Nonlinear MHD and Energetic Particles Hybrid Simulation of Alfvén Eigenmode Bursts

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Outline

- Introduction
  - Alfvén eigenmode (AE) bursts
  - Reduced simulation of AE bursts

- NL MHD effects on AE evolution

- NL MHD simulation of AE bursts
  - simulation model with NL MHD effects and EP source, collision, and loss
  - NL MHD effects on AE bursts
Alfvén Eigenmode Bursts

Results from a TFTR experiment [K. L. Wong et al., Phys. Rev. Lett. 66, 1874 (1991).] (see also DIII-D results [H. H. Duong et al., NF 33, 749 (1993)])

Neutron emission: nuclear reaction of thermal D and energetic beam D -> drop in neutron emission = energetic-ion loss
Mirnov coil signal: magnetic field fluctuation -> Alfvén eigenmodes

• Alfvén eigenmode bursts take place with a roughly constant time interval.
• 5-7% of energetic beam ions are lost at each burst.
Reduced simulation of AE modes and energetic particles

- Spatial profiles and real frequencies of AE modes are given in advance of the simulation.
- Amplitude and phase evolution of each eigenmode is computed in a way consistent with the energy transfer from energetic particles.
- Energetic particle drift kinetic orbits are followed in the EM field = equilibrium field + AE modes field.
Reduced Simulation of Alfvén Eigenmode Bursts

[Todo, Berk, Breizman, PoP 10, 2888 (2003)]

- Nonlinear simulation in an open system: NBI, collisions, losses
- Many aspects of the TAE bursts in the TFTR experiment [Wong et al. PRL 66, 1874 (1991)] were reproduced quantitatively.

Time evolution of energetic-ion density profile.
Time evolution of TAE mode amplitude and stored beam energy

Synchronization of multiple modes due to resonance overlap with time interval 2ms (left).

Stored beam energy is reduced to 40% of the classically expected level due to the 10% drop at each burst (right).
The losses balances with the beam injection when the amplitude of the outermost mode reaches to $6 \times 10^{-3}$. 

![Graph showing the behavior of $\delta B_{n=3}/B$ and Stored Beam Energy over time.](image)
Poincaré plots when particle loss balances the injection: resonance overlap of multiple modes takes place.
Saturation amplitude of AE mode

- inferred from the plasma displacement [Durst et al., (1992)]
  - at the edge region ($\rho \sim 0.8$): $\delta B/B \sim 10^{-3}$
  - at the core region ($\rho \leq 0.6$): plasma displacement is not available

- simulation
  - $\delta B/B \sim 2 \times 10^{-2}$ at the mode peak location

[Durst et al., PoF B 4, 3707 (1992)]
The problem is …

- The significant particle losses take place at $\delta B/B=6 \times 10^{-3}$ in the reduced simulation.

- The resonance overlap leads to the rapid growth of the mode amplitude up to $2 \times 10^{-2}$.

- $\Rightarrow$ Needs some nonlinear mechanism that suppresses the growth. *MHD nonlinearity?*
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Comparison between linear and NL MHD runs (\(j_h\)’ is restricted to n=4)

The viscosity and resistivity are \(\nu = \nu_n = 2 \times 10^{-7} \nu_A R_0\) and \(\eta = 2 \times 10^{-7} \mu_0 \nu_A R_0\).

The numbers of grid points are (128, 64, 128) for (R, \(\phi\), z).

The number of marker particles is 5.2x10^5.
Initial plasma profile and numerical conditions

\[ \beta_h = \beta_{h0} \exp[-(r/0.4a)^2] \]
\[ q = 1 + 2(r/a)^2 \]
\[ a\Omega_h / v_A = 16 \]
\[ R_0 / a = 3.2 \]
\[ v_b = 1.2v_A, v_c = 0.5v_A \]

Initial energetic-particle distribution: slowing down distribution isotropic in velocity space

The viscosity and resistivity are \( \nu = \nu_n = 10^{-6}v_A R_0 \) and \( \eta = 10^{-6}\mu_0 v_A R_0 \).
The numbers of grid points are (128, 64, 128) for (R, \( \phi \), z).
The number of marker particles is \( 5.2 \times 10^5 \). \( 0 \leq \phi \leq \pi/2 \) for the n=4 mode.
TAE spatial profile (n=4)

The main harmonics are m=5 and 6.
Comparison of linear MHD and NL MHD simulations

\( \beta_{h_0} = 1.5\% \)
- Sat. Level (linear) \( \sim 3 \times 10^{-3} \)
- Sat. Level (NL) \( \sim 3 \times 10^{-3} \)

\( \beta_{h_0} = 2.0\% \)
- Sat. Level (linear) \( \sim 1.6 \times 10^{-2} \)
- Sat. Level (NL) \( \sim 8 \times 10^{-3} \)

The saturation level is reduced to half in the nonlinear MHD simulation.
Evolution of total damping rate

The total damping rate ($\gamma_{d\text{ALL}}$) is greater than the damping rate in the linearized MHD simulation ($\gamma_{d\text{ lin}}$).

$\beta_{h0} = 1.7\%$
Sat. Level (linear) $\sim 1.2 \times 10^{-2}$
Sat. Level (NL) $\sim 6 \times 10^{-3}$
Schematic Diagram of Energy Transfer

Energetic Particles

Drive

n=4 TAE

Dissipation

n=0 and higher-n modes

Dissipation

Thermal Energy

Linearized MHD

NL coupling

Thermal Energy

NL coupled modes
Effects of weak dissipation

\[ \beta_{h0} = 1.7\% \]

The viscosity and resistivity are reduced to \( \frac{1}{16} \),

\[ \nu = \nu_n = 6.25 \times 10^{-8} \nu_A R_0 \]

and

\[ \eta = 6.25 \times 10^{-8} \mu_0 \nu_A R_0 \]

with the numbers of grids (512, 512, 128).

The nonlinear MHD effects reduce the saturation level also for weak dissipation.
Spatial profiles of the TAE and NL modes: Evidence for continuum damping of the higher-n (n=8) mode
ZF Evolution and GAM Excitation

After the saturation of the TAE instability, a geodesic acoustic mode is excited.

Evolution of TAE and zonal flow

\[ V_{T,64} / V_A \text{ and } V_{\theta,0,0} / V_A \]

\[ V_{\theta,0,0} / V_A \]
Summary of NL MHD effects on a TAE instability [Y. Todo et al. NF 50, 084016 (2010)]

- Linear and nonlinear simulation runs of a n=4 TAE evolution were compared. The saturation level is reduced by the nonlinear MHD effects.

- The total energy dissipation is significantly increased by the non-linearly generated modes. The increase in the total energy dissipation reduces the TAE saturation level. The dissipation from higher-n modes can be attributed to the continuum damping.

- The zonal flow is generated during the linearly growing phase of the TAE instability. The geodesic acoustic mode (GAM) is excited after the saturation of the instability. The GAM is not directly excited by the energetic particles but excited through MHD nonlinearity.
Questions for AE bursts

- Is the mode amplitude reduced also for the AE bursts?

- Do the significant fast ion losses take place with the NL MHD effects?

-> EP-MHD hybrid code MEGA is extended to simulate with beam injection, collisions, and losses
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**δf simulation with source, loss, collisions and NL MHD**

- **Time dependent** $f_0$ **is implemented to simulate** the formation of the slowing down distribution with source and collisions

- **particle loss**
  - phase space inside the loss boundary should be well filled with the marker particles
  - marker particles can excurse outside the loss boundary and return back to the inside
  - $δf$ particle weight is set to be 0 outside the loss boundary
δf evolution of each marker particle with collisions

\[
\frac{\partial}{\partial t} f + \{f, H\} - \nu \frac{\partial}{v^2 \partial N} \left[ \left( v^3 + v_c^3 \right) f \right] = S(v)
\]

When \( f_0 \) satisfies

\[
\frac{\partial}{\partial t} f_0 + \{f_0, H_0\} - \nu \frac{\partial}{v^2 \partial N} \left[ \left( v^3 + v_c^3 \right) f_0 \right] = S(v),
\]

the evoution of \( \delta f \) is given by

\[
\frac{\partial}{\partial t} \delta f + \{\delta f, H_0 + H_1\} + \{f_0, H_1\} - \nu \frac{\partial}{v^2 \partial N} \left[ \left( v^3 + v_c^3 \right) \delta f \right] = 0.
\]

With a definition \( \frac{d}{dt} \delta f = \frac{\partial}{\partial t} \delta f + \{\delta f, H_0 + H_1\} - \nu \nu \left( 1 + \frac{v_c^3}{v^3} \right) \frac{\partial}{\partial N} \delta f \),

the evolution of \( \delta f \) is expressed by

\[
\frac{d}{dt} \delta f + \{f_0, H_1\} - 3\nu \delta f = 0.
\]
Evolution of phase space volume of each marker particle

The evolution of phase space volume $V$ that each particle occupies should be considered. Comparison of two eqs.:

$$\frac{d}{dt}(fV) = SV$$

and

$$\frac{d}{dt}f - 3vf = S$$

gives

$$\frac{d}{dt}V = -3\nu V$$

We solve the evolution of both $\delta f$ and $V$ of marker particles.
Time-dependent $f_0$

A solution of

$$\frac{\partial}{\partial t} f_0 - v \frac{\partial}{v^2 \partial v} \left[ \left( v^3 + v_c^3 \right) f_0 \right] = S(v),$$

is

$$f_0(v, t) = \frac{1}{v} \frac{1}{v^3 + v_c^3} \left[ \text{erf} \left( \frac{v' - v_b}{\Delta v} \right) - \text{erf} \left( \frac{v - v_b}{\Delta v} \right) \right]$$

with

$$v' = \left[ \left( v^3 + v_c^3 \right) \exp(3v(t + t_{inj})) - v_c^3 \right]^{1/3},$$

$$S(v) = \frac{2}{\sqrt{\pi}} \frac{1}{v^2 \Delta v} \exp \left[ - \left( \frac{v - v_b}{\Delta v} \right)^2 \right],$$

$v_b$: injection or birth velocity of energetic particle

$t_{inj}$: injection starts at $t = -t_{inj} < 0$

Note:

Here we have neglected the finite orbit width effect and $\{ f_0, H_0 \}$ term.
Physics condition

- similar to the reduced simulation of TAE bursts at the TFTR experiment
- parameters
  - $a=0.75\text{m}$, $R_0=2.4\text{m}$, $B_0=1\text{T}$, $q(r)=1.2+1.8(r/a)^2$
  - NBI power: 10MW
  - beam injection energy: 110keV (deuterium)
  - $v_b=1.1v_A$
  - slowing down time: 100ms
  - parallel injection ($v///v=-1$ or 1)
  - no pitch angle scattering
  - particle loss at $r/a=0.8$
Benchmark of the numerical model: slowing down process w/o MHD perturbation

The slowing down process is successfully simulated.
NL MHD effects: reduction of TAE amplitude and beam ion losses

\[ \nu = \frac{\eta}{\mu_0} = \chi = 5 \times 10^{-7} v_A R_0 \]

- \( n=2 \) TAE peak amplitude
- Stored beam energy

**Linear MHD**

**NL MHD**
Linear MHD
Nonlinear MHD
Numerical convergence in numbers of particles and grid points

\[ n=2 \text{ TAE peak amplitude} \]

\[ n=\eta/\mu_0 = \chi = 5 \times 10^{-7} v_A R_0 \]

\[ 10^{21} \text{ particles & (256\times256\times128) grids} \]

\[ 10^{19} \text{ particles & (128\times128\times64) grids} \]
Frequency spectra and TAE spatial profiles

- Frequency spectra at r/a=0.41 (q=1.5) for 0≤t≤10ms
- Nonlinear modes with n=4 and 5 at f=100-120kHz
- Spatial profiles of n=2 and 3TAE modes at t=1.41ms (first burst)
Effects of dissipation coefficients

- Starting from the same condition at $t=10\text{ms}$
- Lower dissipation: steady amplitude $\delta B/B = 2 \times 10^{-3}$ with significant loss
- Higher dissipation $\rightarrow$ bursts with $\delta B/B = 5 \times 10^{-3}$ with 10% loss

$\nu = \eta/\mu_0 = \chi = 10^{-7}v_A R_0$

$n=2$ TAE peak amplitude

stored beam energy
Comparison of EP pressure profiles for different dissipation

- EP pressure profiles are very similar among the different dissipation coefficients.
- Higher dissipation leads to slightly higher EP pressure.

$t=20.0 \text{ ms}$
Simulation with loss boundary at $r/a=1$

- Simulation domain is extended to $1.6a \leq R \leq 4.8a$ and $-1.6a \leq z \leq 1.6a$.

- An MHD equilibrium is constructed with the same $q$ profile $q=1.2+1.8(r/a)^2$.

- Particle weight is set to be 0 at $r/a \geq 1$ (loss condition).
TAE bursts with loss boundary at r/a=1

\[ t \geq 17 \text{ms} \]

\[ \nu = \frac{\eta}{\mu_0} = \chi = 10^{-6} v_A R_0 \]
Saturation amplitude and time interval reduce after \( t=15 \text{ms} \).

Broadened EP pressure profile may account for the reduction in saturation amplitude and time interval.
Amplitude at $r/a=0.8$

- simulation:
  - $\delta B/B \approx 8 \times 10^{-3}$ at the mode peak location
  - $\delta B/B \approx 10^{-3}$ at $r/a=0.8$

- inferred from the plasma displacement [Durst et al., (1992)]
  - $\delta B/B \approx 10^{-3}$ at $r/a \approx 0.8$
Summary of TAE burst simulation with NL MHD effects

- TAE bursts are successfully simulated with NL MHD effects using time-dependent $f_0$.
  - saturation amplitude of the dominant harmonic with significant beam ion loss: $\delta B/B \sim 5-8 \times 10^{-3}$ at the mode peak location and $10^{-3}$ at $r/a=0.8$ (comparable to the TFTR experiment)

- Effects of dissipation
  - Low dissipation: steady amplitude with significant beam ion loss: $\delta B/B \sim 2 \times 10^{-3}$
  - High dissipation: bursts
  - Higher dissipation leads to higher stored beam energy