Nonperturbative simulations of fishbones and kink mode stabilization in ITER

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ABSTRACT

We employ the global NOVA-KN hybrid kinetic-MHD code to study the stability properties of low-n solutions, such as the resonant (fishbone) and non-resonant (ideal branch) nonperturbatively. The nonperturbative approach treats fast ions with their realistic orbits numerically by computing the moments of their perturbed pressure tensors in order to include them into the eigenmode equation. We introduce this technique together with the new conforming velocity space grid to efficiently evaluate the wave-particle interaction matrix. The used method results in both resonant and modified non-resonant branches, which are further studied to understand their stability properties in the presence of energetic ions [Cheng(1992)]. We include the destabilizing effects from energetic beam ions and alpha particles, which seem to be important for the studied instabilities. A model used for beam ion distribution is also presented.
We study the properties of those branches in details. The applications to the modified burning ITER plasma are discussed to understand how far the stability region is in the operating space from its nominal values. The used approach complements earlier analytic approaches to the problem such as in
Ref.[Hu et al.(2006)Hu, Betti, and Manickam].

C. Z. Cheng, Phys. Reports **211**, 1 (1992).

B. Hu, R. Betti, and J. Manickam, Phys. Plasmas **13**, 112505 (2006).

Long history from '74: Does it need anything new? Linear?

$n = 1$ internal MHD mode and fishbone modes (nonpert.)

stabil.	thresh.	EP	EP df	rotat.	new plas.	V&V	period
+	+	+	+/-	+	-	-	-

Should we improve/renew $n = 1$ stability linear codes? Why?

- Need to address sawteeth period. Kin. approach can help?
- Varify new theory elements
- Efficiency in addressing EP df. modification, dispersion
- More accurate damping on continuum
- Efficient/fast computation
- Test bed for finite f simulations

Outline

Use NOVA-K to model/verify $n = 1$ mode stabilisation

- Use analytic input or TRANSP for plasma parameters
- Mode structure is computed within ideal MHD (NOVA)
- Perturbative kinetic mode analysis is performed with NOVA-K code
- Program $n = 1$ mode dispersion for internal kinks only
- Fast ions model includes:
  - finite orbit width (FOW) and FLR effects (Gorelenkov PoP,'99)
  - advanced/flexible EP distribution function models (Gorelenkov NF,'05)
- Damping mechanisms are not included, rely on MHD theory

NOVA-K - perturbative approach

$$\delta K = \delta W \equiv \delta W_{MHD} + \delta W_{kf}, \delta K = \omega^2 \int \rho \xi^2 d^3 r. \quad (1)$$

Generalize dispersion relation (Cheng, PhR 92) to account for EP

$$-i\omega \left(1 + \frac{\omega_T^2}{\omega_s^2 - \omega^2}\right) = \gamma_{MHD} \left(1 + \frac{\gamma_f}{\gamma_{MHD}}\right), \frac{\gamma_f}{\gamma_{MHD}} = -\frac{\Re \delta W_{kf}}{\delta K},$$

where $\omega_T^2 = 2\gamma_s P_c \kappa/\rho$, $\omega_s^2 = (1/2)\gamma_s \beta_c \omega_A$, κ - curvature, $\gamma_s = 5/3$.
Real part of $\Re \delta W_{kf}$ can give stabilization

$$\delta W_{kf} = -(2\pi)^2 e_\alpha c \int dP_\varphi d\mu d\mathscr{E} \tau_b \sum_{m,m',l} \frac{X^*_{m,l}(\omega - \omega_*) X_{m',l}}{\omega - \overline{\omega_d}} \frac{\partial F_f}{\partial \mathscr{E}}, \quad (2)$$

Need $\omega < \omega_d$ for stabilization, $\omega_* = -i \frac{\partial F/\partial P_\varphi}{\partial F/\partial \mathscr{E}} \frac{\partial}{\partial \varphi}$ $X_{m,l}$ - WP interaction.

- $\omega \to \omega - \Omega_E(\psi)$ in the above expression for δW_{kf}.
- If 1/1 mode frequency is $\omega \simeq \Omega_E|_{\psi_{q=1}}$, then shear rotation remains.
- Rotation effect in the denominator in δW_{kh} is dominant.

NOVA-KN - (N)onperturbative advantages

Advantages:
See structure dispersion modification due to EPs
Address instability in the presence of strong drive: balanced to the strong damping can still be unstable
Disadvantages:
Is it required? Which problem(s) would need it?
Modes such as EPMs have to be studied with N.
Old strongly driven modes can also be studied: TAEs, RSAEs ...

incorporate EPs in eigenmode equations, (N)

NOVA-KN approach to include EPs via displacement to pressure coupling

$$\hat{L}\xi + \hat{L}_h \boldsymbol{p}_h = 0$$

$$\delta p_{h\perp} = m_h \int d\mathbf{v} \mu B \hat{g}, \delta p_{h\|} = m_h \int d\mathbf{v} 2(\mathscr{E} - \mu B)\hat{g}$$

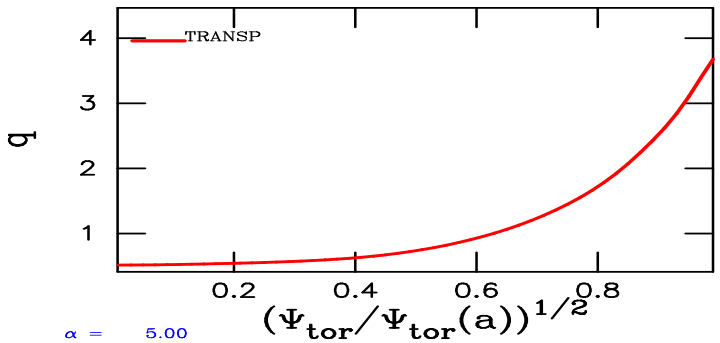
$\hat{g}$ - non-adiabatic perturbed distribution function (Gorelenkov, Phys.Plasmas, '99).
Standard approach using bounce harmonics but put $p = 0$ for $m = 1$, $\omega \ll \omega_b$ studies

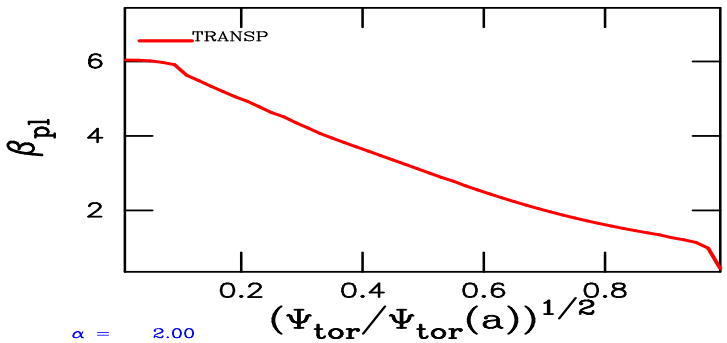
$$\hat{g} = \mathscr{E} \sum_m \frac{(\omega - \omega_*) G_m}{\omega - \overline{\omega_d}} \frac{\partial F_h}{\partial \mathscr{E}} e^{-it\omega + im\theta - in\varphi}, \tag{3}$$

ω_d - precession, G_m - wave-particle interactions.
The method can be extended to arbitrary frequency modes. High m θ-variations may not be captured.

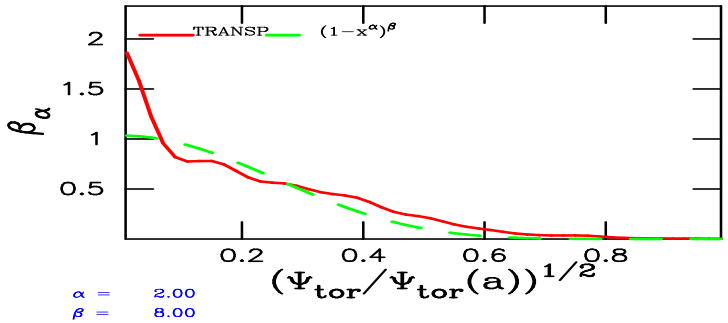
Flat $q_0 < 1$ profile plasma from TRANSP runs

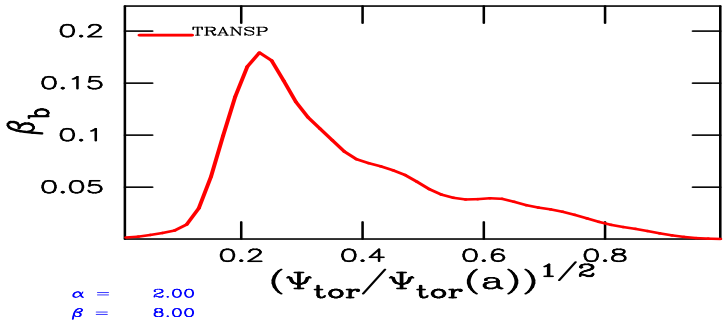
Modify q-profile for unstable $n = 1$ internal kink: lower q_0 only to under 1, keep profile, edge values fixed as per TRANSP values, 20100p07 (Budny, NF'09).





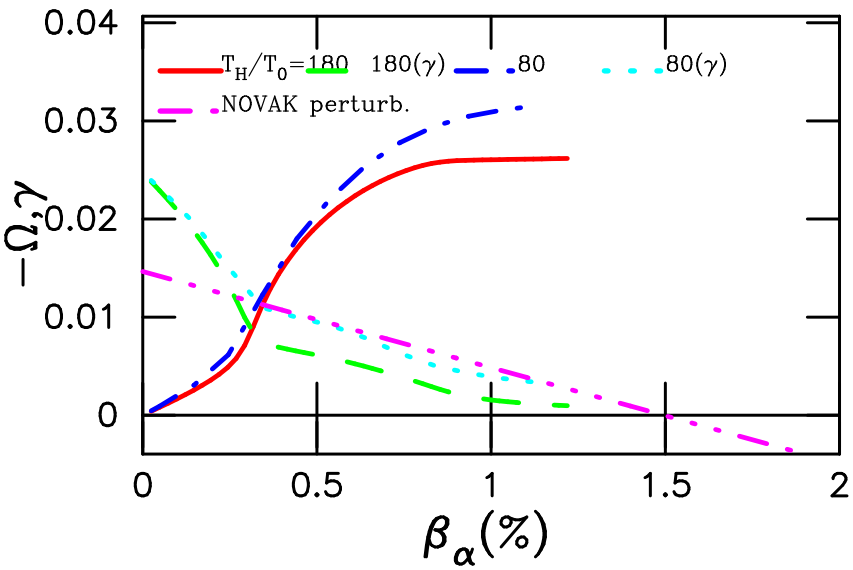
TRANSP computes fusion product and beam ion profiles: substitute with the model profile.

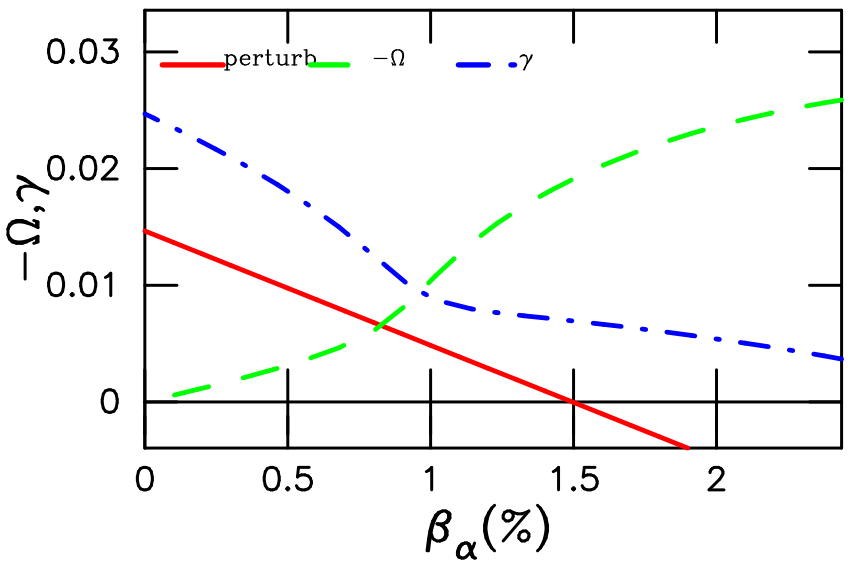




Ideal $m = 1$ mode is stabilized directly by β_α increase

Compare perturbative and nonperturbative stabilization solutions.

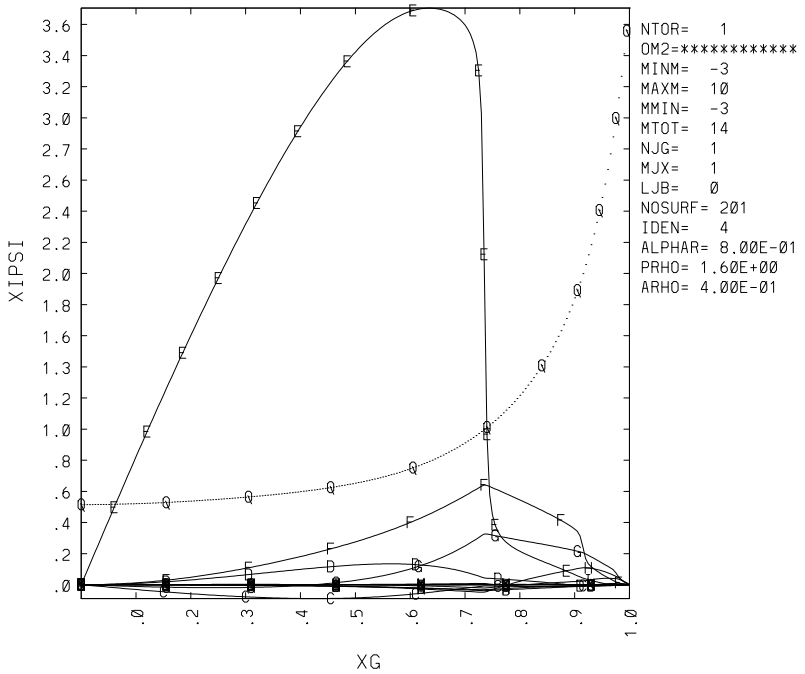




Perturbative and non-perturbative approaches are close. Can not be exact.

Ideal mode structure is computed close to step-function

Both NK and K versions have the same "guess" mode/structure.
NK transforms guess to nonperturbative branch via a series of equilibrium changes.

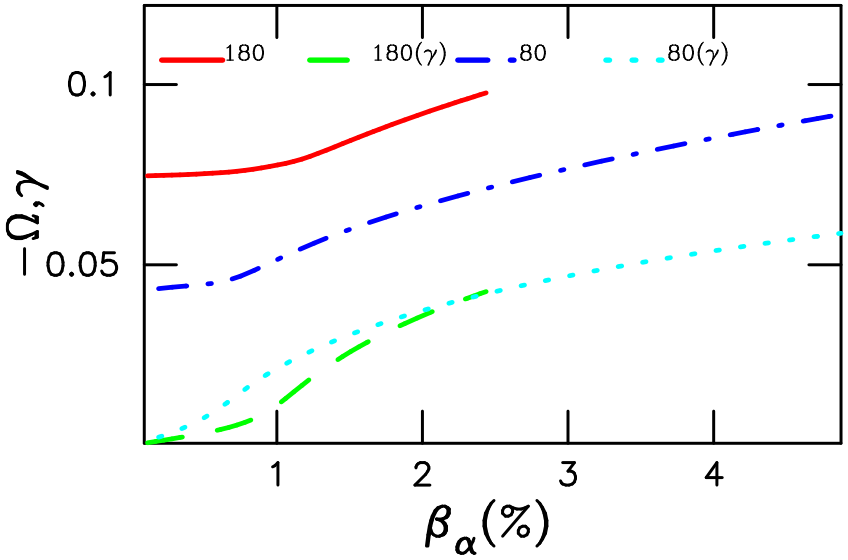


Increase in β_f modifies the nonideal region as expected.

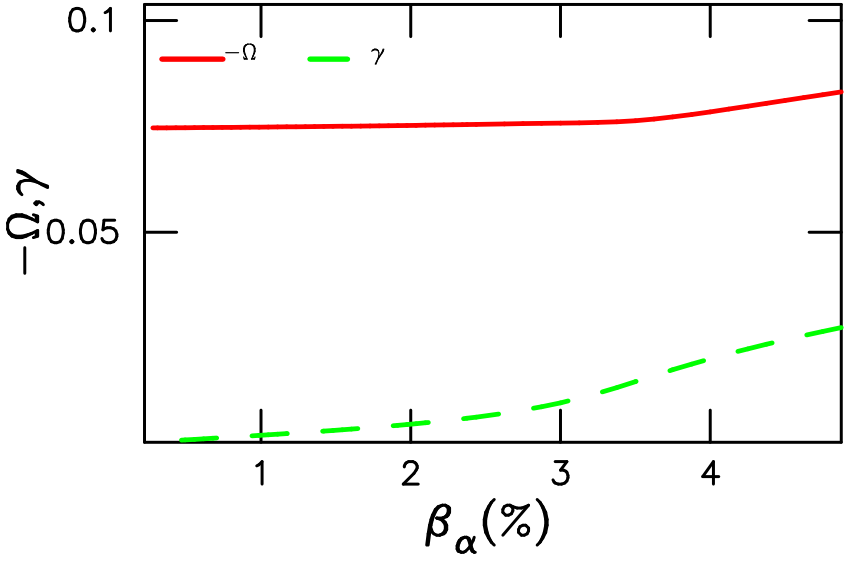
Fishbone mode instability

Destabilization of the fishbone is sensitive to the pitch angle distribution.

Narrow as $\Delta\lambda = 0.05$ or $(\sim 0.05 R_0)$



Almost isotropic, $\Delta\lambda = 5$

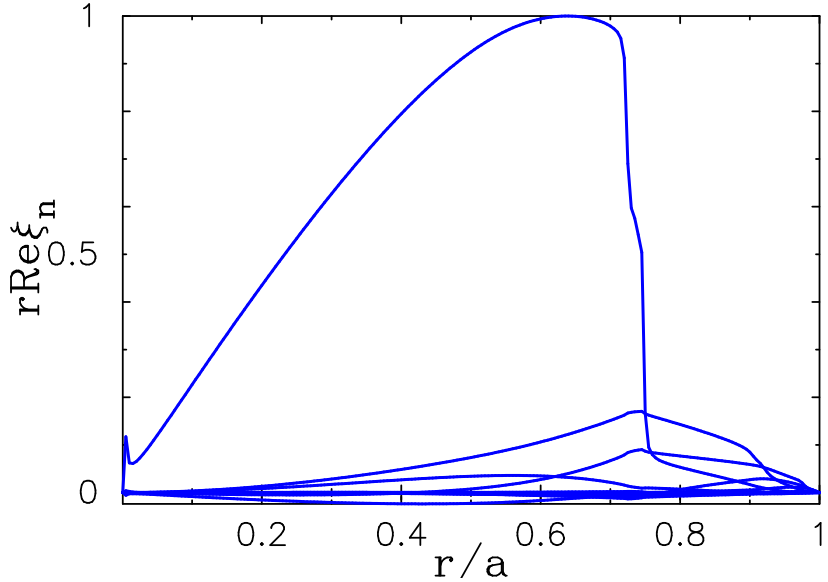


$$F = n_\alpha \exp\left[-\frac{\lambda - \lambda_0}{\Delta\lambda}\right], \lambda_0 = 1.$$

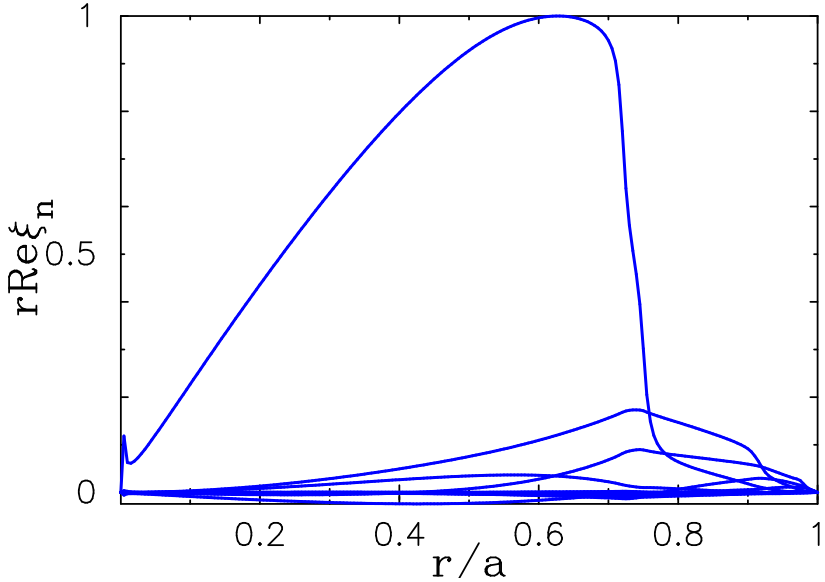
Fishbone mode structure is sensitive to EP component

Double step-function is confirmed numerically [Ödblom, PoP2002].
Take two cases with anisotropic EP pressure.

$\beta_\alpha = 1\%$



$\beta_\alpha = 2.5\%$



More realistic damping rates are expected/incorporated.

Why CG is required

- Success of the hybrid continuum code development depends on the chosen grid/number of points
  - The problem is 2D (fluid part) & 3D (kinetic part)
  - Key for non-perturbative simulations $t_{run} \sim N^{2+3}$, memory $\sim N^{2+3(+1)}$
  - Provides mapping - each initial value code "must have"
    - Good idea for resonance location to load more particles
    - Communicate/interface with other codes: mapping; efficient interface with 2D code, such as NOVA, PEST ...
  - In the *long run* helpful for the quasilinear diffusion theory

Definition

Conforming grid (CG):

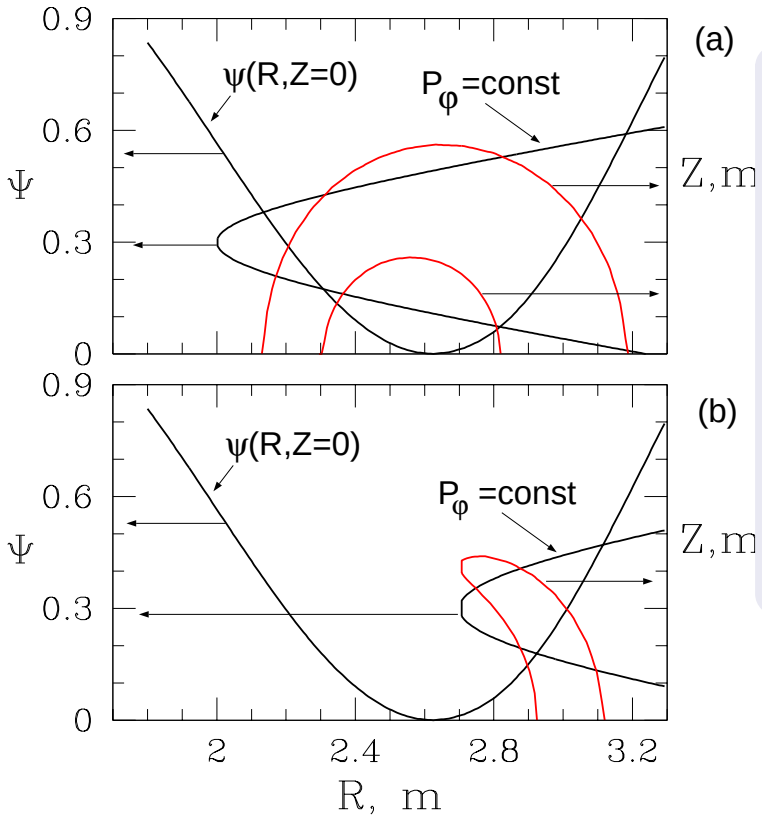
COM conforming grid is aligned with the topological boundaries of the confinement domains in the phase space

- compatible with the flux coordinates
- easy to account for singularities, such as at the trapped/passing separatrix.
- approach separatrix arbitrarily close.
  - avoid amost uncontrollable increase of number of points in Monte Carlo - like simulations, typically reduction is by $\left(10^2\right)^3$. . .

- COM (constants of motion) is natural choice for the phase space integrations - key in WPI theories:

$$d\Gamma = d\mathbf{r}d\mathbf{v} = (2\pi)^2 \sum_{\sigma_\parallel} \frac{\mathscr{E}}{\omega_{ch0}} dP_\varphi d\lambda d\mathscr{E} dt = 2\pi \sum_{\sigma_\parallel} \frac{\mathscr{E}}{\omega_{ch0}\,\omega_{bounce}} dP_\varphi d\lambda d\mathscr{E}$$

From real space to phase space

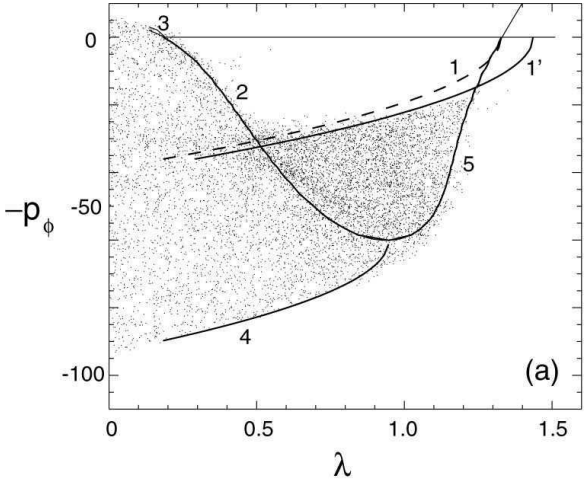


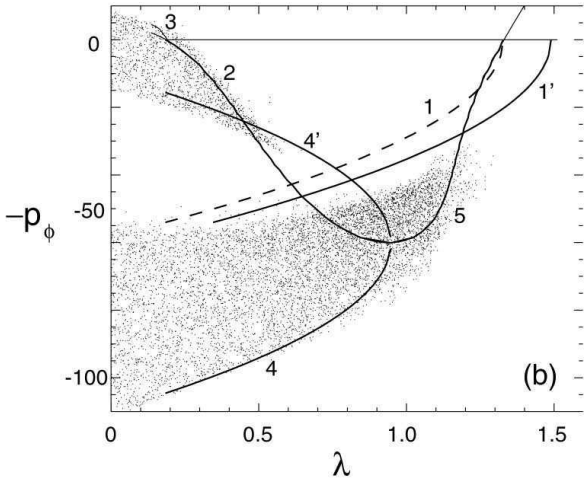
Gorelenkov et.al. PoP'99

- Choose following set of variables: v, λ. P_φ.
  - $\lambda = \mu B_0/\mathscr{E} \sim R_{bounce}$ and P_φ plane is useful to understand orbit topology
- employ the same classification as in White's book or Putvinskii (Rev.Pl.Phys.v.18)
- make use of the common approach:

$$\begin{Bmatrix} P_{\|\boldsymbol{h}} \\ P_{\perp\boldsymbol{h}} \end{Bmatrix} = m_{\boldsymbol{h}} \int \frac{d\mathscr{E}\, d\mu B}{\sqrt{2\mathscr{E} - 2\mu B}} F_{\boldsymbol{h}}\left(P_\varphi, H, \mu\right) \begin{Bmatrix} 2\mathscr{E} - 2\mu B \\ \mu B \end{Bmatrix}$$

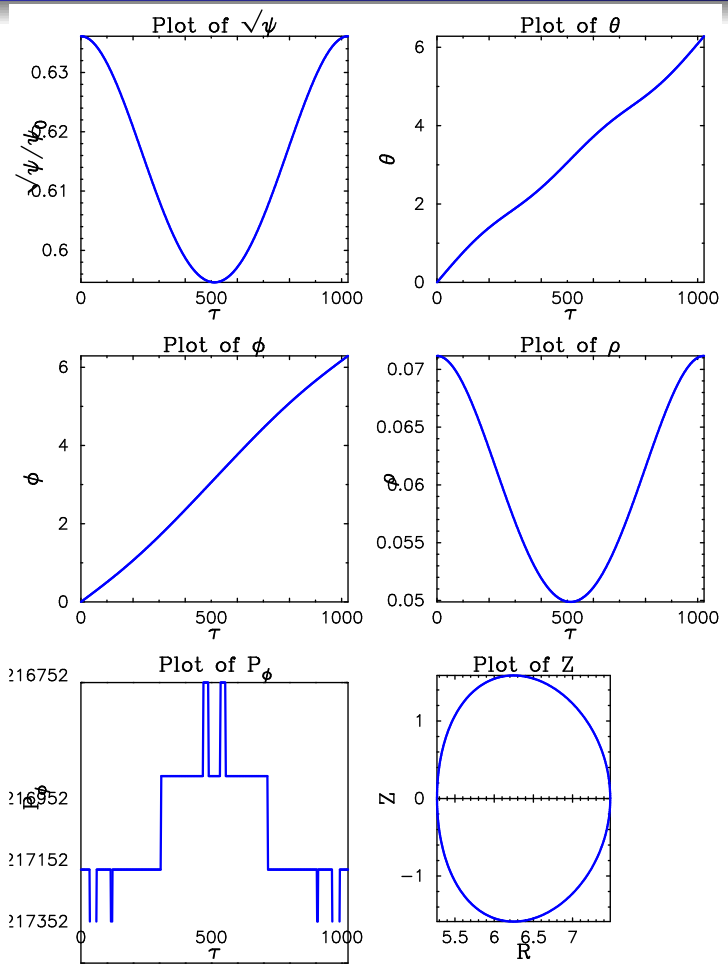
HYM particles (E. Belova)



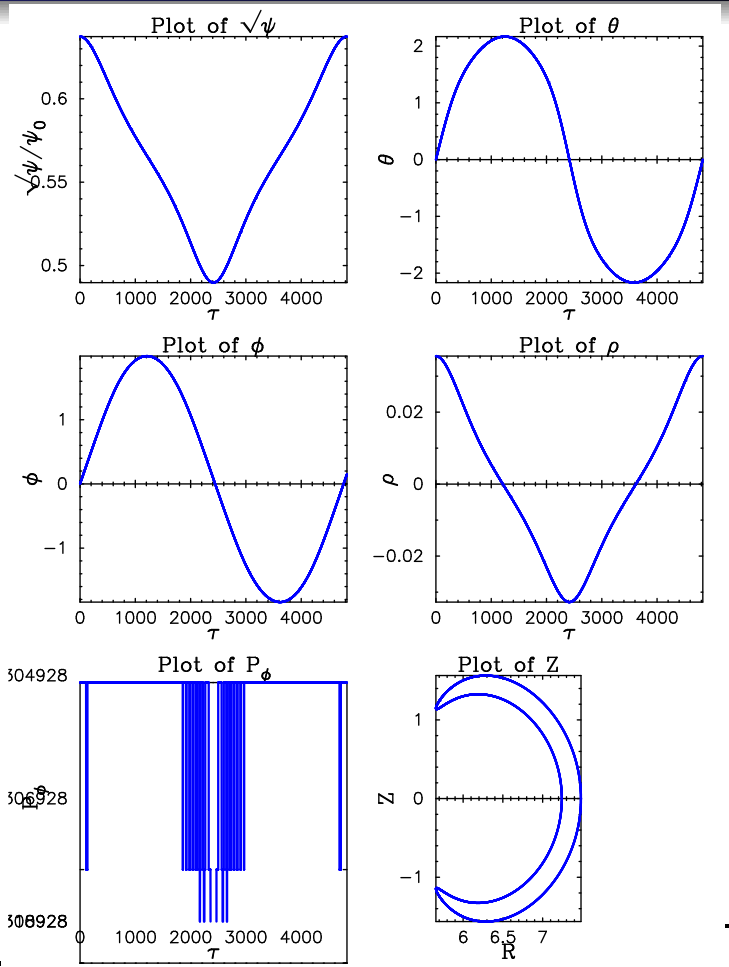


- Two graphs correspond to $v = 2v_A$ and $v = 3v_A$
- "Monte-Carlo" particle loads
- some noise at the boundaries is to avoid

Direct output from NOVA-KN



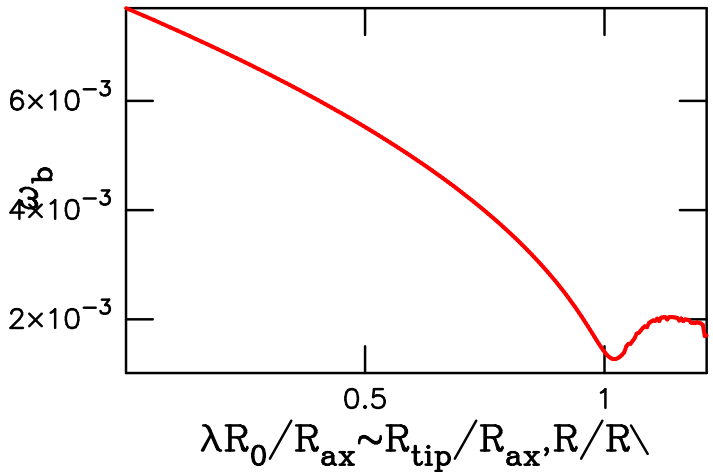
Pass.



Trap.

Orbit integrator is incorporated in the code:
expand orbit via finite elements for easy subsequent solution.

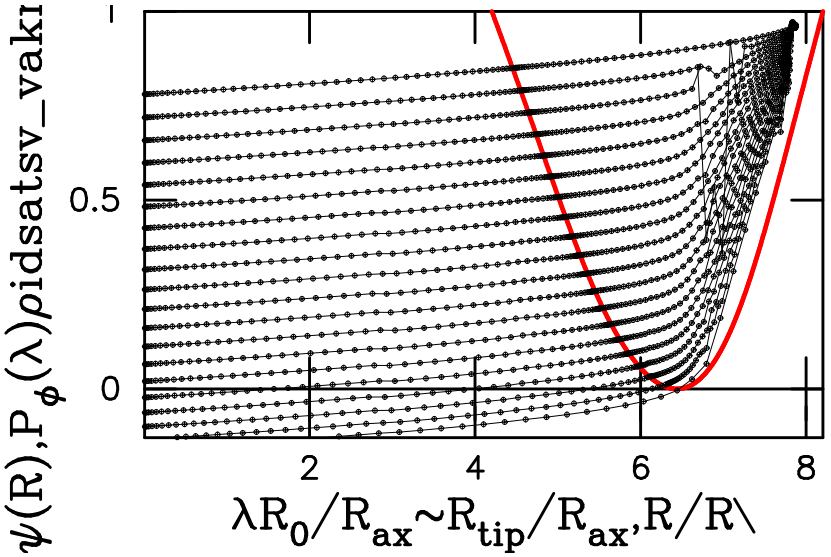
Natural confinement domains are introduced



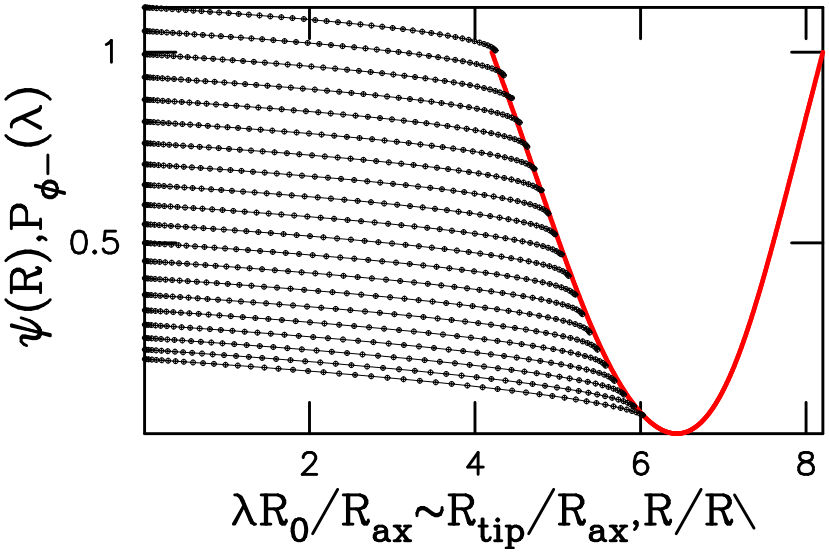
- Two domains: passing and trapped are separated by the boundary with (often) singularity in ω_b
  - have to deal with the integrable singularity at the separatrix $\omega_b \sim \ln|R - R_{sep}|$.

Computing conforming grids for ITER plasma

cogoing/trapped



countergoing



- Grid points are adjusted to the singularities in the transforming Jacobian (ω_b)
  - Non-uniform grid addresses singular point basing on the numerical results.
  - Approaches each boundary with given accuracy ($10^{-5}\psi_0$ in this case)

Summary

- Nonperturbative NOVA-KN code is modified to include α-particles with realistic FOW effects, drift trajectories. α effects are incorporated in the MHD mode equation directly
- Simulations of $n = 1$ internal mode with α-particles is done. Fishbones are addressed with the same technique
- Preliminary results agree with previous simulations. Ideal $n = 1$ mode is stabilized by alphas.
- Fishbone branch is close to being unstable, but is near threshold. Could be destabilized by NBI. Multispecie version is to be developed.
- Further code development with special conforming grid (CG) is presented
- A finite element expansion scheme of the EP pressure is proposed
- A conforming grid mapper to the COM phase space has been developed
- A new equilibrium solver will allow to compute self-consistently EP instabilities, in particular with the strong fast ion pressure