Hybrid MHD-Gyrokinetic codes: extended models, new implementations and forthcoming applications

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Outline

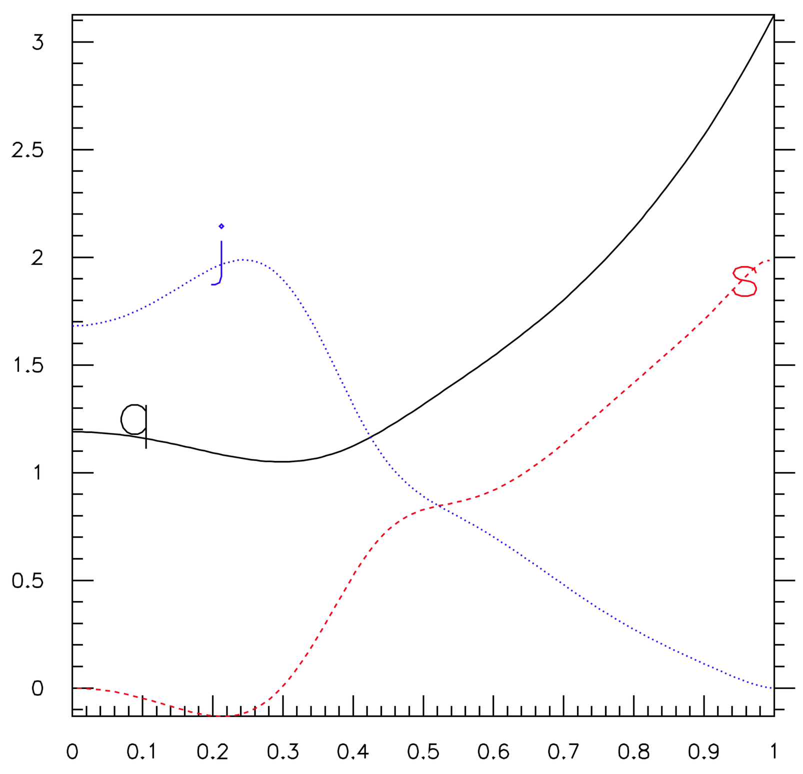
- The hybrid MHD-Gyrokinetic model has been proven to be very successful in describing the coupling between Alfvén waves and energetic particles and their mutual interaction in toroidal devices.
- HMGC, the nonlinear MHD-Gyrokinetic code originally developed at the Frascati laboratories, is being currently extended to include new physics.
- In this paper we will present the first simulations of an electron fishbone mode using the extended HMGC code.
- We will also present some benchmarks of the new hybrid code HYMAGYC (linear resistive MHD in general curvilinear geometry plus fully nonlinear gyrokinetic description, $k_\perp \rho_H \sim 1$, of the energetic particles) with the results obtained by HMGC and an analytical expression of the energetic particle response.

Electron fishbone simulations using the extended version of HMGC

- The simple physical model, originally used in HMGC [1] ($O(\varepsilon^3)$ nonlinear reduced MHD equations, circular shifted magnetic surface equilibrium, zero bulk plasma pressure, and drift-kinetic fast ions), has been recently extended to include new physics, which are currently under implementation and/or benchmarking [2]. These extensions include both thermal ion compressibility and diamagnetic effects, in order to account for thermal ion collisionless response to low-frequency Alfvénic modes driven by energetic particles (e.g., KBAEs), and finite parallel electric field due to parallel thermal electron pressure gradient, which enters the parallel Ohm's law and generalizes it, accounting for the kinetic thermal plasma response. Moreover, HMGC is now able to treat two independent particle populations kinetically, assuming different equilibrium distribution functions (as, e.g., bulk ions, energetic particles accelerated by NB, IRCH, fusion generated alpha particles, etc.).
- Internal kink instabilities exhibiting fishbone like nature have been observed in a variety of experiments where a high energy electron population was present (e.g., DIII-D, Compass-D, HL-1M, FTU and Tore Supra, ...).
- The relevance of the electron fishbones is primarily related to the fact that suprathermal electrons are characterized by relatively small width orbits, when compared with those of fast ions, similarly to the case of alpha particles in burning plasmas: thus, electron fishbones offer the opportunity to study the coupling between energetic particles and MHD like modes in burning plasma relevant conditions even in present machines.

FTU-like Equilibrium

$\varepsilon = a/R_0 = 0.35$;
$q_0 \approx 1.2$, $q_{min} \approx 1.05$, $r_{q\text{-}min}/a \approx 0.3$, $q_a \approx 3.1$;
$B_T = 5$ T;
$n_{i0} = 1 \times 10^{20}$ m^{-3}, $n_i(\psi)/n_{i0} = (1-\psi)^{1/2}$;
$T_{i0} = 2$ keV, $T_i(\psi)/T_{i0} = (1-\psi)$.



Bulk ions

- Bulk ions treated kinetically to describe ion Landau damping and finite compressibility: isotropic Maxwellian
- Note: strong time step subcycling required in order to describe properly electrons and ions simultaneously!

Energetic electrons

- Energetic electrons treated kinetically to describe resonant excitation
- strongly anisotropic Maxwellian (as, e.g., produced by Lower Hybrid heating)

$$f_{\rm electrons} \propto \frac{\hat n_{Ee}(\psi)}{\tau_{Ee}(\psi)^{3/2}} \Theta(\alpha;\alpha_0,\Delta) e^{-E/T_{Ee}(\psi)} \equiv \frac{\hat n_{Ee}(\psi)}{\tau_{Ee}(\psi)^{3/2}} \hat f_{\rm electrons}$$

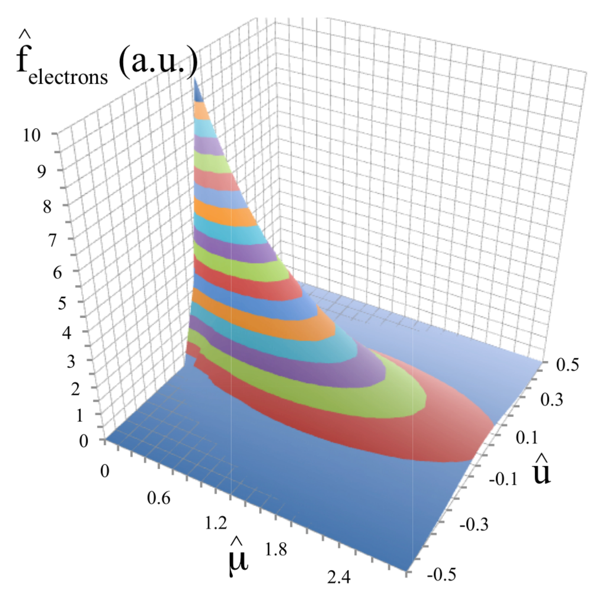
$$\Theta(\alpha;\alpha_0,\Delta) \equiv \frac{4}{\Delta\sqrt{\pi}} \frac{\exp\left[-\left(\frac{\cos\alpha-\cos\alpha_0}{\Delta}\right)^2\right]}{{\rm erf}\left(\frac{1-\cos\alpha_0}{\Delta}\right)+{\rm erf}\left(\frac{1+\cos\alpha_0}{\Delta}\right)}$$

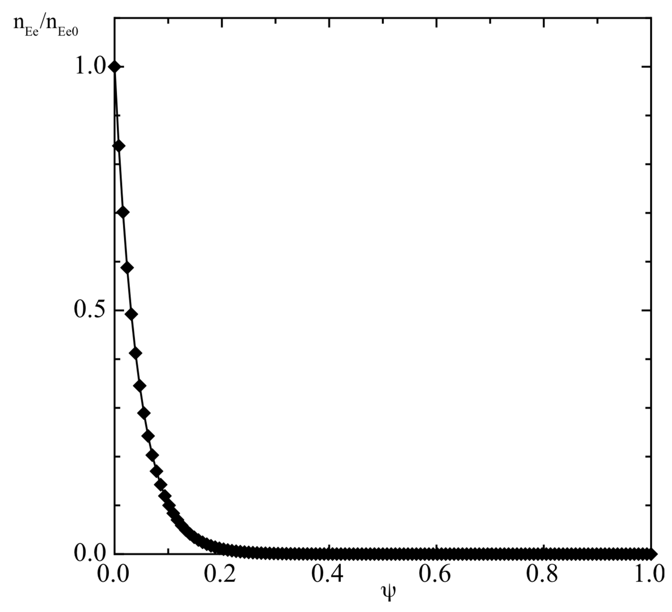
$$E = \frac{1}{2} m_e u^2 + \mu\Omega_{ce} \qquad \sin^2\alpha \equiv \frac{\mu\Omega_{ce}}{E} \qquad \cos\alpha \equiv \frac{u}{\sqrt{2E/m_e}}$$

$$\hat u \equiv u/v_{th0}\,, \quad v_{th0} = \sqrt{T_{Ee0}/m_e} \qquad \tau_{Ee}(\psi) \equiv T_{Ee}(\psi)/T_{Ee0}$$

$$\hat\mu \equiv \mu\Omega_{ce0}/T_{Ee0} \qquad T_{Ee} = T_{Ee0} = 50\,{\rm keV}$$

$$\cos\alpha_0 = 0\,, \quad \Delta = 0.1$$





- Note: the energetic electrons distribution function is assigned in terms of variables (E, α, ψ) which are not all constant of unperturbed motion: to prevent its relaxation in time, the (∇B) drift contribution to the source term in the equation that evolves the weight of the particles has been neglected, thus forcing the equilibrium distribution function to be constant in time

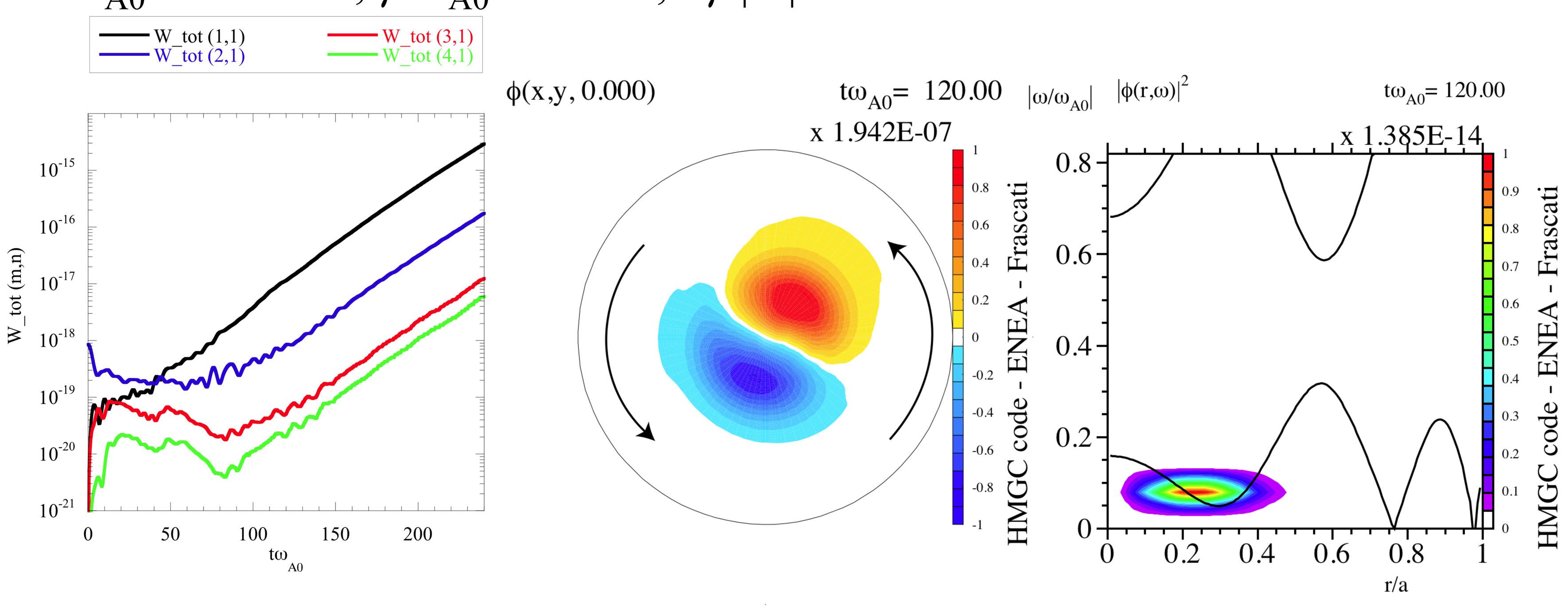
[1] S. BRIGUGLIO, G. VLAD, F. ZONCA, and C. KAR, Phys. Plasmas, 2, (1995) 3711.
[2] X. WANG, S. BRIGUGLIO, L. CHEN, G. FOGACCIA, G. VLAD, and F. ZONCA, Physics of Plasmas, 18, (2011) 052504.
[3] F. ZONCA et al., Nucl. Fusion 49 (2009) 085009.

Electron fishbone simulation results

- mode numbers: n=1, m=1,...,4
- $n_{Ee0}/n_{i0} \approx 0.055$

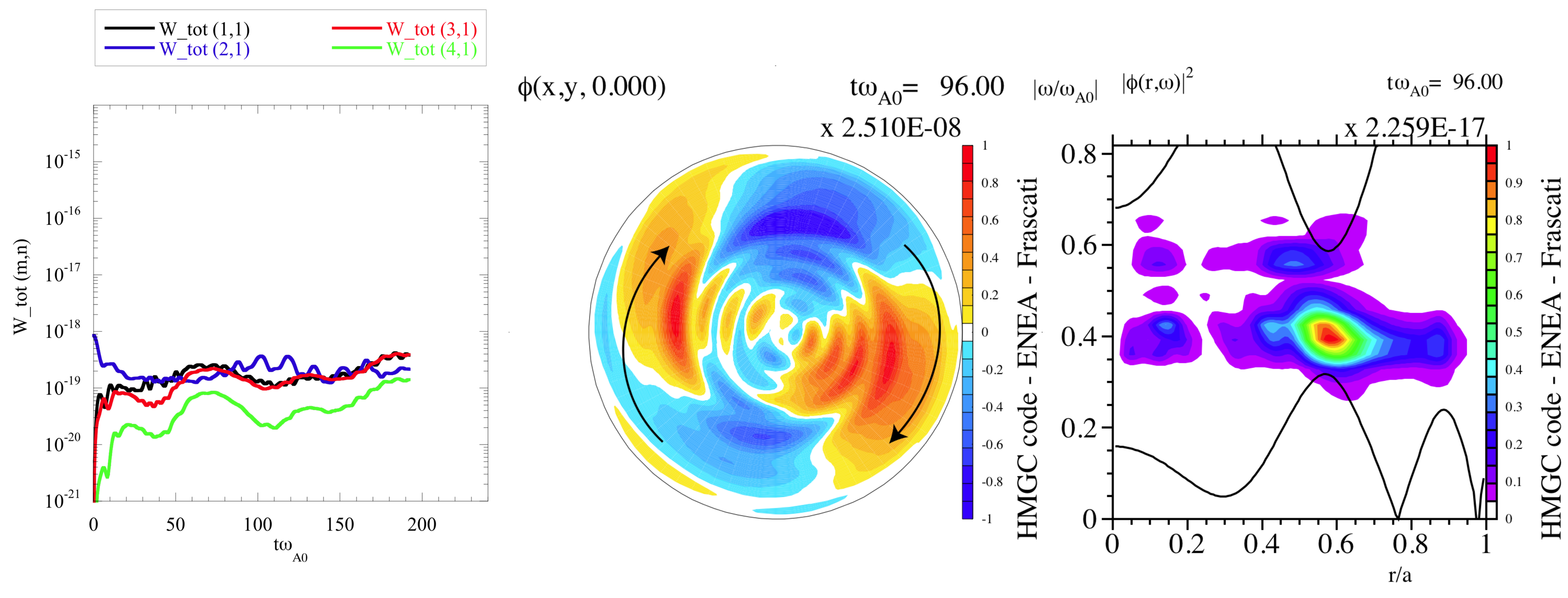
full simulation

- almost m=1 “step function”
- mode rotates in the direction of the diamagnetic velocity of the suprathermal electrons (counterclock-wise)
- $\omega/\omega_{A0} \approx$ -0.0815, $\gamma/\omega_{A0} \approx$ 0.024, $\gamma/|\omega| \approx$ 0.29



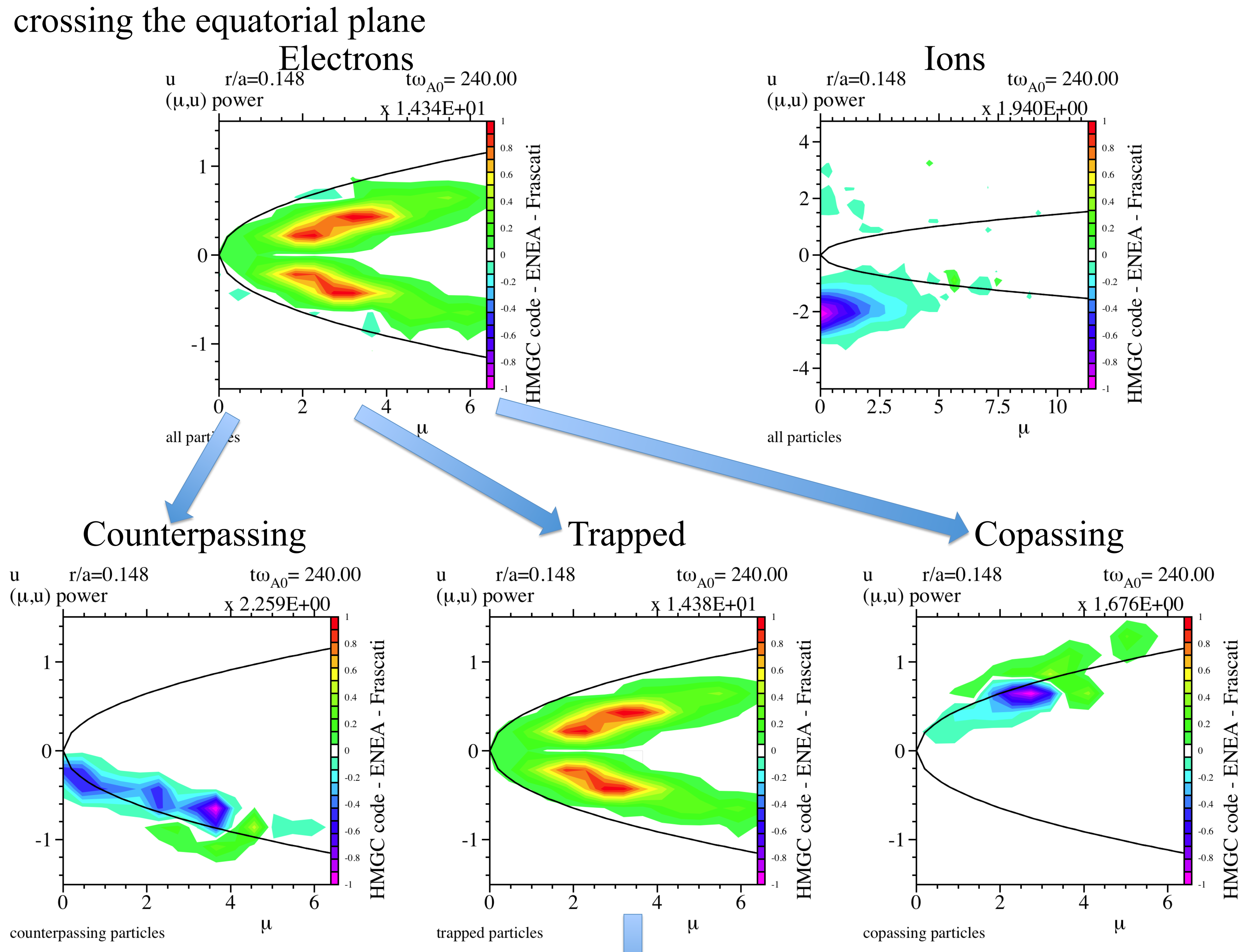
mirroring term off

- almost stable mode
- mode rotates in the opposite direction (clock-wise)



Power transfer between particles and wave at the radial position where the power exchange is maximum (r/a ≈ 0.15)

- note: each particle contribution is referred to the value of u value of that particle when crossing the equatorial plane

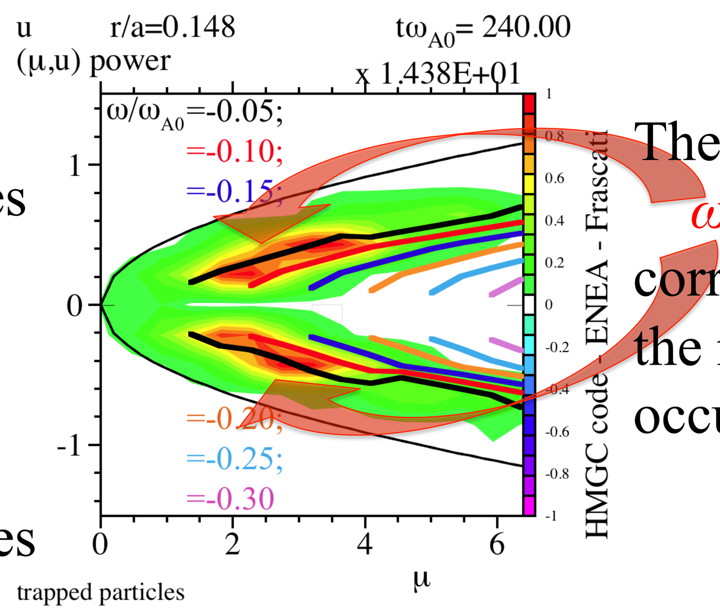


- While performing a numerical simulation, the HMGC code can also be used to evolve a set of test particles, and single particles characteristic frequencies can be computed
- A set of test particles has been initialized at the radial location where the power exchange is maximized, sampling the same region of space (μ, u) used for the power exchange plots

Trapped particle power exchange with the curves corresponding to

$\omega_{precession} - \omega = 0$

superimposed, for various values of ω; $\omega_{precession}$ (μ, u) isobtained by the test particles



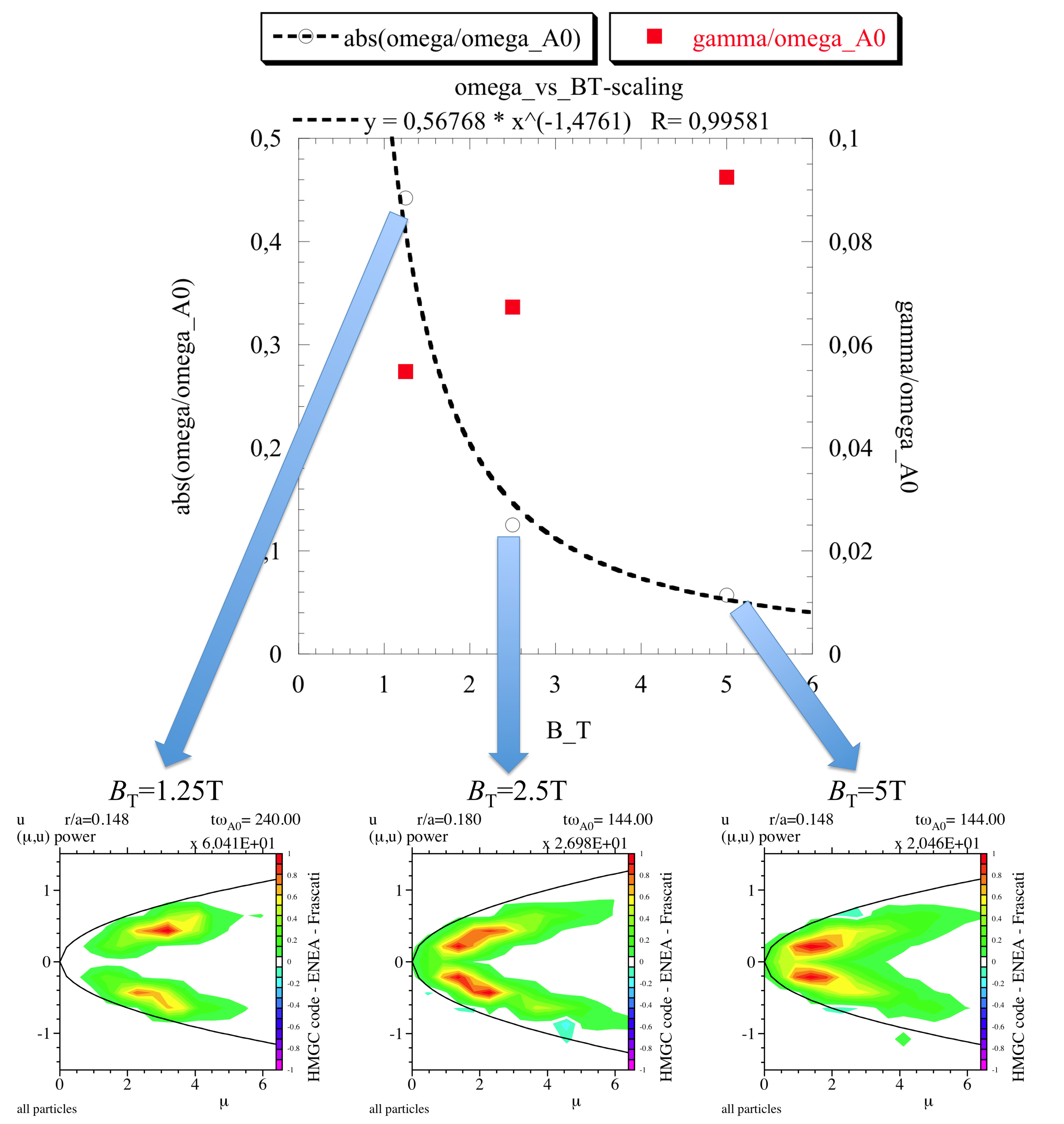
The simulation value:

$\omega_{sim}/\omega_{A0} \approx$ - 0.0815

corresponds to the region where the maximum power exchange occurs

Dependence of the mode frequency and growth rate of the e-fishbone on the magnetic field B_T

- Keep $\beta_{Ee,i}$ and $T_{Ee,i}$ constant
- use n_{Ee0}/n_{i0} = 0.2
- the mode frequency scales as $\omega/\omega_{A0} \propto 1/B_T^{1.5}$ (somewhat weaker than expected from an analytical scaling (see [3], Eq.(9)) where only deeply trapped particles were considered ($\omega/\omega_{A0} \cong \langle\omega_{dEe}\rangle/\omega_{A0} \propto 1/B_T^2$, with $\langle\omega_{dEe}\rangle$ the bounce averaged precession frequency of the energetic electrons.

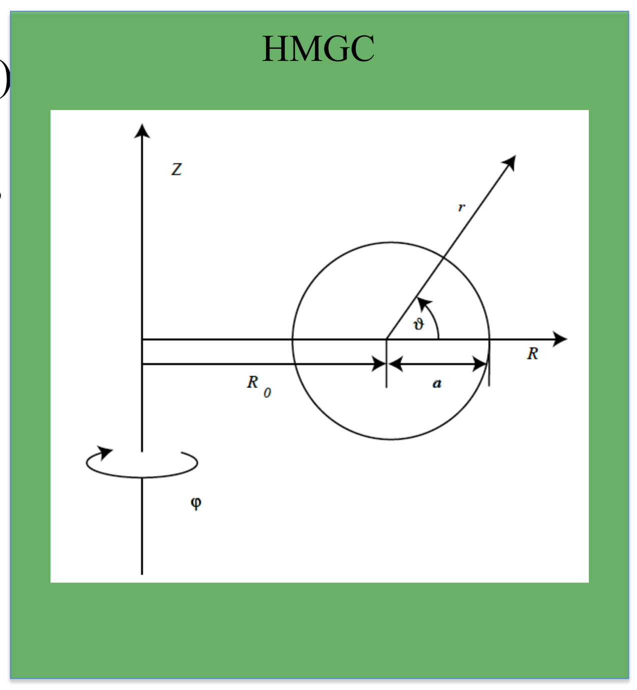


The region of highest power exchange moves along the resonance curves according to: $\mu \propto B_T^{-1/2}$

Benchmarks of HYMAGYC

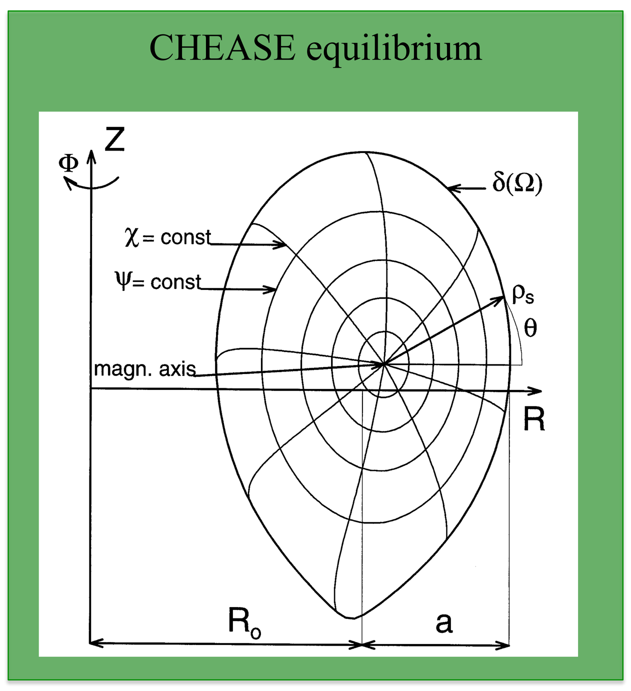
From present Frascati hybrid MHD-Gyrokinetic code: HMGC ...

- Thermal (core) plasma:
  - described by reduced $O(\varepsilon_0^3)$ visco-resistive MHD equations in the limit of zero pressure ($\varepsilon_0 \equiv a/R_0$ being the inverse aspect ratio of the torus; this model allows to investigate equilibria with shifted circular magnetic surfaces only).
  - MHD fields: ψ (poloidal flux function), ϕ (e.s. potential)
- Energetic-ion population:
  - described by the nonlinear gyrokinetic Vlasov equation, expanded up to order $O(\varepsilon)$ and $O(\varepsilon_B)$ with $\varepsilon \sim \rho_E / L_n$ (gyrokinetic ordering parameter, ρ_E being the energetic ion Larmor radius and L_n the equilibrium density scale length) and $\varepsilon_B \sim \rho_E / L_B < \varepsilon$ (L_B being the equilibrium magnetic field scale length), and in the $k_\perp \rho_E$ <<1 limit (with $k_\perp$ the component of the wave vector perpendicular to the magnetic field)
  - energetic particle pressure: $\Pi_\perp$, $\Pi_\parallel$,
  - magnetic drift orbit widths fully retained,
  - solved by particle-in-cell (PIC) techniques.
- Coordinates system (r, θ, φ)



... to the new Frascati hybrid MHD-Gyrokinetic code:

- Thermal (core) plasma:
  - described by full, resistive MHD linear equations
  - e.m. potentials required by Gyrokinetic module: $\mathbf{A}$, ϕ
  - Fluid nonlinearities will not be retained
- Energetic-ion population:
  - particle gyrocenter-coordinates are evolved by solving gyrokinetic eqs. up to order $O(\varepsilon^2)$ and $O(\varepsilon\varepsilon_B)$
  - perturbed quantities satisfy the nonlinear gyrokinetic ordering of Frieman-Chen, Phys. Fluids (1982) 23, 502 $\omega/\Omega_E \approx k_\parallel \rho_E = O(\varepsilon)$, $\quad k_\perp \rho_E = O(1)$ (with Ω_E the Larmor frequency and $k_\parallel$ the component of the wave vector parallel to the magnetic field)
  - returns energetic particles pressure tensor Π^{ij} computed in terms of the particle distribution function in gyrocenter coordinates
- Flux coordinates system (s, χ, φ)

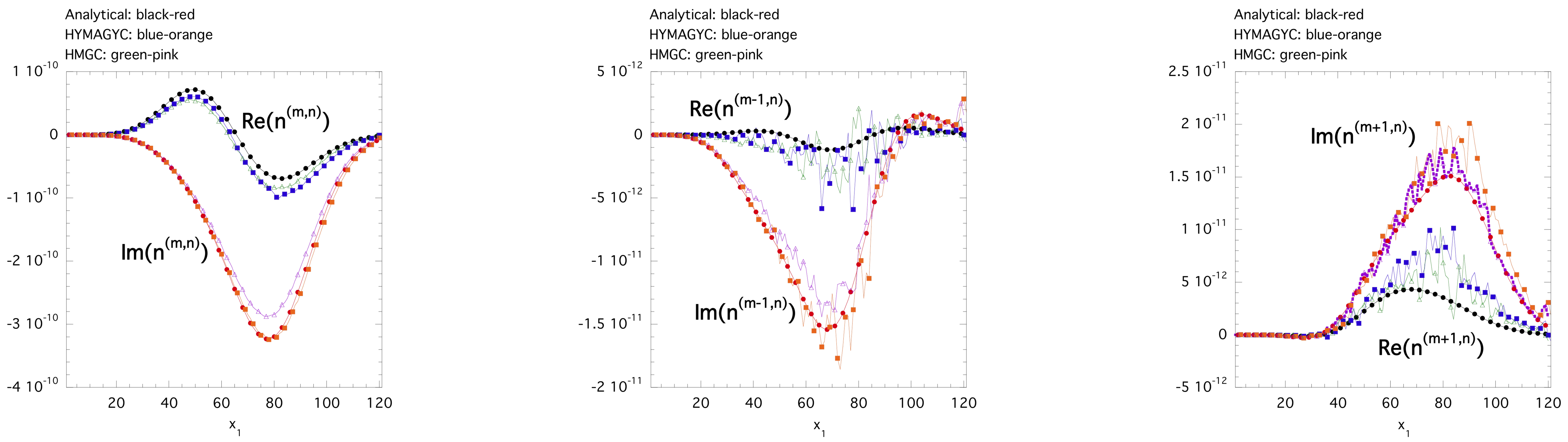


- In this section, we present some validation tests that have been performed on the gyrokinetic module newly written for HYMAGYC. To this purpose, the particles response to assigned time varying e.m. fields computed by HYMAGYC has been compared with an analytical solution and with the solution provided by HMGC.
- The analytical solution, which is well in the limit of validity also of HMGC, has been derived assuming:
  - ✓circular, concentric magnetic surfaces, small inverse aspect ratio $\varepsilon = a/R_0$= 0.01, uniform safety factor profile, and small Larmor gyroradius ρ_E/a=0.01.
  - ✓Only the parallel (to the equilibrium magnetic field) component of the vector potential and the electrostatic potential have been considered, with assigned real frequency and growth-rate, and a single Fourier component:

$$\phi^{(m,n)}(r,t) = \phi_0 (r/a)^m e^{-\left(\frac{r-r_0}{\Delta_0}\right)^2} e^{-i\omega t},$$
$$A_\parallel^{(m,n)}(r,t) = A_{\parallel,0} (r/a)^m e^{-\left(\frac{r-r_1}{\Delta_1}\right)^2} e^{-i\omega t}$$
$$\phi_0 = A_{\parallel,0} = 10^{-10}, \quad r_0/a = 0.3, \quad r_1/a = 0.6, \quad \Delta_0/a = \Delta_1/a = 0.2, \quad \omega/\omega_{A0} = (0.3 + i0.01)$$

  - ✓The distribution function for the energetic particles is bi-Maxwellian with uniform radial profiles and $T_{E,\perp}/T_{E,\parallel}$ = 0.01 and a normalized density profile $n_E(r)/n_{E0}$= exp[-2.5$(r/a)^2$].
  - ✓The analytical derivation follows the one obtained in [1], and has been obtained by neglecting the mirroring term in the parallel velocity equation of motion, and considering only unperturbed particles motion.

Radial profiles at a specific time



Time evolution at r/a = 0.5