{\partial}{\partial t}\rho + \mathbf{v}\cdot\nabla\rho = -\nabla\cdot\mathbf{v}\rho$$

Thermal pressure response

$$\frac{\partial}{\partial t}P_{th} + \mathbf{v}\cdot\nabla P_{th} = -\gamma\nabla\cdot\mathbf{v}P_{th}$$

Energetic particle response

$$P_{\|h} + P_{\perp h} = \int d^3v(mv_\|^2 + \frac{1}{2}mv_\perp^2)f_h$$

Parameters and Profiles

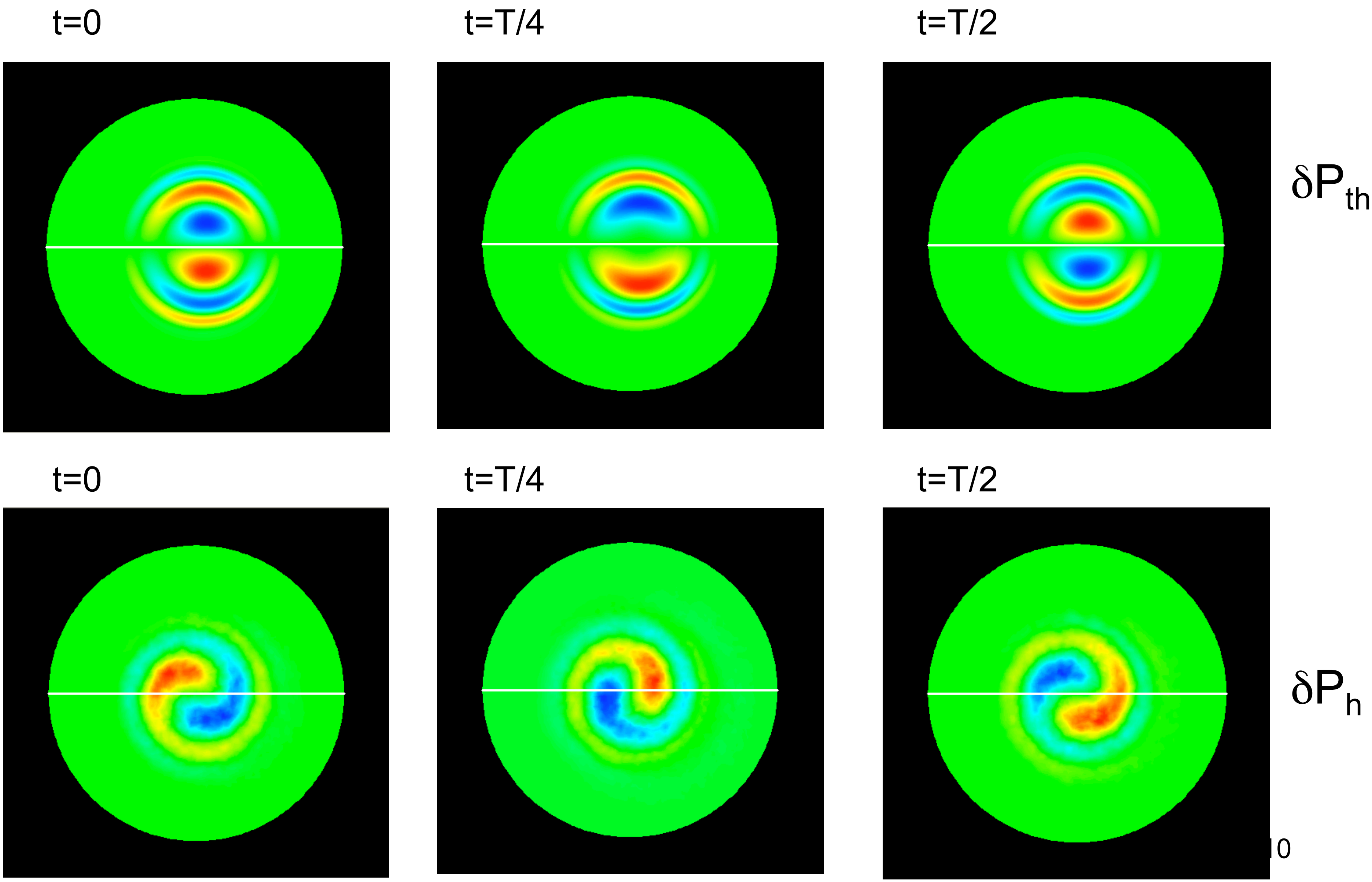
$R/a=3$, $q_{min}=4.0$

$P=P_0\,(1-\psi)^2$ $\rho=\text{constant}$

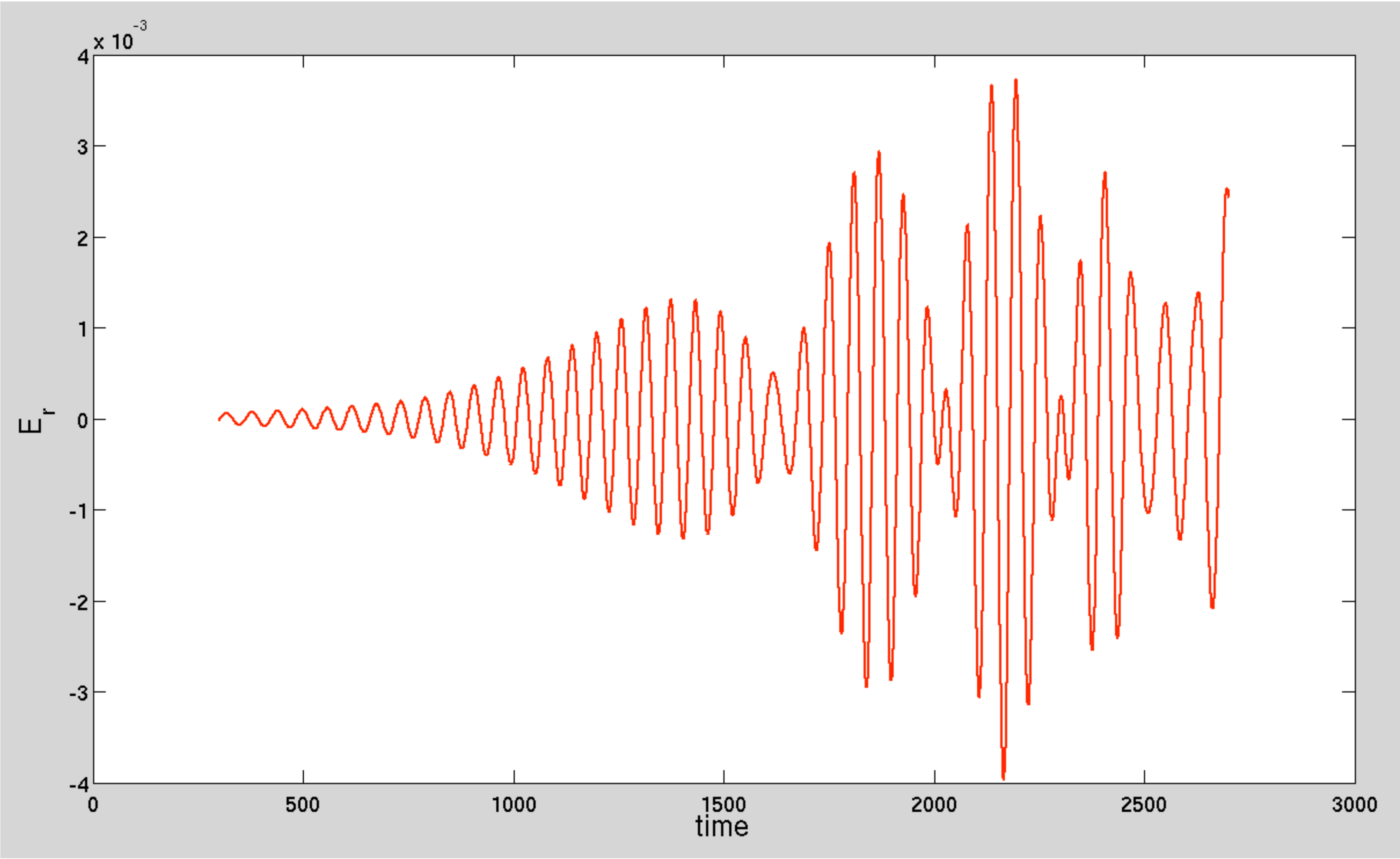
$$f = \frac{1}{v^3 + v_{\mathrm{crit}}^3}\exp\left[-\frac{P_\phi}{e\Delta\Psi} - \left(\frac{\Lambda - \Lambda_0}{\Delta\Lambda}\right)^2\right]$$

$\Lambda_0=0.5$, $\Delta\Lambda=0.2$

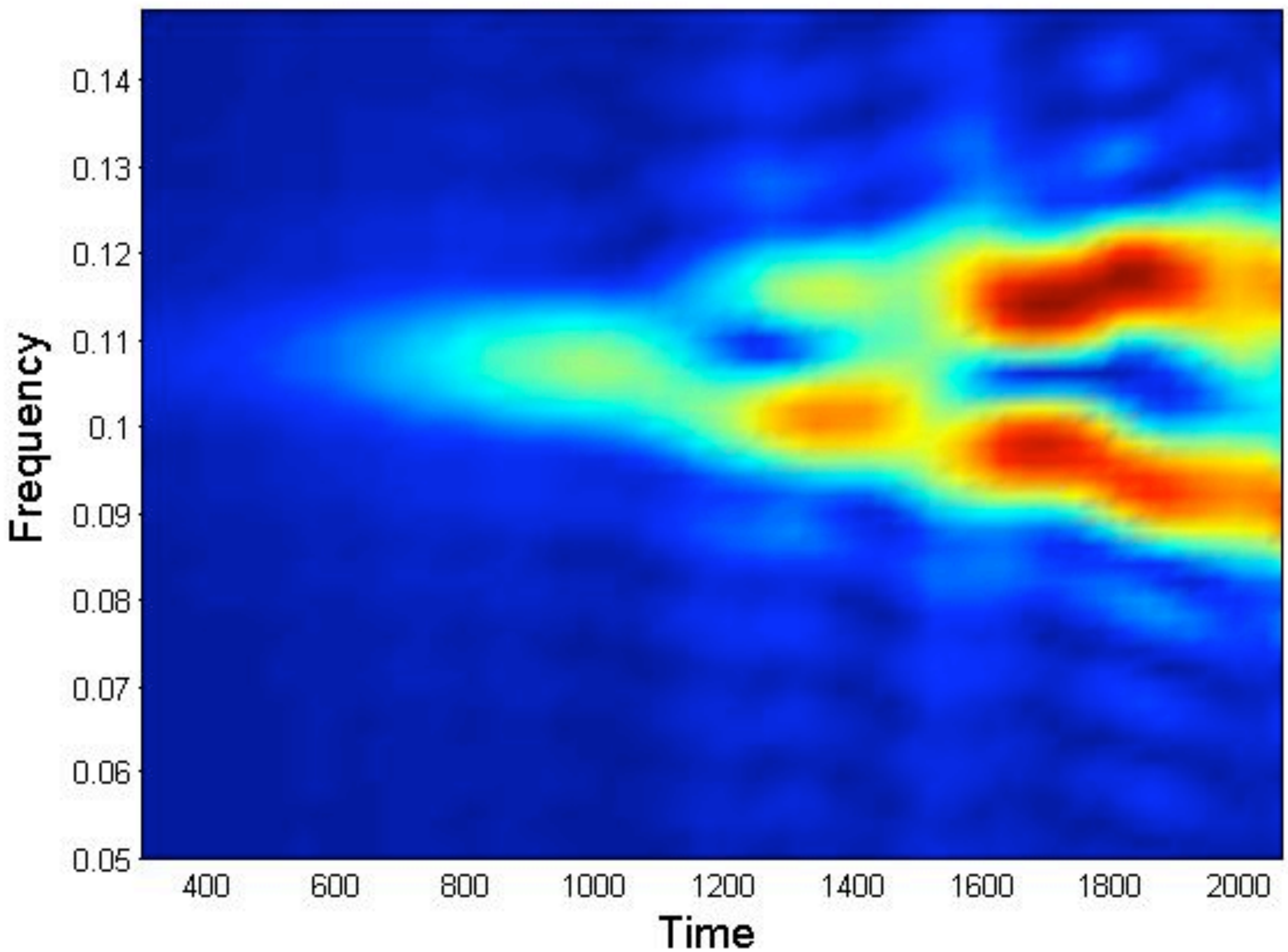
Linear evolution of perturbed pressures



Nonlinear simulations show bursting behavior



Nonlinear simulation shows frequency chirping of EGAM



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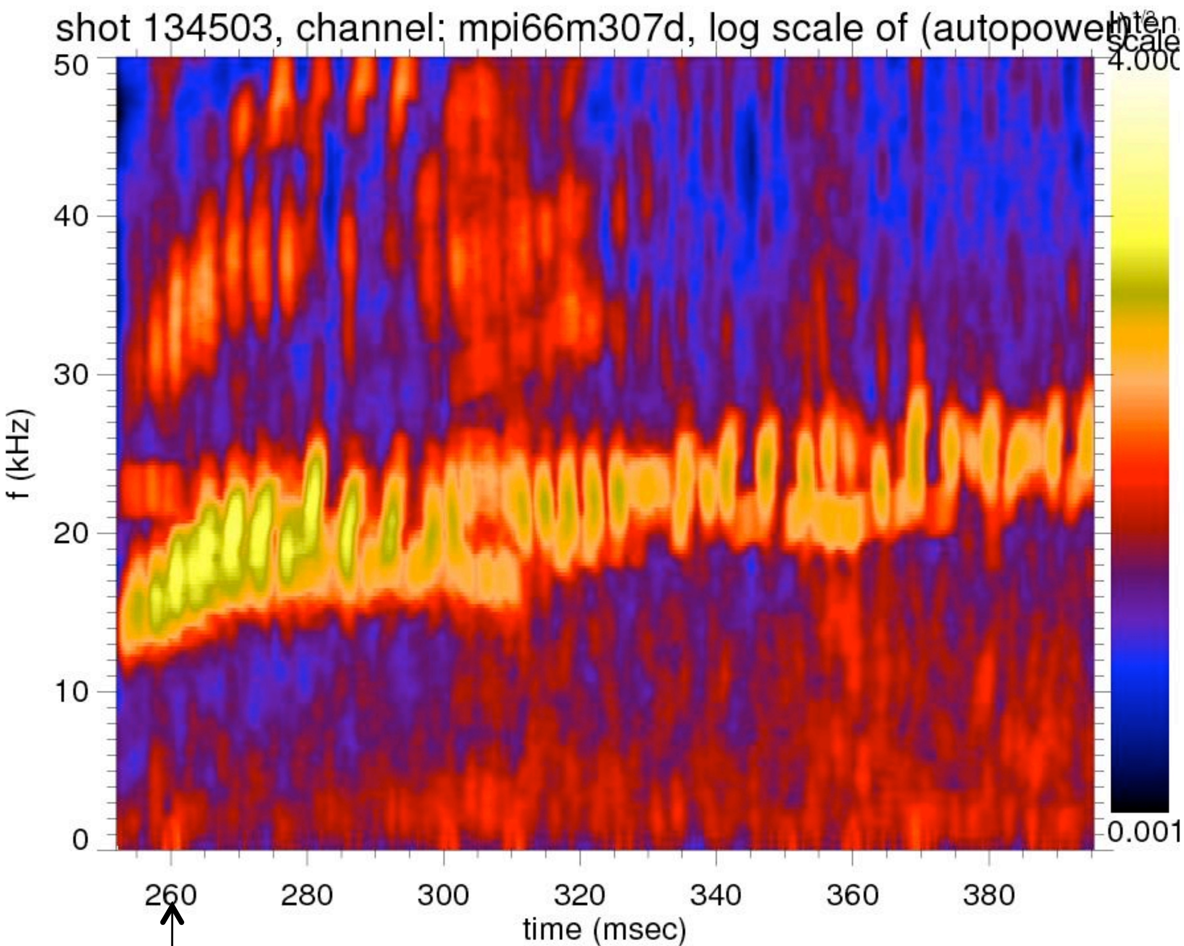
Strong beam-driven GAM was observed in DIII-D (shot #134503)



R. Nazikian

simulations

Nonlinear Simulation Model of Beam-driven EGAM in DIII-D

- Assume n=0 electric static perturbation (Er only);
- Use hybrid model --- fluid model for thermal species and drift-kinetic model for beam ions
- Realistic beam distribution from NUBEAM code including both Co and Counter beams and all energy components.

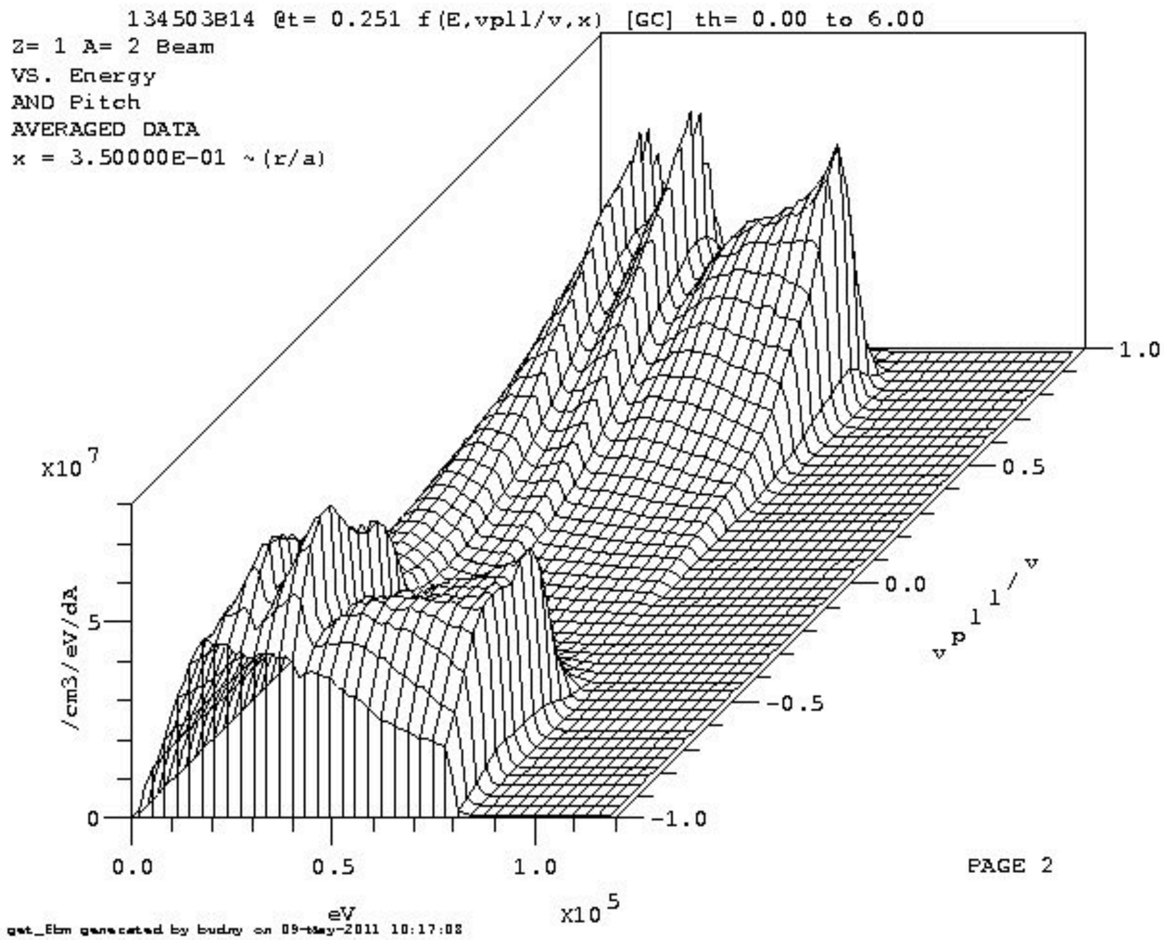
$$\left\langle \frac{\rho |\nabla r|^2}{B^2} \right\rangle \frac{\partial}{\partial t} E_r = - \left\langle G(r,\theta)(2P_c + P_{\parallel h} + P_{\perp h}) \right\rangle$$

$$\frac{\partial}{\partial t} P_c = 2\gamma G(r,\theta) E_r P_c \qquad G(r,\theta) = -\frac{B_\varphi R}{J B^3} \frac{\partial B}{\partial \theta}$$

$$P_{\parallel h} + P_{\perp h} = \int m(\mathbf{v}_\parallel^2 + \frac{1}{2}\mathbf{v}_\perp^2) f \, d^3\mathbf{v}$$

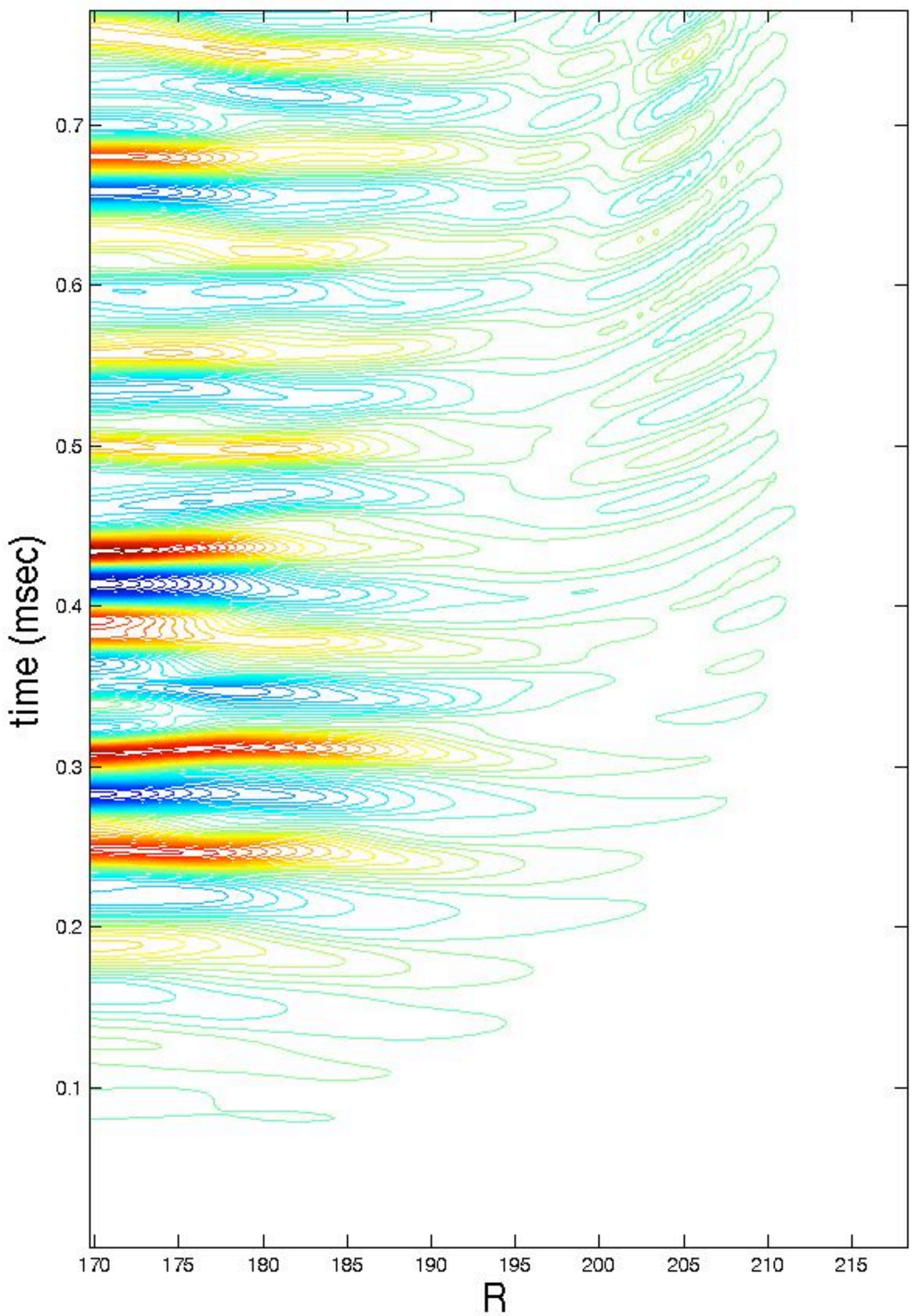
Beam Ion Distribution from TRANSP/ NUBEAM (r/a~0.3)

R. Budny



Parameters and Profiles (DIII-D shot # 134503 at t=260msec)

- R=1.64m, a=0.62m
- B=2.06T, n_e=1.4x10^{13}cm^{-3} , T_e=0.6kev
- q_{min}~5.0
- β_{th}=0.15%, β_{beam}=0.16%



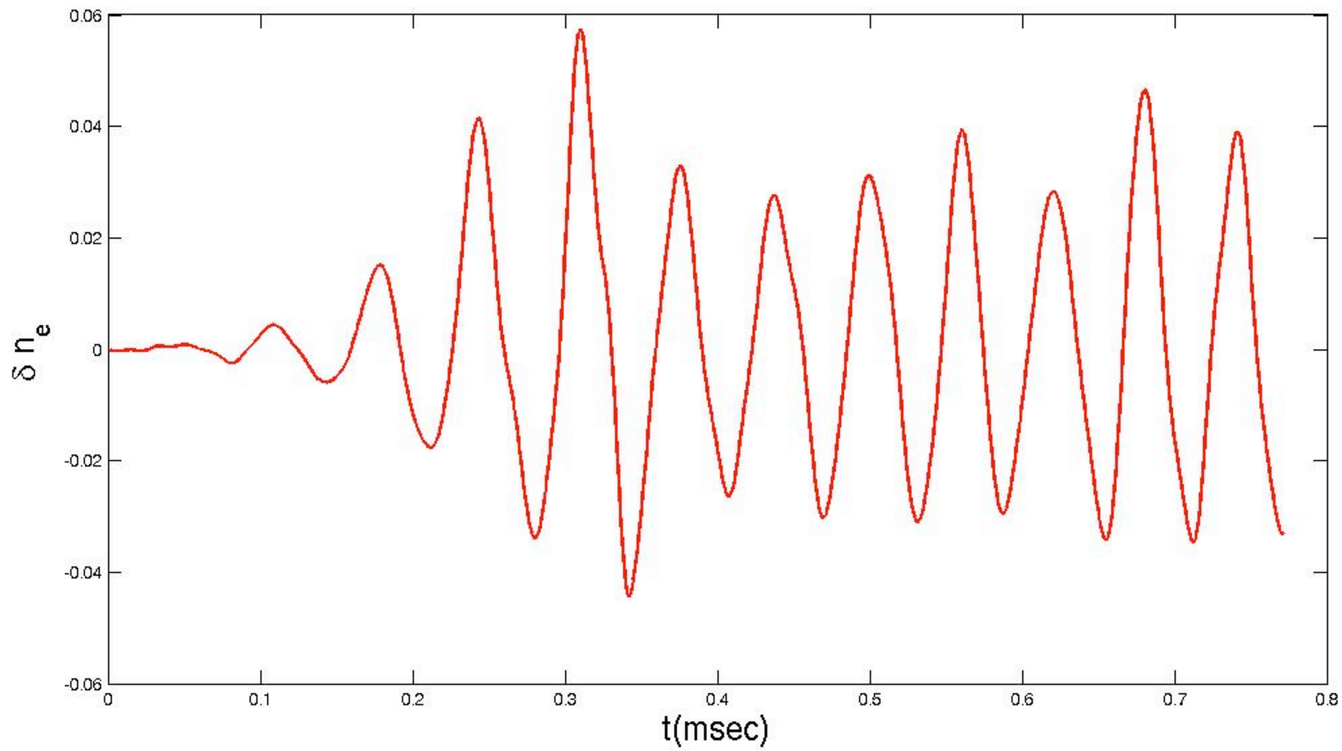
Evolution of
density fluctuation
From t ~ 260msec
At Z=3.6cm.

f ~ 16 kHz

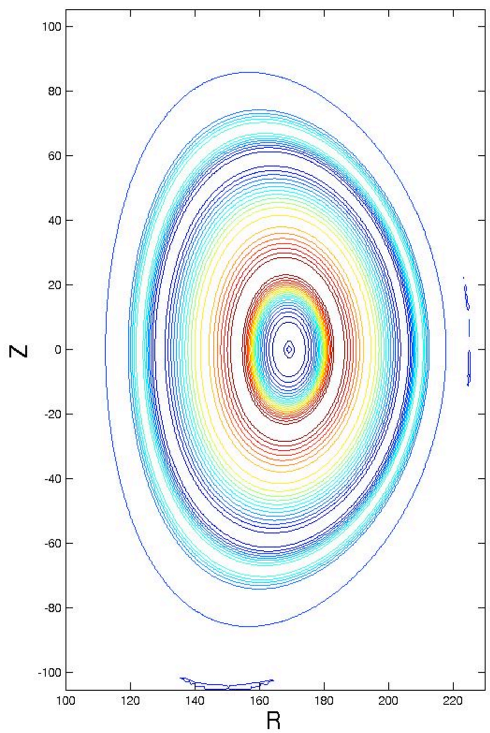
δn_e ~ 3%

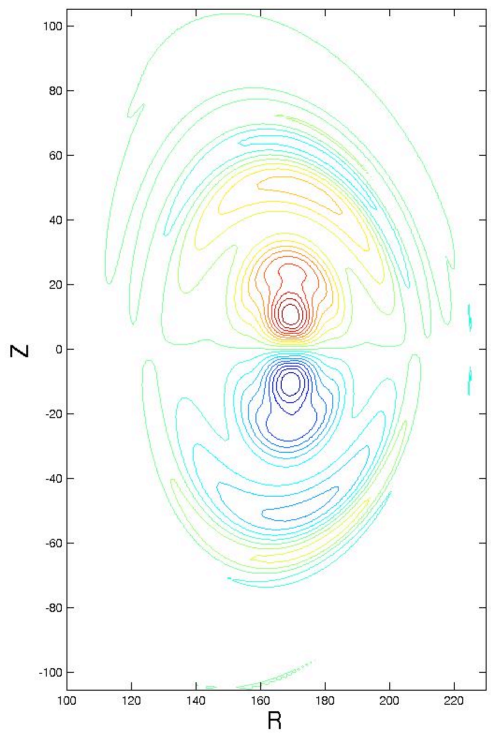
outward propagation!

Density fluctuation evolution at R=1.88m



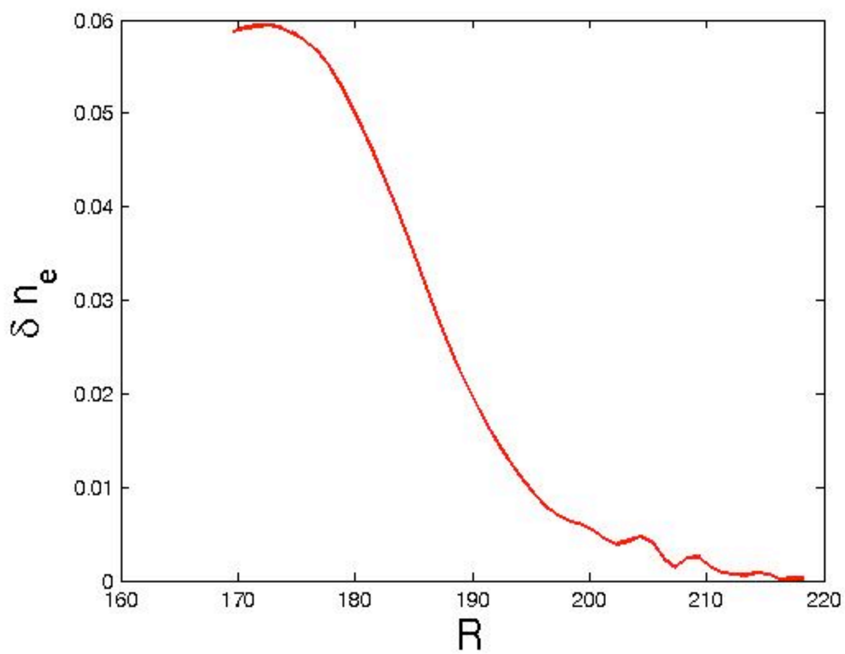
E_r and δn_e (2D mode structure)

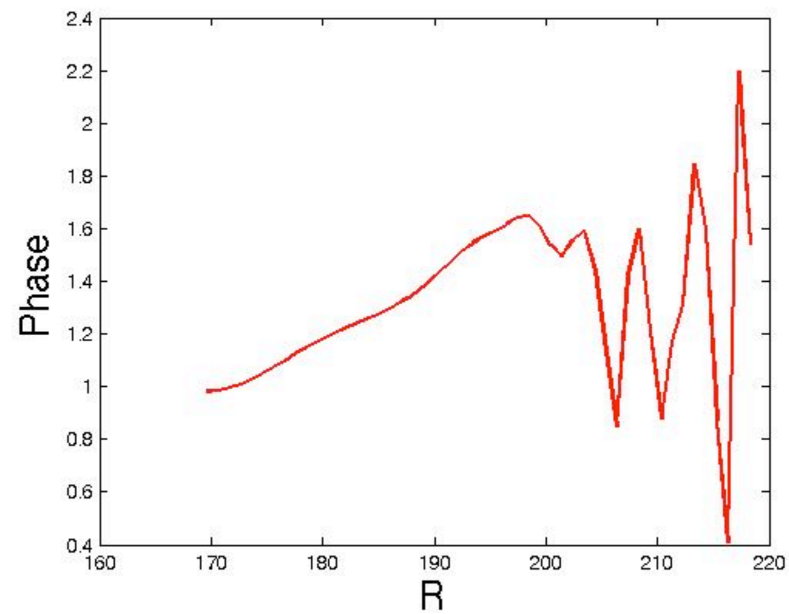




Up-down asymmetric density fluctuation !

Density fluctuation amplitude and phase





Summary: Simulation of beam-driven EGAM in DIII-D

- Carried out hybrid simulations with realistic beam distribution function (full f PIC method).

- Nonlinear simulations yield mode frequency, mode amplitude and outward radial propagation consistent with experimental measurement of density fluctuation from BES.

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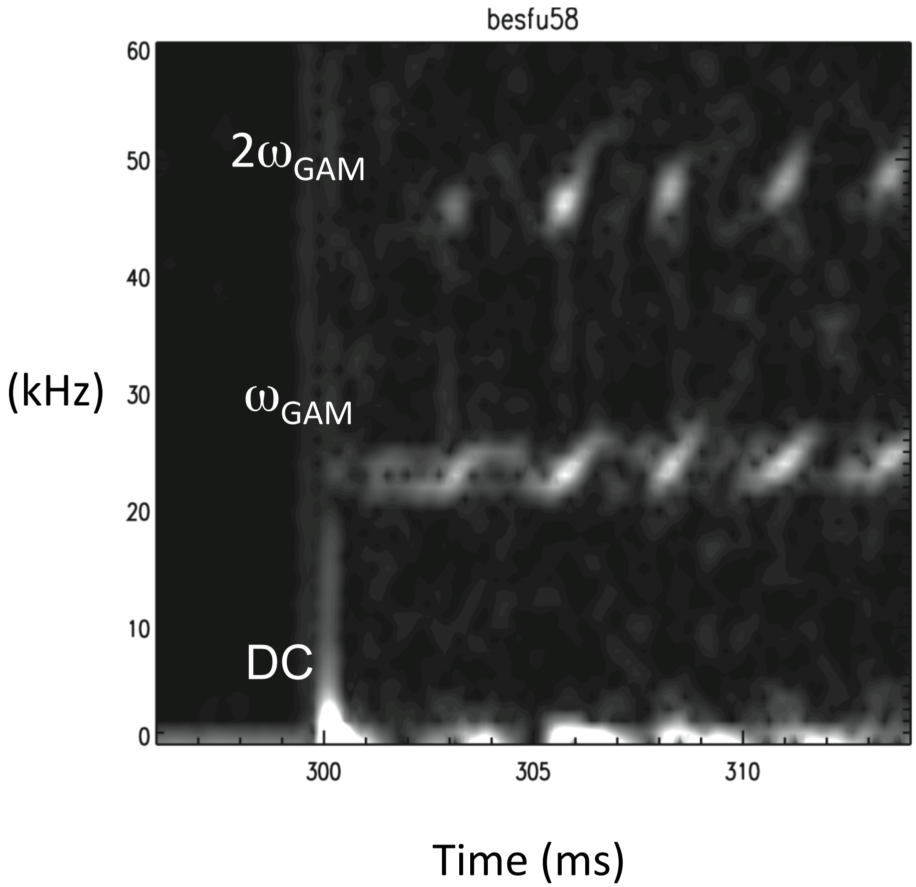
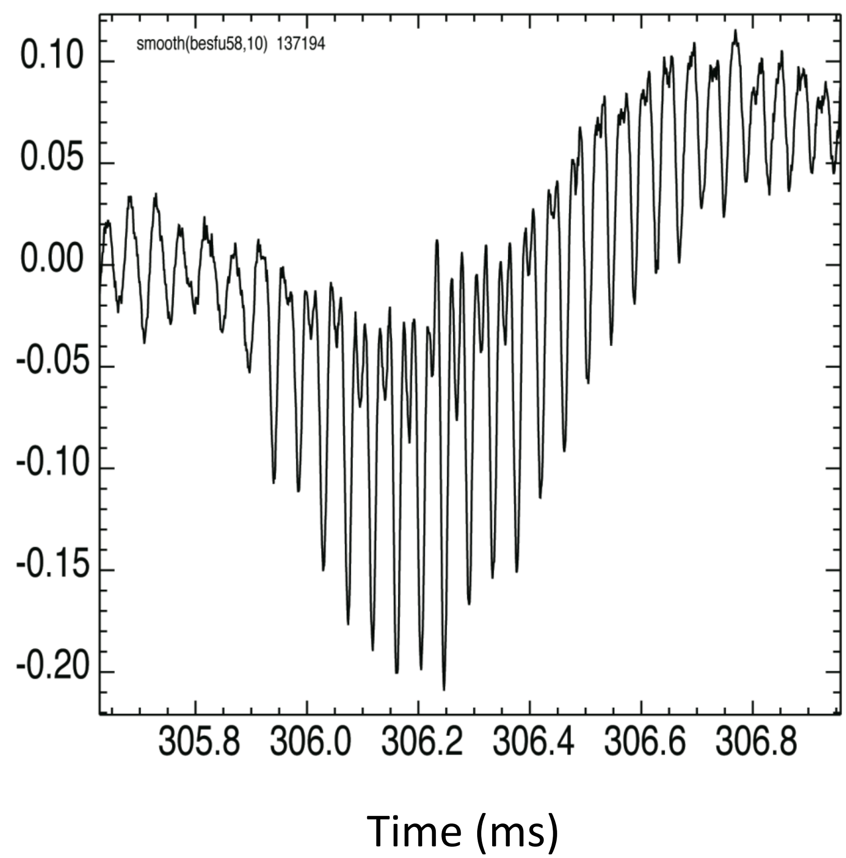
This Work

- Consider nonlinear generation of the second harmonic in density fluctuation due to beam-driven GAM;
- Motivated by recent DIII-D data which show that there is a large 2nd harmonic in the measured density fluctuation associated with the beam-driven GAM (R. Nazikian, 2009).

G.Y. Fu, "On Nonlinear Self-interaction of Geodesic Acoustic Mode Driven by Energetic Particles", PPPL report #4527, *J. Plasma Physics*, 2011


besfu58
(kHz)
60
50
40
30
20
10
0
$2\omega_{GAM}$
ω_{GAM}
DC
300
305
310
Time (ms)


smooth(besfu58,10) 137194
0.10
0.05
0.00
-0.05
-0.10
-0.15
-0.20
305.8
306.0
306.2
306.4
306.6
306.8
Time (ms)

R. Nazikian

Kinetic/MHD Hybrid Equations

$$\rho(\frac{\partial}{\partial t}\mathbf{v} + \mathrm{v}\cdot\nabla\mathrm{v}) = -\nabla P_c - \nabla\cdot\mathrm{P_h} + \mathrm{J}\times\mathrm{B}$$

$$\frac{\partial}{\partial t}\rho + \mathrm{v}\cdot\nabla\rho = -\nabla\cdot\mathrm{v}\rho$$

$$\frac{\partial}{\partial t}P_c + \mathbf{v}\cdot\nabla P_c = -\gamma\nabla\cdot\mathbf{v}P_c$$

$$\mathrm{E} + \mathrm{v}\times\mathrm{B} = 0$$

E_r Equation

$$\rho(\frac{\partial}{\partial t}\mathbf{v} + \mathbf{v}\cdot\nabla\mathbf{v}) = -\nabla P_c - \nabla\cdot\mathbf{P}_h + \mathbf{J}\times\mathbf{B}$$

$$\downarrow$$

$$< \frac{\rho\,|\nabla r|^2}{B^2} > \frac{\partial}{\partial t}E_r + < \frac{\rho}{B^2}(\nabla r\times\mathbf{B})\cdot(\mathbf{v}\cdot\nabla\mathbf{v}) >= -< \frac{\nabla r\times\mathbf{B}}{B^2}\cdot(\nabla P_c + \nabla\cdot\mathbf{P}_h) >$$

$$\downarrow$$

$$< \frac{\rho\,|\nabla r|^2}{B^2} > \frac{\partial}{\partial t}E_r + < \frac{\rho}{B^2}(\nabla r\times \mathrm{B})\cdot(\mathrm{v}\cdot\nabla\mathrm{v}) >= -< G(r,\theta)(2P_c + P_{\parallel} + P_{\perp}) >$$

$$G(r,\theta) = -\frac{B_\varphi R}{JB^3}\frac{\partial B}{\partial\theta} \approx -\frac{1}{BR}\sin(\theta)$$

Linear Fluid Equations

$$< \frac{\rho |\nabla r|^2}{B^2} > \frac{\partial}{\partial t} E_{r1} = -2 < G(r,\theta) P_{c1} >$$

$$\rho \frac{\partial}{\partial t} \mathrm{v}_{\|\mathbf{1}} = -\mathbf{b} \cdot \nabla P_{c1}$$

$$\frac{\partial}{\partial t} P_{c1} = -\gamma (B \cdot \nabla (\frac{v_{\|1}}{B}) - 2G(r,\theta) E_{r1}) P_c$$

$$\frac{\partial}{\partial t} P_{c1} = 2\gamma P_c (L^{-1} G(r,\theta)) E_{r1}$$

$$L = 1 + \frac{\gamma P_0}{\rho \omega^2} \mathbf{B} \cdot \nabla \frac{1}{B^2} \mathbf{B} \cdot \nabla$$

GAM dispersion relation

$$< \frac{\rho |\nabla r|^2}{B^2} > \frac{\partial}{\partial t} E_{r1} = -2 < G(r,\theta) P_{c1} >$$

$$\frac{\partial}{\partial t} P_{c1} = 2\gamma P_c (L^{-1} G(r,\theta)) E_{r1}$$

$$\omega^2 = \omega_{GAM}^2 \equiv \frac{4\gamma P}{\rho} \frac{< G L^{-1} G >}{< |\nabla r|^2 / B^2 >} \approx \frac{2\gamma P}{\rho R^2} (1 + \frac{1}{2q^2})$$

Nonlinear Fluid Theory: second order radial electric field is zero

$$< \frac{\rho\,|\nabla r|^2}{B^2} > \frac{\partial}{\partial t} E_{r2} = < \frac{\rho_1\,|\nabla r|^2}{B^2} > \frac{\partial}{\partial t} E_{r1} - < \frac{\rho}{B^2} (\nabla r \times \mathbf{B}) \cdot (\mathbf{v_1} \cdot \nabla \mathbf{v_1}) > -2 < G(r,\theta) P_{c2} >$$

$$\frac{\partial}{\partial t} P_{c2} = -\mathbf{v_1} \cdot \nabla P_{c1} - \gamma \nabla \cdot \mathbf{v_1} P_{c1} - \gamma \nabla \cdot \mathbf{v_2} P_c$$

$$P_{c2} = P_{21} + P_{22} E_{r2}$$

P_{21} is an even function of θ $\longrightarrow$ $E_{r2} = 0$

This result is the same as the gyrokinetic result of H.S. Zhang et al. 2009 Nucl. Fusion **49, 125009**

Nonlinear Fluid Theory: second order density perturbation

$$\frac{\partial}{\partial t}\rho_2 = -\mathrm{v}_1 \cdot \nabla\rho_1 - \nabla \cdot \mathrm{v}_2\rho_0 \approx -E_{r1}\frac{\nabla r \times \mathrm{B}}{B^2}\cdot\nabla\rho_1$$

$$\rho_1 = 2G\int E_{r1}(r,\tau)d\tau \approx -\frac{2}{BR}\sin(\theta)\int_0^t E_{r1}(r,\tau)d\tau$$

$$\frac{\rho_2}{\rho_0} = -\frac{r}{R}\cos(\theta)\left(\frac{1}{rB}\int_0^t E_{r1}(r,\tau)d\tau\right)^2 \qquad E_{r1}(r,t) = \hat{E}_{r1}(r,t)\cos(\omega t)$$

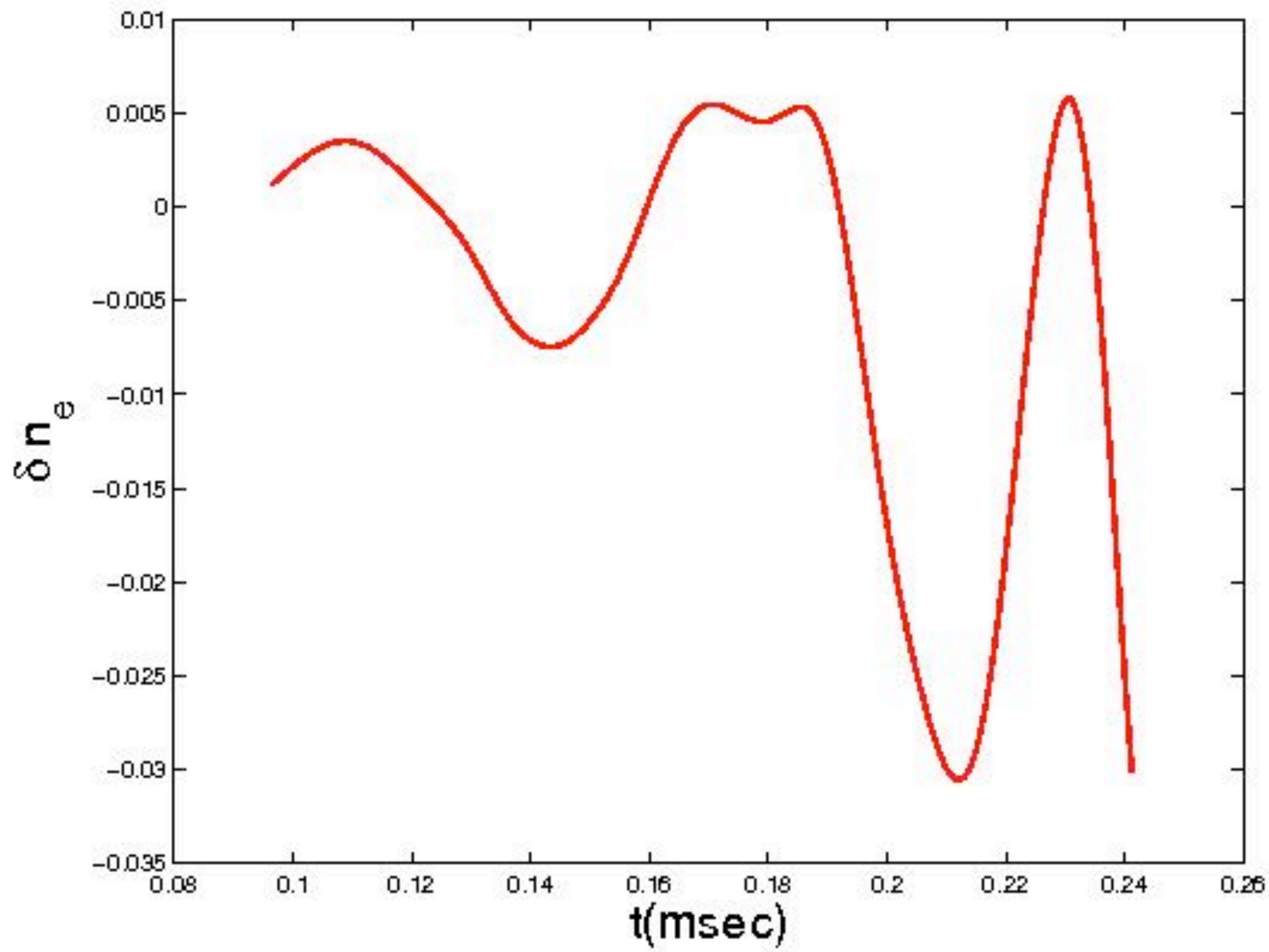
$$\approx -\frac{r}{2R}\cos(\theta)\left(\frac{1}{rB\omega}\hat{E}_{r1}(r,t)\right)^2\left(1-\cos(2\omega t)\right)$$

Properties of the second order density perturbation

$$\frac{\rho_2}{\rho_0} \approx -\frac{r}{2R}\cos(\theta)\left(\frac{1}{rB\omega}\hat{E}_{r1}(r,t)\right)^2\left(1-\cos(2\omega t)\right)$$

- Zero frequency and second harmonic have equal amplitude;
- Second order perturbation breaks up-down asymmetry;
- Second order density perturbation is negative on low field side;
- Near mid-plane, ρ_2 is comparable to ρ_1 since $\rho_1 \sim \sin\theta$

Simulation of density fluctuation including fluid nonlinearity shows clear negative perturbation



R=1.88m

Kinetic/MHD hybrid equations

$$< \frac{\rho |\nabla r|^2}{B^2} > \frac{\partial}{\partial t} E_r + < \frac{\rho}{B^2} (\nabla r \times \mathrm{B}) \cdot (\mathrm{v} \cdot \nabla \mathrm{v}) > = - < G(2P_c + P_{\| h} + P_{\perp h}) >$$

$$P_{\| h} + P_{\perp h} = \int m(\mathrm{v}_{\|}^2 + \frac{1}{2}\mathrm{v}_{\perp}^2) f \, d^3\mathrm{v}$$

$$\frac{\partial f}{\partial t} + (\mathrm{v}_{\|} b + \mathrm{v}_d + \frac{\mathrm{E} \times \mathrm{B}}{B^2}) \cdot \nabla f + \frac{dE}{dt} \frac{\partial f}{\partial E} = 0$$

$$\frac{dE}{dt} = (m\mathrm{v}_{\|}^2 + \frac{1}{2} m\mathrm{v}_{\perp}^2) G(r, \theta) E_r$$

Nonlinear expansion of distribution function

$$< \frac{\rho |\nabla r|^2}{B^2} > \frac{\partial}{\partial t} E_{r2} = - < G(r,\theta)(2P_{c2} + P_{\parallel h2} + P_{\perp h2}) >$$

$$f = f_0 + f_1 + f_2 + ...$$

$$\frac{df_1}{dt} \equiv \frac{\partial f_1}{\partial t} + (\mathrm{v}_{\parallel} b + \mathrm{v}_d) \cdot \nabla f_1 = -(m\mathrm{v}_{\parallel}^2 + \frac{1}{2} m\mathrm{v}_{\perp}^2) G(r,\theta) E_{r1} \frac{\partial f_0}{\partial E}$$

$$\frac{df_2}{dt} = -\frac{\mathrm{E}_1 \times \mathrm{B}}{B^2} \cdot \nabla f_1 - (m\mathrm{v}_{\parallel}^2 + \frac{1}{2} m\mathrm{v}_{\perp}^2) G(r,\theta) \left(E_{r1} \frac{\partial f_1}{\partial E} + E_{r2} \frac{\partial f_0}{\partial E} \right)$$

Energetic particles can generate a second harmonic in radial electric field

$$f_{1,non-res} = \frac{m\mathrm{v}_{\parallel}^2 + \frac{1}{2}m\mathrm{v}_{\perp}^2}{2BR}\hat{E}_r \frac{\partial f_0}{\partial E}\left(\frac{\cos(\omega t - \theta)}{\omega - \omega_b} - \frac{\cos(\omega t + \theta)}{\omega + \omega_b}\right)$$

$$f_2 = \frac{3(m\mathrm{v}_{\parallel}^2 + \frac{1}{2}m\mathrm{v}_{\perp}^2)}{2B^2 rR}\hat{E}^2 \frac{\partial f_0}{\partial E}\left(\frac{\omega\omega_b}{(4\omega^2 - \omega_b^2)(\omega^2 - \omega_b^2)}\right)\sin\theta\cos 2\omega t + \ldots$$

$$E_{r2} = -\frac{\omega_h^2}{4\omega^2 - \omega_{EGAM}^2}\frac{\hat{E}^2}{\omega Br}\sin(2\omega t)$$

Energetic particle effects can generate second harmonic in E_r

$$\omega_h^2 = -\frac{3}{2\rho R^2}\int\left(\frac{\omega^3\omega_b}{(4\omega^2 - \omega_b^2)(\omega^2 - \omega_b^2)}\right)(m\mathrm{v}_{\parallel}^2 + \frac{1}{2}m\mathrm{v}_{\perp}^2)^2 \frac{\partial f_0}{\partial E} d^3\mathrm{v}$$

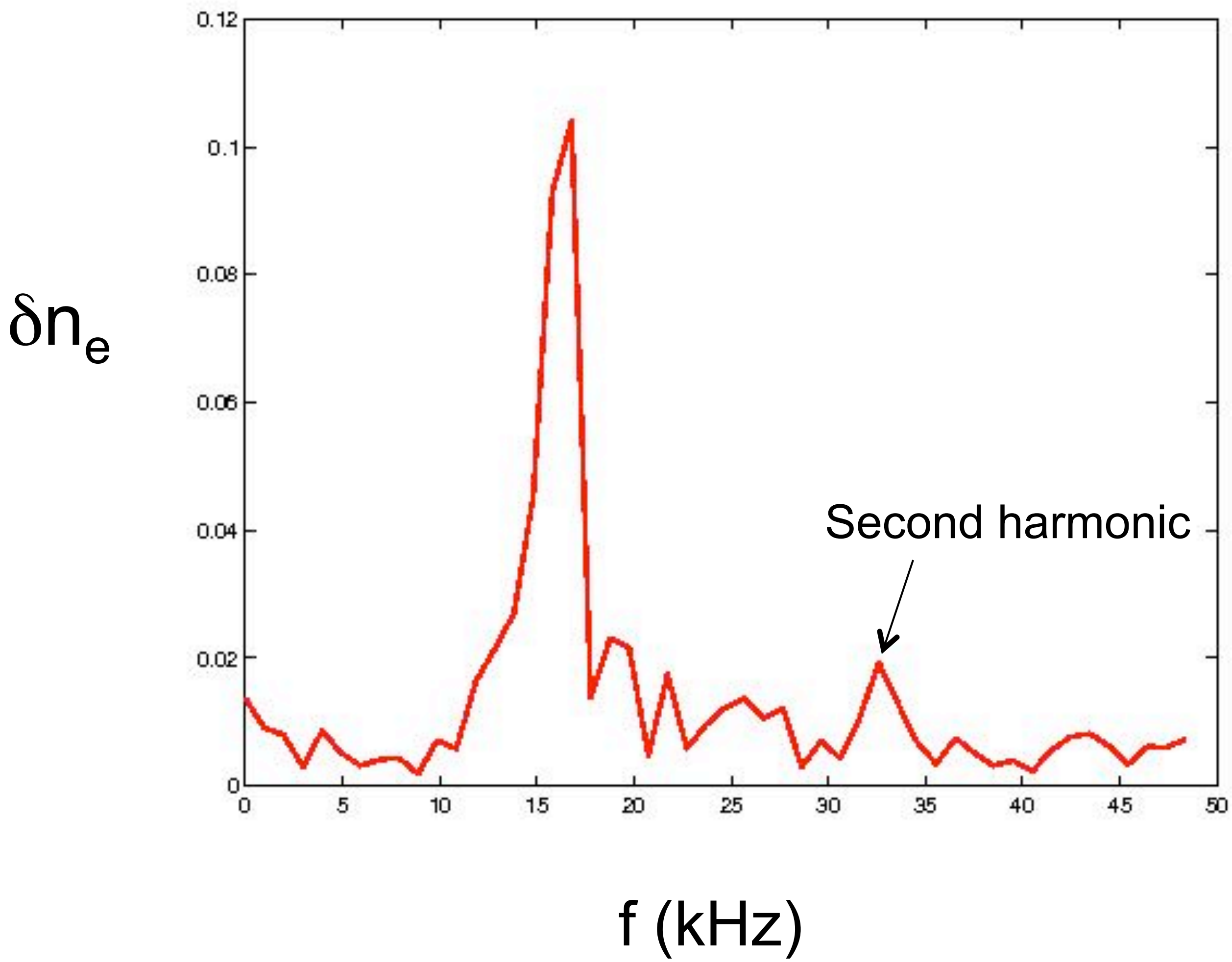
Second order density perturbation due to energetic particles

$$\frac{\rho_{2,h}}{\rho_0} = -\frac{\omega_h^2}{4\omega^2 - \omega_{EGAM}^2} \frac{\hat{\mathbf{E}}^2}{\omega^2 B^2 R r} \sin(\theta)\cos(2\omega t)$$

$$\frac{\rho_{2,h}}{\rho_{2,f}} = -\frac{\omega_h^2}{4\omega^2 - \omega_{EGAM}^2} \frac{\sin(\theta)}{\cos(\theta)} \sim \frac{P_{\|\mathrm{h}} + P_{\perp h}}{P_c} \frac{\sin(\theta)}{\cos(\theta)}$$

Spatial structure of energetic particle-induced second harmonic is the same as the fundamental harmonic!

Simulations without fluid nonlinearity shows evidence of second harmonic due to energetic effects



R=1.74m
Z=0.18m

Summary: Nonlinear theory of generation of second harmonic

- Fluid Model:
  - GAM self-interaction cannot generate a second harmonic in the radial electric field;
  - A second harmonic of density fluctuation is generated by convective nonlinearity. A DC component is also present.
  - The density perturbation is negative near the mid-plane for strong instability.
  - These results are consistent with DIII-D experiments.
- Energetic Particle Effects:
  - Energetic particle effects can generate a second harmonic in radial electric field;
  - The energetic particle-induced second harmonic of the density perturbation scales as $\sin\theta$. The EP contribution is small near the mid-plane.