EQUATIONS FOR DRIFT-ALFVÉN AND DRIFT-SOUND EIGENMODES IN TORIODAL PLASMAS

Ya.I. Kolesnichenko, B.S. Lepiavko, Yu.V. Yakovenko

Institute for Nuclear Research, Kyiv, Ukraine

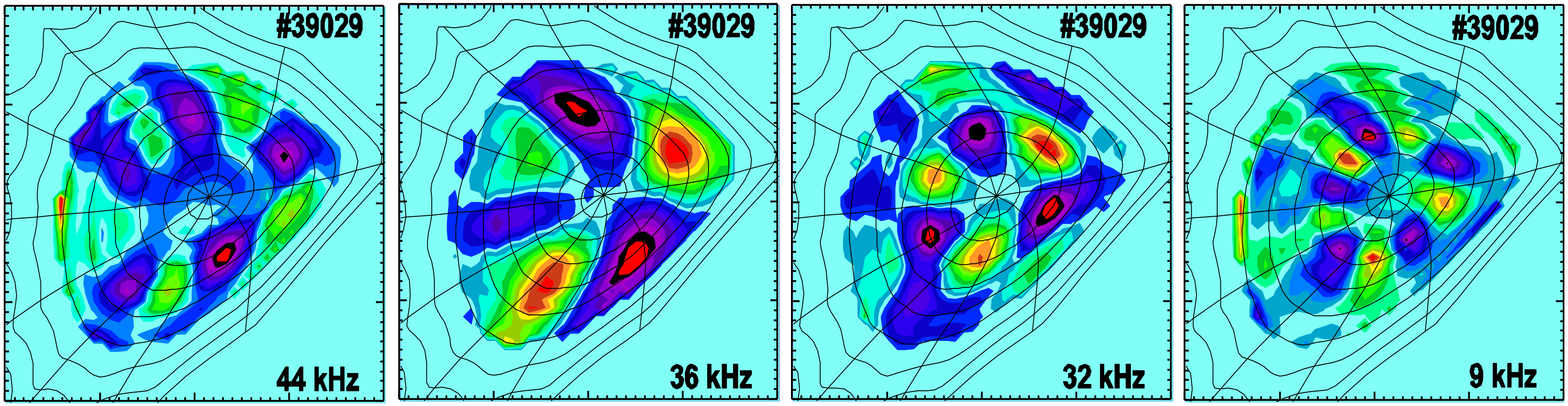
12th IAEA Technical Meeting on Energetic Particles in Magnetic Confinement Systems, Austin, USA

Outline

- Introduction
- Equations for drift-Alfvén and drift-sound waves
- Modelling HSX experiment

Instabilities in the W7-AS stellarator*

Discharge #39029



m=5 at 9 kHz and 44 kHz m=3 at 32 kHz and 36 kHz

Several low frequency instabilities occured simultaneously

*Weller A. et al., Physics of Plasmas, 2001, **8,** 931

Low frequency instabilities in W7-AS

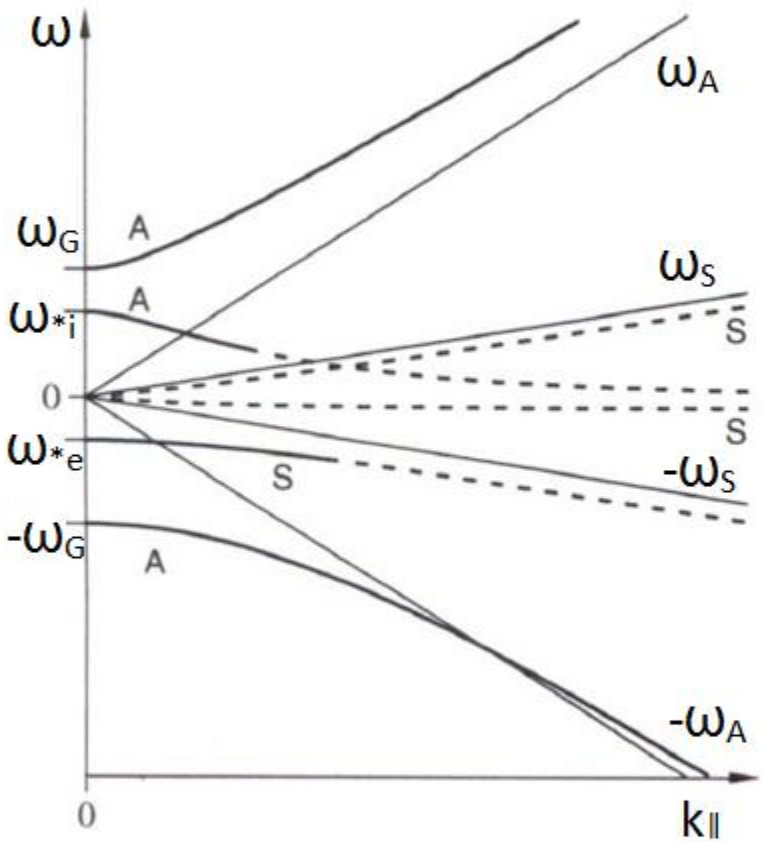
Using ideal MHD, the instabilities with 44 kHz and 32 kHz were identified as GAE modes, 36 kHz – NGAE *

The nature of the 9 kHz instability remained unclear: weakly damped ideal MHD modes have frequencies above the frequency of the geodesic acoustic modes, $\omega > \omega_{GAM}$, but $\omega_{GAM} \sim 20\,\mathrm{kHz}$, i.e., $\omega_{GAM} > 9\,\mathrm{kHz}$.

Ideal MHD may be not sufficient for the description of sub-GAM modes!

*Kolesnichenko Ya.I. et al., Phys. Plasmas, 2007, **14,** 102504

Drift-Alfvén and drift-sound continua [1)]



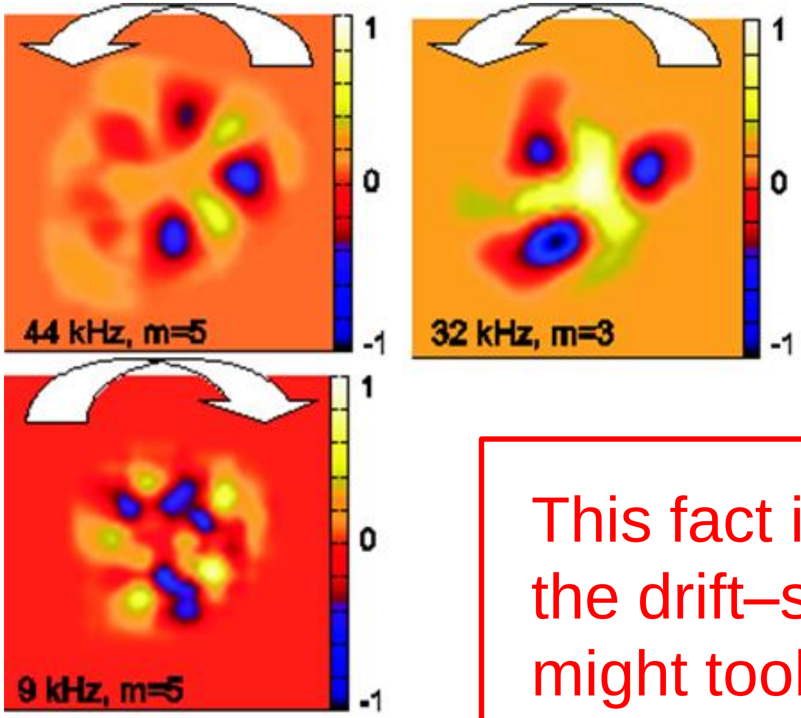
"A" labels drift-Alfven branches,
"S" labels drift-sound branches.
ω_{*e} , ω_{*i} are diamagnetic drift frequencies,
ω_A and ω_s are Alfvén and sound continuum branches,
ω_G is geodesic acoustic frequency.

$$\omega_G^2 = c_s^2 < K_G^2 >, K_G = \frac{2}{B}[K \times B]\cdot\nabla r$$

This picture is valid for the case of $\omega_G > \omega_{*i,e}$

[1)] Kolesnichenko Ya.I. et al., Europhysics Letters, 2009, **85,** 25004

Tomographical reconstructions of W7-AS discharges*

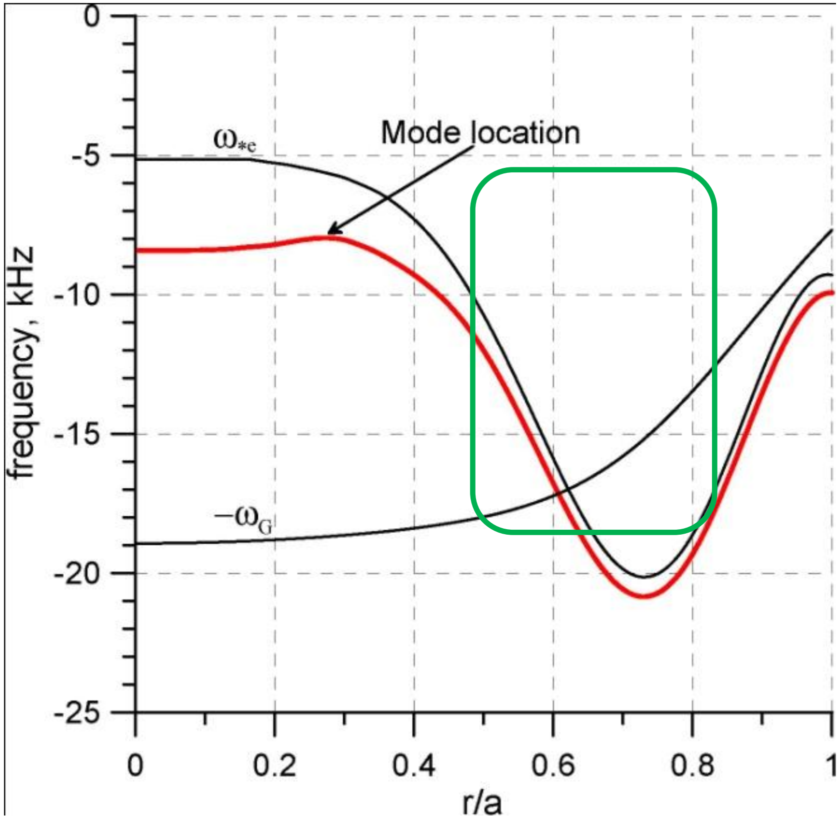


High-frequency modes rotate in the ion direction, while the low-frequency one in the opposite, electron direction. It corresponds with theory *

This fact indicates that an instability of the drift–sound type
might took place in W7-AS

*Kolesnichenko Ya.I. et al., Europhysics Letters. 2009, **85,** 25004

Modelling of the 9 kHz instability in the W7-AS discharge #39029



Drift-sound continuum (red curve), ω_{*e} and ω_G.

Note: there is a region (green frame), where $\omega_G > \omega_{*e}$

The observed instability can be a drift-sound eigenmode.

*Kolesnichenko Ya.I. et al., Europhysics Letters. 2009, **85,** 25004

Aim of this work

1. To extend equations of Ref.[1] for the case of arbitrary ratio of $\omega_G/\omega_{*i,e}$
2. To take into consideration inhomogeneity of the plasma temperature
3. To apply the equations for interpretation of the experimental data from HSX and W7-AS

Model used

Two-fluids collisionless hydrodynamics [2].
Plasma compressibility and finite values of ω_{*e} , ω_{*i} were taken into account.

$$v_{th,i} \ll \frac{\omega}{k_\parallel} \ll v_{th,e}$$

Different sets of equations for electrons and ions are to be used.

1. Kolesnichenko Ya.I. et al., Europhysics Letters. 2009, **85,** 25004
2. Ramos J.J. , Physics of plasmas. 2005, **12**, 052102

Basic equations for the ions

$$\left[\frac{\partial}{\partial t} + (\mathbf{v}_i \cdot \nabla)\right] n_i + n_i \nabla \cdot \mathbf{v}_i = 0,$$

$$M_i n_i \left[\frac{\partial}{\partial t} + (\mathbf{v}_i - \mathbf{u}_{*i}) \cdot \nabla\right] \mathbf{v}_i = -\nabla p_{i\perp} - (\mathbf{B} \cdot \nabla)\left(\frac{p_{i\|} - p_{i\perp}}{B^2}\mathbf{B}\right) + e_i n_i \left(\mathbf{E} + \frac{1}{c}\mathbf{v}_i \times \mathbf{B}\right)$$

$$\left[\frac{\partial}{\partial t} + (\mathbf{v}_i \cdot \nabla)\right] p_{i\|} + p_{i\|} \nabla \cdot \mathbf{v}_i + \frac{2p_{i\|}}{B^2}\mathbf{B} \cdot (\mathbf{B} \cdot \nabla)\mathbf{v}_i = -2\nabla \cdot \mathbf{q}_{B\perp} + 4\boldsymbol{\kappa} \cdot \mathbf{q}_{B\perp},$$

$$\left[\frac{\partial}{\partial t} + (\mathbf{v}_i \cdot \nabla)\right] p_{i\perp} + 2p_{i\perp} \nabla \cdot \mathbf{v}_i - \frac{p_{i\perp}}{B^2}\mathbf{B} \cdot (\mathbf{B} \cdot \nabla)\mathbf{v}_i = -\nabla \cdot \mathbf{q}_{T\perp} - 2\boldsymbol{\kappa} \cdot \mathbf{q}_{B\perp},$$

$\mathrm{p}_{\mathrm{i}\|,\perp}$ ion paral./perp. pressure, $u_{i*} = -\frac{c}{e_i n_i}\nabla \times \frac{2p_{i\perp}}{B^2}$

$\mathbf{q}_{\mathrm{B}\perp}, \mathbf{q}_{\mathrm{T}\perp}$ are perpendicular heat fluxes, parallel fluxes neglected

Ramos J.J. , Physics of plasmas. 2005, **12**, 052102

Basic equations for the electrons

$$e_e n_e \left(\mathbf{E} + \frac{1}{c} \mathbf{v}_e \times \mathbf{B} \right) - \nabla p_e = 0$$

$$\mathrm{n_i = n_e} \qquad\qquad \mathbf{B} \cdot \nabla \mathrm{T_e} = 0$$

$v_{i,e}$ is ion/electron fluid velocity, p_e, electron pressure

Derived equations for drift-Alfvén and drift-sound modes

$$\hat{\omega}^3\left(\hat{\omega}-\hat{\omega}_{en}\right)W_{\parallel}+c_e^2\hat{\omega}\left[(1+3\tau)\hat{\omega}-2\tau\hat{\omega}_{en}-\hat{\omega}_{iT}\right]\nabla_{\parallel}^2 W_{\parallel}$$

$$=-c_e^2\left[2(1+2\tau)\hat{\omega}+\hat{\omega}_{in}-2\hat{\omega}_{iT}-\tau\hat{\omega}_{en}\right](\hat{\omega}-\hat{\omega}_{*i})\nabla_{\parallel}^2\frac{c}{B^2}(\mathbf{B}\times\boldsymbol{\mathcal{K}})\cdot\nabla_{\perp}\Phi.$$

$$\nabla\cdot\frac{\hat{\omega}^2}{v_A^2}(\hat{\omega}-\hat{\omega}_{*i})(\hat{\omega}-\hat{\omega}_{*e})\left(\hat{\omega}+\frac{1}{\tau}\hat{\omega}_{*i}\right)\nabla_{\perp}\Phi+\hat{\omega}^3\left(\hat{\omega}+\frac{1}{\tau}\hat{\omega}_{*i}\right)B_0\nabla_{\parallel}\left\{\frac{1}{B_0^2}\nabla\cdot\left[B_0\nabla_{\perp}\left(\frac{1}{B_0}\nabla_{\parallel}\Phi\right)\right]\right\}=$$
$$\nabla\cdot\frac{2c_e^2}{cv_A^2}(\mathbf{B}_0\times\boldsymbol{\mathcal{K}})\left[2(\hat{\omega}-\hat{\omega}_{*i})(\hat{\omega}-\hat{\omega}_{*e})\left(\hat{\omega}+\frac{7}{4}\tau\hat{\omega}+\frac{3}{4}\hat{\omega}_{*i}\right)\frac{c(\mathbf{B}_0\times\boldsymbol{\mathcal{K}})}{B_0^2}\cdot\nabla_{\perp}\Phi\right.$$
$$\left.+(1+\tau+\tau(\hat{\omega}-\hat{\omega}_{*e}))\left(\hat{\omega}+\frac{1}{\tau}\hat{\omega}_{*i}\right)\hat{\omega}W_{\parallel}\right]$$

$\tau = \mathrm{T_i}/\mathrm{T_e}$ $\quad c_e = \sqrt{\mathrm{T_e}/\mathrm{M_i}}$

Φ is the scalar potential of the electromagnetic field

$W_{\parallel} = \nabla_{\parallel}\tilde{v}_{\parallel}$ represents plasma compressibility

ω_{in}, ω_{en}, ω_{iT} are ion/electron drift frequencies:

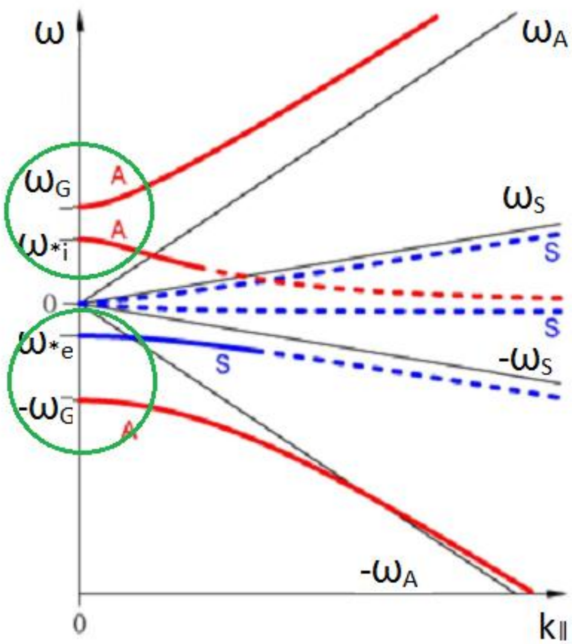
$$\omega_{in}\sim\frac{\partial n}{\partial r}\qquad \omega_{en}\sim\frac{\partial n}{\partial r}\qquad \omega_{iT}\sim\frac{\partial T}{\partial r}$$

Drift-Alfvén and drift-sound continua

$$(\omega-\omega_{*i})(\omega-\omega_{*e})\left(\omega+\frac{1}{\tau}\omega_{*i}\right)+v_A^2k_{mn}^2\left(\omega+\frac{1}{\tau}\omega_{*i}\right)=0 \quad \text{A}$$

$$\omega^2(\omega-\omega_{en})+c_e^2k_{mn}^2(\omega(1+3\tau)-\tau\omega_{en}-\omega_{iT})=0 \quad \text{S}$$

$\omega_G \gg \omega_{*i,e}$



$$\omega_G=\frac{c_{ei}}{R}\sqrt{\frac{2}{\delta_0}\left(1+\frac{7}{4}\tau\right)}$$

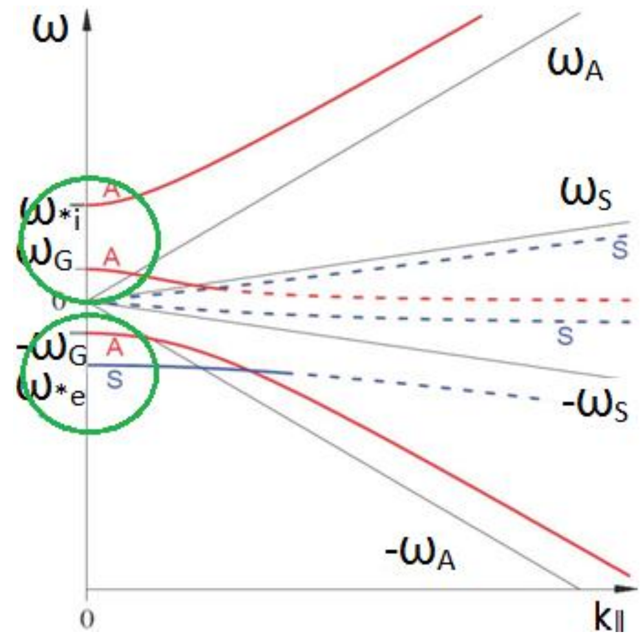
$$\omega_{*i}=\frac{mcp_i'}{e_in_iBr}$$

$$\omega_{*e}=\frac{mcT_en_e'}{e_in_iBr}$$

$$k_{mn}=\frac{m\iota-n}{R}$$

* Kolesnichenko Ya.I. et al., Europhysics Letters. 2009, **85,** 25004

$\omega_G \ll \omega_{*i,e}$



$$\omega_G=\frac{c_{ei}}{R}\sqrt{\frac{3\tau}{2\delta_0}}$$

Instabilities in HSX

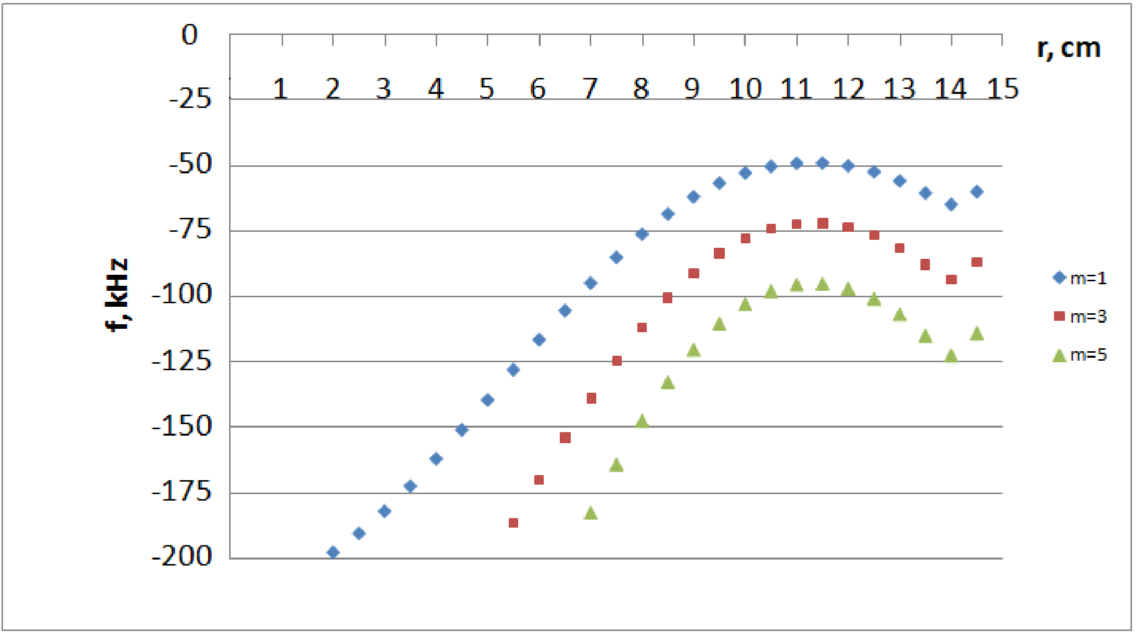
In Ref. [1] instabilities with n = 1 and odd m in the frequency range 50-90 kHz were reported:

$f_{m=5}$ = 50 kHz
$f_{m=7}$ = 72 kHz
$f_{m=9}$ = 94 kHz

It is of interest to see whether these instabilities can be identified by means of the derived equations. In this work, only the first step was done in this direction: drift-sound continuum branches were calculated.

1. Deng C.B. et al., Phys. Rev. Lett. 2009, **103**, 025003

Calculated drift-sound continua



HSX parameters used:

$$N = 1{,}56 \times \left[1 - \left(\frac{r}{a}\right)^2\right]^2 + 0{,}02 \quad \times 10^{12}\,\mathrm{cm}^{-3}$$

$$T_e = 950 \times \left[1 - \left(\frac{r}{a}\right)^2\right]^6 + 78\ eV$$

$T_e \gg T_i$ $\quad \iota = 1$

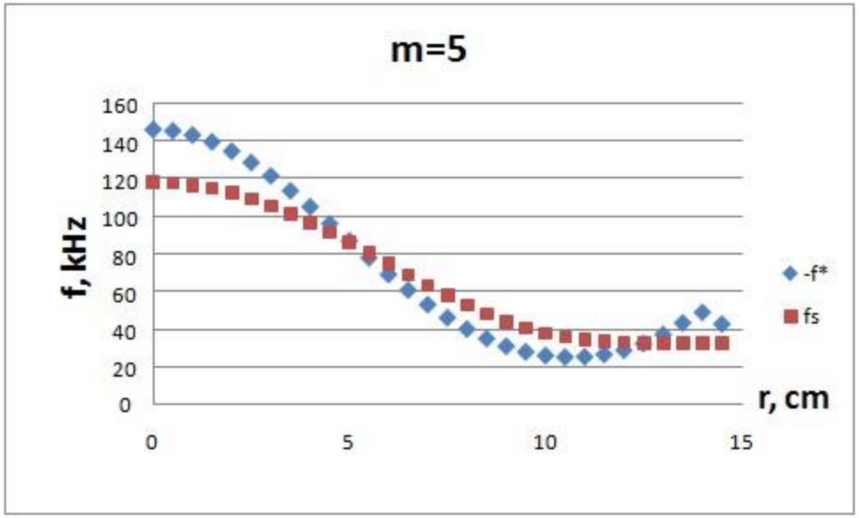
Frequencies of continuum maxima are:

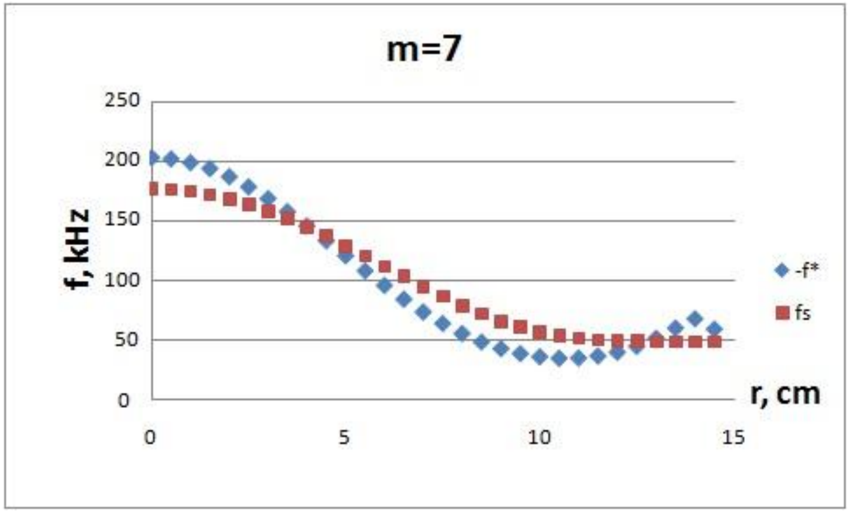
$f_{m=5}$ = 49,3 kHz, $f_{m=7}$ = 72,2 kHz, $f_{m=9}$ = 95,0 kHz

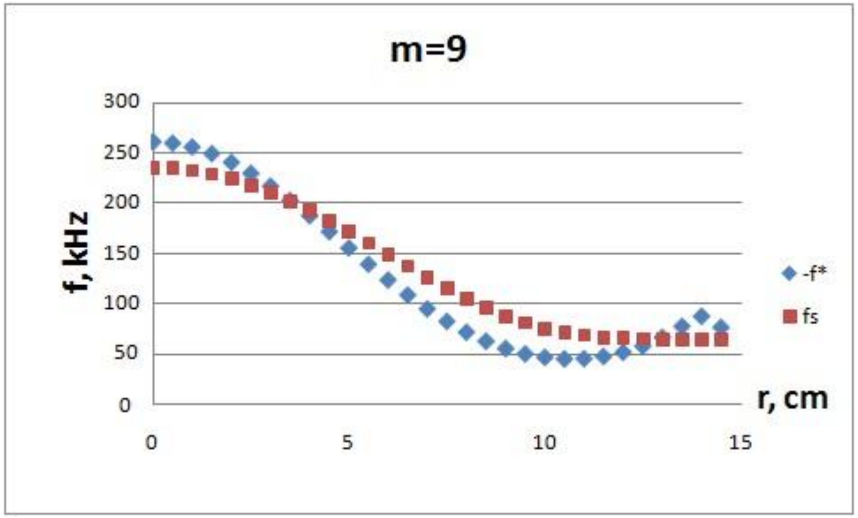
These frequencies are close to experimental values:

$f_{m=5}$ = 50 kHz, $f_{m=7}$ = 72 kHz, $f_{m=9}$ = 94 kHz

Role of finite diamagnetic drift frequency







Contributions of drift (f*) and sound (fs) frequencies to the continua with n = 1, m = 5,7,9. We conclude that extrema in the continua appear due to drift frequency.

Conclusions

- Equations for drift-sound and drift-Alfvén eigenmodes in toroidal plasmas are derived. Plasma compressibility, inhomogeneity, and finite values of diamagnetic drift frequencies are taken into account.

- These equations are applicable to tokamaks and stellarators.

- Preliminary analysis indicates that the modes observed in HSX can be identified as drift-sound modes. It is found that the diamagnetic drift frequency plays a crucial role in formation of extrema of continuum branches.